Version MPFS/3

EGT3
ENGINEERING TRIPOS PART IIB

Thursday 1 May 2014 2 to 3:30

Module 4C2

DESIGNING WITH COMPOSITES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Module 4C2 Designing with Composites data sheet (6 pages).
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) In the context of stacking sequences for fibre composite laminates, state briefly what is meant by:

(i) tensile-shear interaction;

(ii) a balanced laminate;

(iii) a symmetric laminate. [15%]

(b) Show that a cross-ply [90/0] laminate, with a ply thickness *t*, does not deform in shear when loaded parallel to one of the plies. [15%]

(c) An epoxy / glass composite is being considered as a replacement for the steel drive shaft of a helicopter rotor. The shaft, which is a tube of outside diameter 100 mm, must sustain an axial tensile load of 100 kN and a torque of 5 kN m. The composite tube is to be made of a $[45/{-45}]_S$ symmetric angle-ply laminate composed of 0.2 mm thick unidirectional glass/epoxy laminae with the following stiffness matrix [*Q*]

$$[Q] = \begin{pmatrix} 15 & 1 & 0 \\ 1 & 10 & 0 \\ 0 & 0 & 5 \end{pmatrix} \text{GPa}$$

(i) Calculate the laminate extensional stiffness [*A*] and coupling stiffness [*B*]. Comment on their form. [35%]

(ii) Determine the strains in the wall of the tube due to the combination of the applied tensile and torque loads. [35%]

2 (a) Briefly describe appropriate tests to assess impact performance of composites. [20%]

(b) Why are porosity and crack-like flaws of concern for composites? Describe a non-destructive testing method to find such flaws. [20%]

(c) Describe with the help of a sketch compression moulding of composites. What would be a typical application and material for this manufacturing process. [20%]

(d) Describe the testing pyramid methodology widely used for aerospace composites applications. What are the pros and cons of such a methodology? [30%]

(e) Why might it be inadvisable to send CFRP tennis racquets in a strung condition on a container ship? [10%]

3 Figure 1 illustrates a laminate made of Scotchply/1002 GFRP composite (properties on the datasheet). The plate has an initial radius of curvature R and is made up of three layers each containing N plies of equal thickness t. The plies in the outer layers are orientated in the 0° direction along the x direction in the plate while the central layer consists of 90° plies running in the y direction. The thickness of the plate is small compared with its in-plane dimensions and radius of curvature, so that it can be modelled using laminate plate theory. The stiffness and compliance matrices [S] and [Q] for each ply are given by

$$[S] = \begin{bmatrix} 0.026 & -0.0067 & 0 \\ -0.0067 & 0.12 & 0 \\ 0 & 0 & 0.24 \end{bmatrix} \text{GPa}^{-1} \text{ and } [Q] = \begin{bmatrix} 39.2 & 2.2 & 0 \\ 2.2 & 8.4 & 0 \\ 0 & 0 & 4.1 \end{bmatrix} \text{GPa}$$

while the elastic stiffness matrices [A] and [D] for the plate are given by

$$[A] = Nt \times \begin{bmatrix} 86.7 & 6.54 & 0 \\ 6.54 & 56.0 & 0 \\ 0 & 0 & 12.42 \end{bmatrix} \text{GPa} \text{ and } [D] = (Nt)^3 \times \begin{bmatrix} 85.6 & 4.9 & 0 \\ 4.9 & 21.4 & 0 \\ 0 & 0 & 9.3 \end{bmatrix} \text{GPa}$$

(a) The plate is subject to bending moments M_x and $M_y = 0.0572M_x$ per unit length as illustrated in Fig. 1. Show that the resulting curvature κ_y is effectively zero. Derive expressions in terms of M_x, N and t for the curvature κ_x of the plate and for the in-plane strains (in the x–y plane of the plate) both at the top and bottom of the top layer of 0° plies (i.e. at locations A and B marked on Fig. 1). [35%]

(b) Derive expressions for the bending moments M_x and $M_y = 0.0572M_x$ associated with in-plane failure in the 0° plies using a maximum stress failure criterion. You may assume that N is large. [30%]

(c) Derive expressions for the through-thickness stresses (i.e. in the z direction) between the central layer and each of two outer layers. [35%]

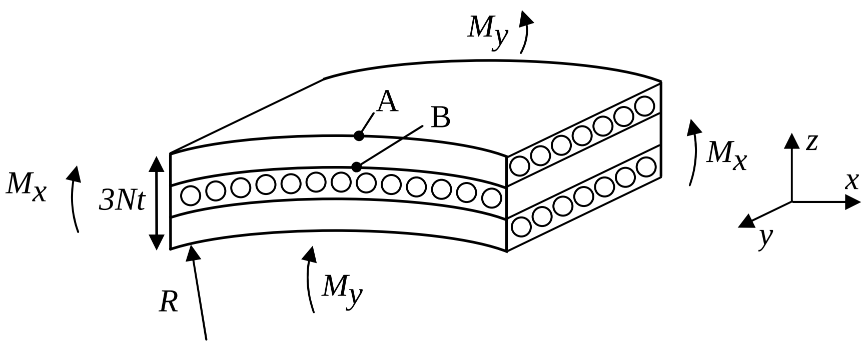

Fig. 1

4 (a) Outline how shear lag theory can be used to model toughness of long fibre composites. [20%]

(b) Figure 2 illustrates the cross section of an idealisation of the joint between a unidirectional CFRP composite propeller blade and a steel hub. The hub is assumed to be rigid, while the composite blade is modelled as a plate of thickness t and axial modulus E embedded a distance L into the hub. The blade is attached to the hub on its upper and lower surfaces by an adhesive, but there is no adhesive on the embedded end face of the blade. The blade depth D (out of the plane of the figure) is large compared with its thickness. A remote tensile load F is applied to the blade. There is relative slip between the hub and blade surfaces which is characterised by a displacement δ which depends on the distance x from the embedded end of the blade as illustrated in Fig. 2.

(i) The adhesive connecting the blade and hub is modelled as rigid-perfectly plastic with shear failure stress τ. Derive an expression for the failure load of the joint. Sketch the variation of displacement δ along the joint both just below the failure load and at 50% of the failure load, deriving expressions for salient points on your curves. [40%]

(ii) Now consider the case where the adhesive behaves in a rigid-perfectly plastic manner as for part (i) up to a critical displacement δ_C, but supports no shear stress for shear displacements greater than δ_C. Sketch the corresponding variation of displacement δ along the joint just below the failure load. [25%]

(iii) What failure modes would you expect in the joint area of the composite blade? [15%]

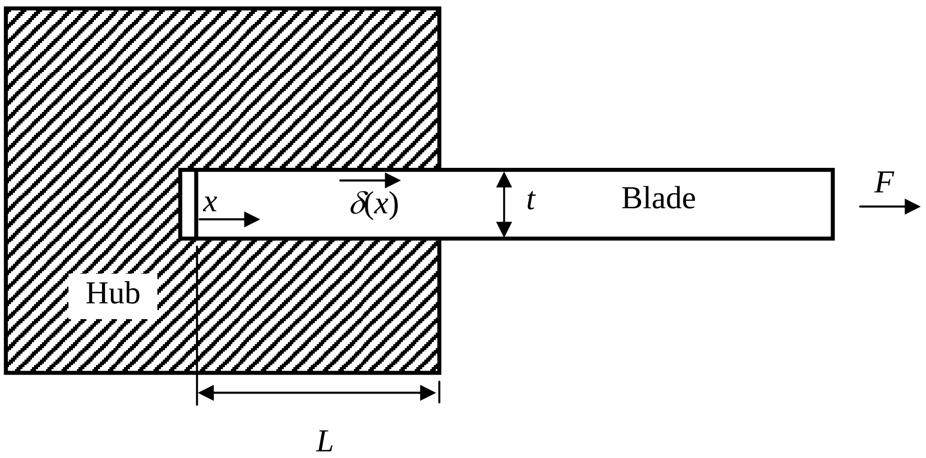

Fig. 2

END OF PAPER

Engineering Tripos Part IIB: Module 4C2
Designing with Composites

Numerical answers - 2013/14

1. (c) (ii) $\varepsilon_x = 0.035$, $\varepsilon_y = -0.0052$, $\gamma_{xy} = 0.069$

3. (a) $\kappa_x = 0.0117 \frac{M_x}{(Nt)^3} \frac{1}{\text{GPa}}$

(b) $M_x = 717 (Nt)^2 \, \text{MPa}$

(c) $\sigma_z = 0.458 \frac{M_x}{NtR} \, \text{GPa}$