1) a) Piezo materials are use in the form of thin sheets meaning that a large field or strain is only seen across the thickness $\sigma_{i,2} + \epsilon_{i,2} \neq 0$

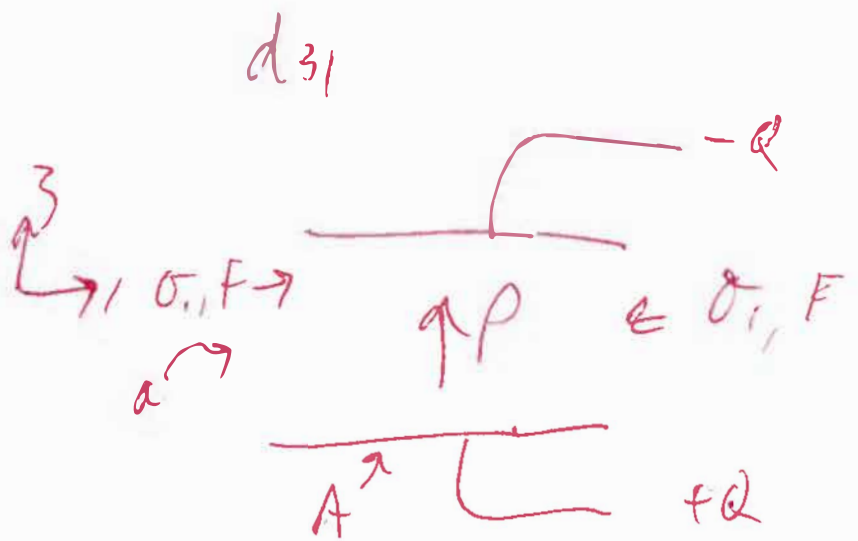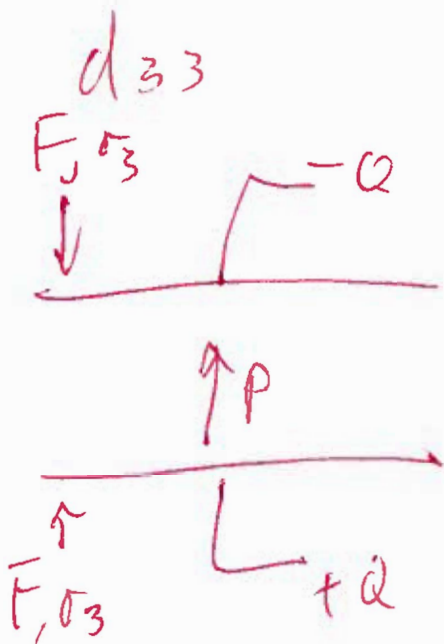In this geometry the only non-zero co-efficients are therefore d_{33}, d_{31} and d_{15}.

d_{33} — thickness mode

d_{31} — longitudinal mode

d_{15} — shear mode

This question was well answered by almost all candidates

(one of the below required)

d_{33}

d_{31}

d_{15}

-Q

$\downarrow \frac{Q}{5}$

+Q

$\hat{1} \rightarrow 3$

6) A ferroelctric material the direction of polarization can be change through the application of a "poling" voltage. This means polycrystalline materials can be used and "poled" in the manufacturing process
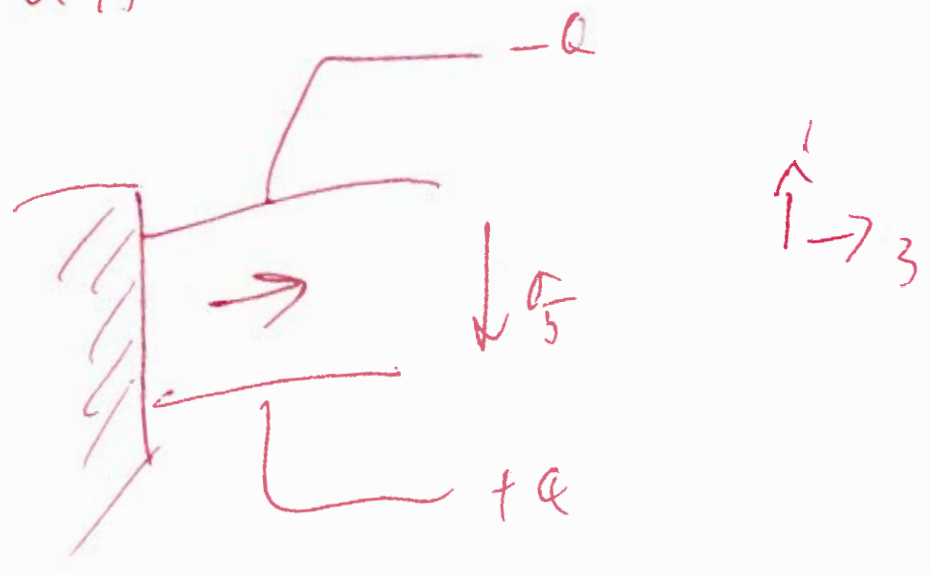
While generally well answered a few candidates waffled and didn't get to the key point that you can pole Ferroelectric materials

c) i) Not shear and as load is on same face as electrodes this is "thickness" mode

A few candidates thought the field was applied horizontally, it is important to look at diagrams carefully

ii) The manufacturer poles the device so the direction of "p" in each layer alternates — so for a given applied voltage the strain in all layers has the same sign

Not that many candidates mentioned the key point that the device only works if the layers are poled in alternating directions
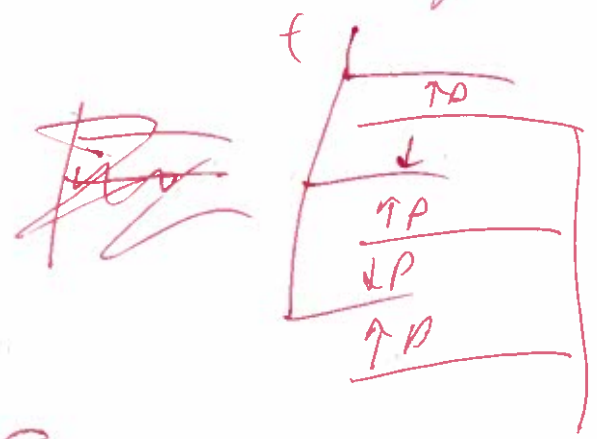
iii) Converse effect $S = dE$ in this

case $S = d_{33} E$ $d_{33} = S/E$

Total strain is $0.5 \mu m$ so strain per element is

$0.05 \mu m$ $\dfrac{0.00005}{0.1} = 0.5$ microstrain / 5×10^{-4}

On one element V is $100V$ so $E = \dfrac{100}{1 \times 10^{-4}} = 1 \times 10^{6}$ V/m

so $d_{33} = \dfrac{5 \times 10^{-4}}{1 \times 10^{6}} = 500 \, pCN^{-1}$ (typical for PZT)

This was well answered, a few silly mistakes were not heavily penalised (such as forgetting that the absolute extension per element is a tenth of the overall extension).

iv) Direct effect $P = d\sigma$ or (multiply by area) ④

$$Q = dF$$

From databook or memory in a capacitor $Q = CV$

Total C is $10_n F$ so (as in parallel) each element

C is $1_n F$

$$CV = dF$$

$$F = \frac{CV}{d}$$

— V on each segment
  = V on total

$$= \frac{1 \times 10^{-9} \times 100}{500 \times 10^{-12}}$$
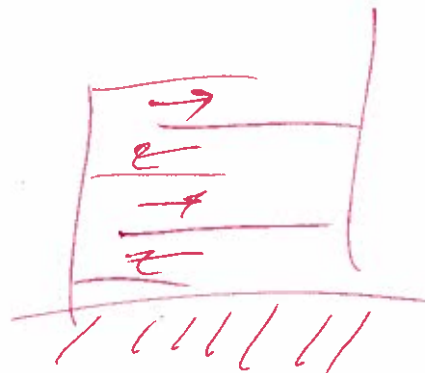
Well answered

$$= 200 \, N$$

v) If poled sideways alternating
it would operate in
shear $(d_{15} = d_{24})$ mode
and give a lateral
displacement

Not many candidates realised that this would be
a shear mode of operation

Q2

(a) Heisenberg's energy-time uncertainty principle:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Thus $\Delta E \approx \hbar/2\Delta t$ with $\Delta E = h\Delta f$
and $\Delta f = 1/(4\pi\Delta t) = 8 \times 10^6$ Hz

$\Delta f/f = 10^{-8}$, the emitted radiation can be considered monochromatic.

(b) (i) Hydrogenic model

$$R = R_0 m^*/\varepsilon_r^2$$

Where $R_0 = 13.6$ eV. Thus lowest acceptor level 0.13 eV above valence band edge. For p doping N must substitutionally replace Se.

(ii) Ionisability of dopant state (compared to 25 meV for kT at room temperature) can be discussed as issue here. Main doping challenge for wide band gap materials arises from so called compensation, where it becomes energetically favourable to create a defect state that compensates the doping effect i.e. where the hole falls into.

(c) The bandgap of black phosphorous decreases for increasing layer number, and this can be rationalised by simple coupled quantum well model with nearest neighbour interlayer hopping for e and h:
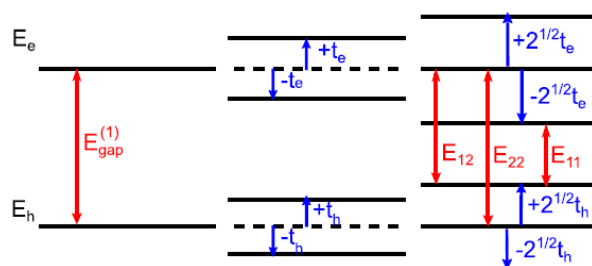

Band diagram for AB stacked bilayer:

1-BP   2-BP

E(eV)

— Tight-binding mode
---- Continuum model

AB stacking
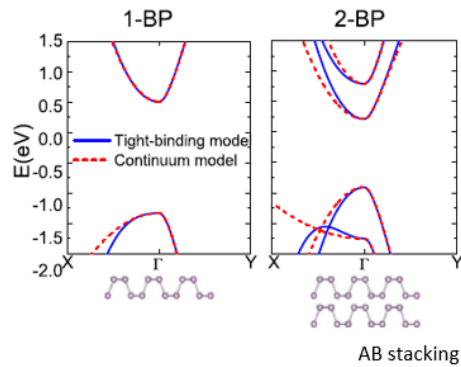
(d) Suitable dielectric would be h-BN, which is insulating, very flat, and has high band gap. Gate contact could be made by monolayer graphene, which gives sufficient conductivity but shows only about 2% absorption in the visible range. As these are all 2D materials, the device stack can be made via exfoliation and stamp transfer. Difficult to grow these materials though with standard techniques like ALD, hence standard materials might be considered but then optical transparency and interfacing to black phosphorous become major challenge. Also carbon nanotubes might be considered as gate contact material.

(e) The absorption features arise from excitonic effects, ie bound electron-hole pairs. Due to the low screening, excitonic binding energies can be large in 2D materials. The bound electron holes states can be modelled by the hydrogenic model, hence the 1s and 2s labels.

c) i) Field arises from current in wire

resistance of copper leads to heating which limits maximum feasible field

In solenoid field from all turns superposes
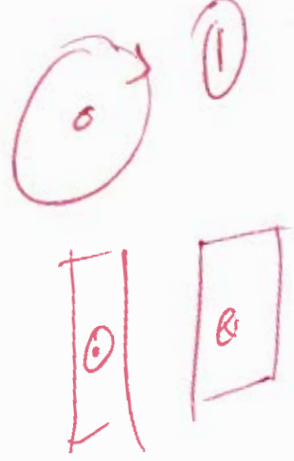
①

ii)

One loop of current flows in disc

limited by J_c of the superconductor

iii) Field arises from spin of electrons in atomic orbitals. There is a maximum number of μ_b avalle' so when all spins aligned maximum magnetisation is reached.
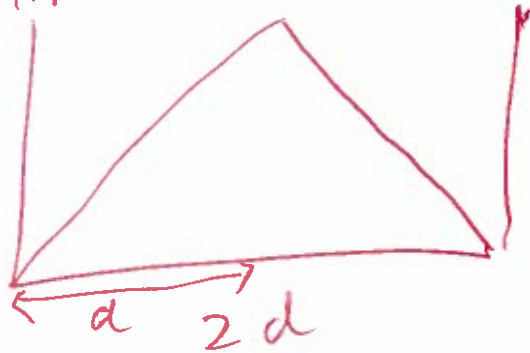
b) i) $\text{curl } H = J + \left(\dfrac{dD}{dt} \right)$ ← no time dependence so = 0

$\text{curl } B = \mu_0 J$

in a thin slab can neglect as $\dfrac{dB}{dy} = 0$ and $\dfrac{dB}{dz} = 0$

$B_x = B_3 = 0$

$\text{curl} B = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & 0 & 0 \\ & & B_2 \end{vmatrix}$

So $\dfrac{dB}{dx} = \mu_0 J_c$

so slope gien of field given by

$\mu_0 J_c$

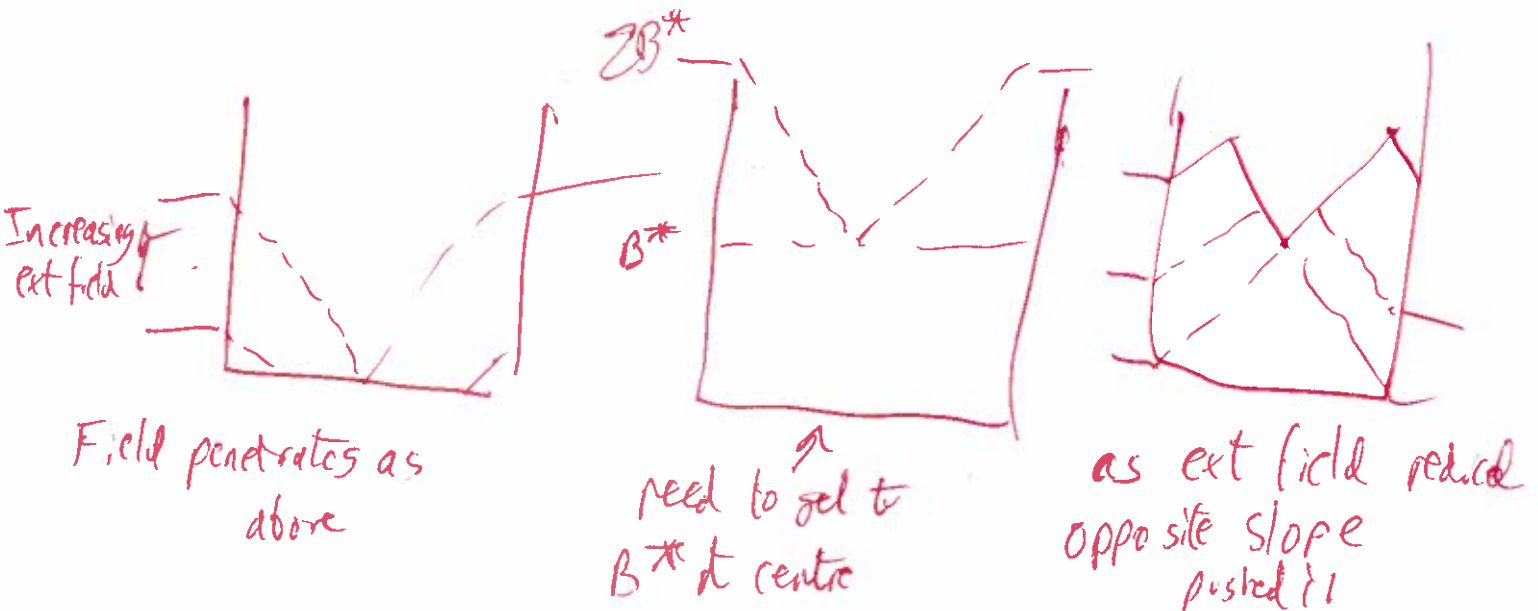Max trapped field corresponds to:

d $\quad$ $2d$

Some candidates did not look carefully at the
diagram and were out by a factor of 2

So $B^* = \mu_0 J_c d$

ii) Bean model tells us that $J = +J_c, -J_c$ or 0

so slope of $\dfrac{dB}{dx}$ is $\pm \mu_0 J$ or 0

$2B^*$

B^*

Increasing
ext field

Field penetrates as
above

need to get to
B^* at centre

as ext field reduced
opposite slope
pushed in

So need to apply $2B^*$ to get
full magnetisation

(3)

c) i) From definition $\vec{M} = \dfrac{\vec{m}}{V}$

$\vec{m} = 2.5 \times 10^{-1} \ Am^2$

$V = 0.001 \times 0.01 \times 0.02 = 2 \times 10^{-7} \ m^3$

so $\vec{M} = \dfrac{2.5 \times 10^{-1}}{2 \times 10^{-7}} = 1.25 \times 10^{6} \ A/m$

$(\mu_0 \vec{M}) = 1.57 \ T$

$M = \dfrac{(\mu_0 M)}{\mu_0}$

ii) as field distribution must be

as fully charged $B^* = 2 \times$ or

so $B^* = \qquad 3.14 \ T$

iii) Thickness = 1 mm = $2d$

½ thickness = 0.5 mm = d

So as $B^* = \mu_0 J_c d$

$$J_c = \frac{B^*}{\mu_0 d}$$

$$= \frac{3.14}{1.257 \times 10^{-6} \times 0.0005}$$

$$= \frac{3.14}{6.285 \times 10^{-10}}$$

$$= 5 \times 10^{9} \text{ A/m}^2$$

$$or \; 5 \times 10^{5} \text{ A/cm}^2$$

As this leads directly from an earlier question this was generally well answered. Candidates were not penalised twice where error arose in the earlier question.

Most candidates showed good understanding of effective mass and amorphous semiconductor properties. Not many though realised that mobility gap is larger than Egap of crystalline semiconductor. Sketching of optical absorption was bit scattered, lacking connection to theory. Candidates showed good knowledge on material and device structures in final part, although some answers lacked reasoning and detail.

Q4

(a) (i) The curvature of the extremities of the valence and conduction bands is captured by the effective masses m*. Assuming parabolic band dispersion:

$$m^* = \hbar^2 \left(\frac{d^2 E}{dk^2} \right)^{-1}$$

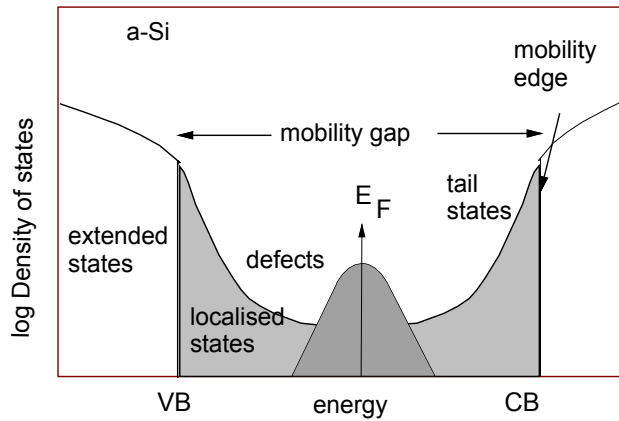In a simple kinetic model the carrier mobility is given by

$$\mu = e\tau/m^*$$

Hence a larger curvature (smaller radius of curvature) is reflected by a smaller m* and thus larger mobility. Thus the electron mobility can be expected to be higher.

(ii) The band gap is indirect but absorption will start to rise at Eg = 1.1 eV and then get stronger for direct transitions at higher energy.
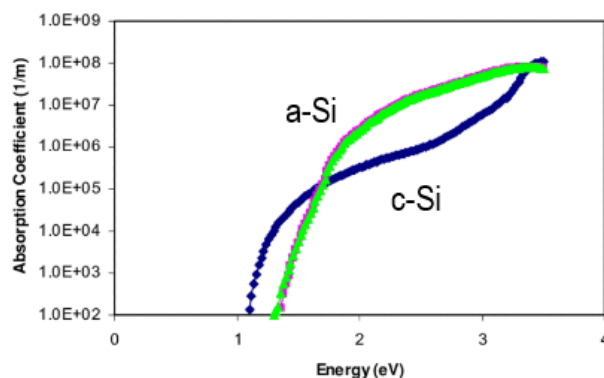
(iii)

For amorphous film the abrupt band edges of a crystal are replaced by a broadened tail of states extending into the forbidden gap. These tail states are localised, in contrast to the extended states in the conduction and valence bands. The so called mobility gap is larger than the crystalline Eg. Taking example of a-Si, the mobility gap is 1.7 eV.

(iv) see sketch above. Lower onset of absorption for a-Si but stronger absorption at higher energy.

(b) PV applications demand high absorption matched to the spectrum of solar radiation. For single junction device band gap around 1.4 eV is desirable. The band gap should be direct and ideally tunable. Superior materials could be GaAs or hybrid perovskites like MAPbI3. The latter offer bandgap tunability via the X and A site of the ABX3 lattice, and show very strong optical absorption, defect tolerance and charge transport properties. They are also easy to produce over large areas.

(c) Emission of white light needs direct bandgap of around 3eV. The bandgap of Fig.1 is indirect and too small. A suitable better material would be GaN. White light can be created by (1) using 3 different LEDs, (2) down-converting from a blue source, or (3) down converting from a UV source. Option (2) is cheapest and widely used.