EGT3
ENGINEERING TRIPOS PART IIB

Wednesday 30 April 2014 2 to 3.30

Module 4C6

ADVANCED LINEAR VIBRATIONS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C6 Advanced Linear Vibration data sheet (10 pages).
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 An instrumented impulse hammer is to be used for modal testing. The total mass of the hammer head is m and the stiffness of the hammer tip is k.

(a) Sketch a typical instrumented hammer and identify its principal parts. [10%]

(b) The impulse-force wave form can be idealised by the function

$$f(t) = F_0 \cos(\Omega t) \;, \qquad -\frac{\pi}{2\Omega} \le t < \frac{\pi}{2\Omega}.$$

(i) Sketch the impulse wave form $f(t)$ and explain how the duration of the impulse b is influenced by m and k. [20%]

(ii) Using the definition of the Fourier transform

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t) e^{-i\omega t}\, dt$$

show that the spectrum of the impulse is

$$F(\omega) = B\frac{\cos\dfrac{\pi\omega}{2\Omega}}{\Omega^2 - \omega^2}$$

and determine the value of B in terms of F_0 and Ω. [40%]

(c) Sketch the impulse spectrum $F(\omega)$ showing clearly the significance of the impulse duration b and the distribution of zeros. With reference to your sketch explain the "rule of thumb" that an impulse does not excite vibration at frequencies (in Hz) above $1/b$. [30%]

2 A pinned-pinned beam of length L has mass-per-unit-length m and bending-stiffness EI. It undergoes bending vibration, and the mode shapes can be assumed to be

$$u_n(x) = \sin\frac{n\pi x}{L}, \qquad n = 1,2,3...$$

where x is the distance from one end of the beam.

(a) Using energy expressions from the Data Sheet, obtain an expression for the corresponding natural frequencies ω_n by Rayleigh's principle. [10%]

(b) Explain briefly how Rayleigh's principle can be used to estimate modal damping factors for a lightly-damped system with material damping represented via complex elastic moduli. [10%]

(c) A partial damping treatment is applied to the beam so that over part of the length the Young's modulus becomes $E(1+i\eta)$ where η is a constant that can be assumed to be small. The damping treatment does not change the mass or stiffness of the beam. If the complex natural frequencies are $\bar{\omega}_n^2$ such that

$$\bar{\omega}_n^2 = \omega_n^2(1 + i\eta_n)$$

obtain an approximate expression for the modal damping factor η_n in terms of an integral over the region of the beam where the damping treatment has been applied. [40%]

(d) The total length of the damping treatment is limited to $L/10$. With the aid of sketches, discuss how the damping of the first *three* modes of the beam would be influenced by the placement of this short damping patch. Where should the damping treatment be placed in order to have a good effect on *all three* of these modes. [40%]

3 (a) A laminated leaf spring, as commonly used in heavy vehicle suspension systems, is sketched in Fig. 1. A stack of steel beams is pinned together at the centre, where the wheel load *F* is applied, while the ends of the spring are connected to the chassis. List as many mechanisms as you can think of, by which energy might be dissipated during dynamic loading or transient vibration of this compound spring. For each mechanism explain carefully how the dissipation occurs, using sketches where appropriate. [40%]

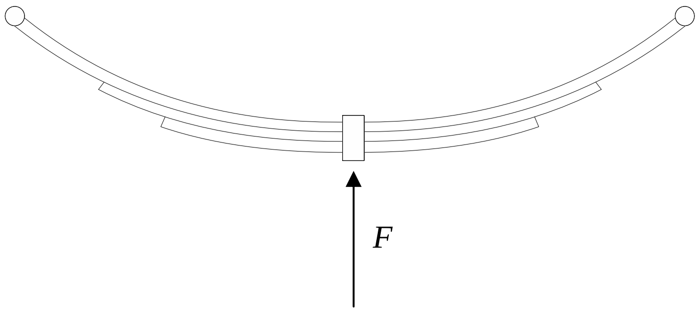

Fig. 1

(b) (i) Describe in qualitative terms how a Helmholtz resonator works, including the concept of "end correction". What factors influence the resonant frequency? Explain how a Helmholtz resonance is different from a standing-wave resonance as in an organ pipe. [35%]

(ii) Using information from the Data Sheet, calculate an approximate value for the Helmholtz resonance frequency of an empty 2-litre plastic bottle as used for carbonated drinks, estimating the relevant geometrical dimensions and stating your assumptions. Assume that the speed of sound in air is 340ms^{-1}. [25%]

4 (a) A stretched string with tension P and mass per unit length m can undergo small transverse vibration with displacement $w(x,t)$ where x is distance along the string and t is time. A point mass M is attached to the string at the point $x = a$. Show that the condition

$$M \left.\frac{\partial^2 w}{\partial t^2}\right|_{x=a} = P\left\{ \left.\frac{\partial w}{\partial x}\right|_{x=a+} - \left.\frac{\partial w}{\partial x}\right|_{x=a-} \right\}$$

must be satisfied, and give the second boundary condition that applies at $x = a$. [10%]

(b) A stretched string as in part (a) has length $2L$, and the mass M is attached at its mid-point. The two ends of the string are fixed to rigid supports. Use the governing equation from the Data Sheet together with the various boundary conditions to obtain one or more equations that determine the natural frequencies of the weighted string. You may find it helpful to consider symmetric and antisymmetric modes separately. [40%]

(c) Explain carefully what the interlacing theorem has to say about the natural frequencies for this problem. Using a graphical approach, show that the results of part (b) are consistent with the interlacing theorem, whatever the value of M may be. Discuss the limiting cases of very small and very large M. [40%]

(d) Without detailed calculations, explain how the general pattern of behaviour of the natural frequencies would change if the mass was not attached at the mid-point but rather at some other position on the string. [10%]

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