EGT3
ENGINEERING TRIPOS PART IIB

Friday 2 May 2014 9.30 to 11:00

Module 4C7

RANDOM AND NON-LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Candidates may bring their notebooks to the examination
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A structural member in part of an off-shore platform is stressed by pounding waves and the stress $s(t)$ can be approximated by a narrow-band random process with mean–square spectral density $S_S(\omega)$ as shown in the Fig. 1.

(a) Calculate the mean stress and the standard deviation of the stress, and estimate the standard deviation of $\dot{s} = ds/dt$. [10%]

(b) Assuming that s and $\dot{s}$ are independent Gaussian random processes, calculate the frequency of positive–slope crossing of the level $s = a$. You may assume:

$\nu_a^+ = \int_0^\infty \dot{s}\, p(a,\dot{s})\, d\dot{s}$, where $p(a,\dot{s})$ is the joint probability distribution. [20%]

(c) Show that the probability distribution of the peaks in stress is

$$p_p(s) = \frac{s}{2S_0\Delta\omega}\exp\left(-\frac{s^2}{4S_0\Delta\omega}\right).$$ [20%]

(d) What is the expected number of cycles with peaks in the range s to $s + ds$ during time T? [20%]

(e) If the number of cycles to failure N_f at stress level s is given by

$$N_f(s) = Cs^{-k}$$

where C and k are constants, determine an integral expression for the expected life of the member and evaluate it for $k = 1$. [30%]

You may assume that Miner's Rule applies, ie failure occurs when $\sum \frac{n(s)}{N_f(s)} = 1$,

where $n(s)$ is the number of stress cycles at level s.

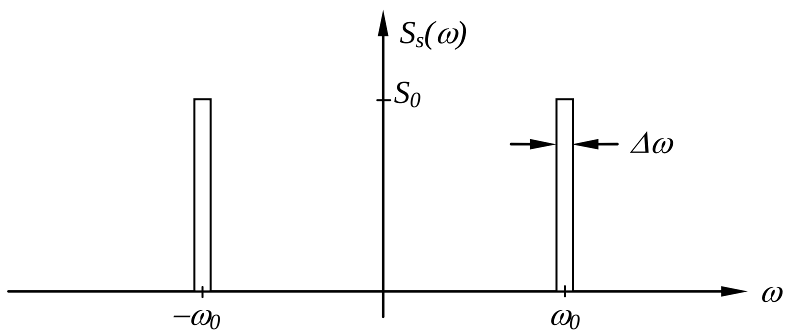

Fig. 1

2 The dynamic system in Fig. 2 comprises a massless trolley with horizontal displacement $y(t)$, which is excited by an ergodic, Gaussian random force $f(t)$. The trolley is attached to a rigid support by two springs of stiffness k_1 and k_2 and a linear viscous dashpot of damping coefficient λ.

(a) The autocorrelation function of $f(t)$ is given by

$$R_f(\tau) = Ae^{-b|\tau|}$$

(i) Determine the probability that $f(t)$ exceeds $2\sqrt{A}$. [20%]

(ii) Show that the mean square spectral density of f is given by

$$S_f(\omega) = \frac{S_0}{1+(\omega/\omega_0)^2}$$

and give the values of S_0 and ω_0. [40%]

(b) Determine the mean square displacement of the trolley when $f(t)$ is a stationary random signal with mean–square spectral density of the form given in (a). [40%]

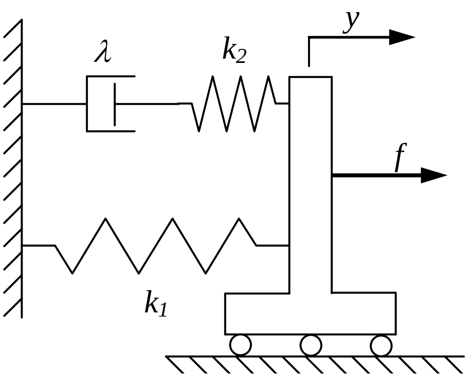

Fig. 2

3 Consider the system shown in Fig. 3 where a mass m slides on a smooth surface restrained by a linear spring of stiffness k, and unstretched length $\hat{\ell}$, anchored a distance ℓ below the mass.

(a) Derive the equation of motion for this system for free oscillations about x=0. [10%]

(b) Show that the equation of motion can be written in the following form:

$$\ddot{u} + 2u(1+u^2)^{-1/2}[(u^2+1)^{1/2} - \lambda] = 0$$

[10%]

where the new independent variable $\tau = \omega t$ and u, λ and ω are appropriately expressed in terms of the original system parameters. What is the physical significance of λ=1? [30%]

(c) Expand the equation of motion above for small u retaining only the first higher order non-linear term for the case λ=1. [20%]

(d) The system is driven by an external force excitation $f(\tau) = a\cos\tau$. Use the method of harmonic balance to search for a solution of the form:

$$u = A\cos\tau + B\cos 3\tau$$

Derive (but do not solve) suitable equations for the constants A, B in terms of the excitation amplitude a. [30%]

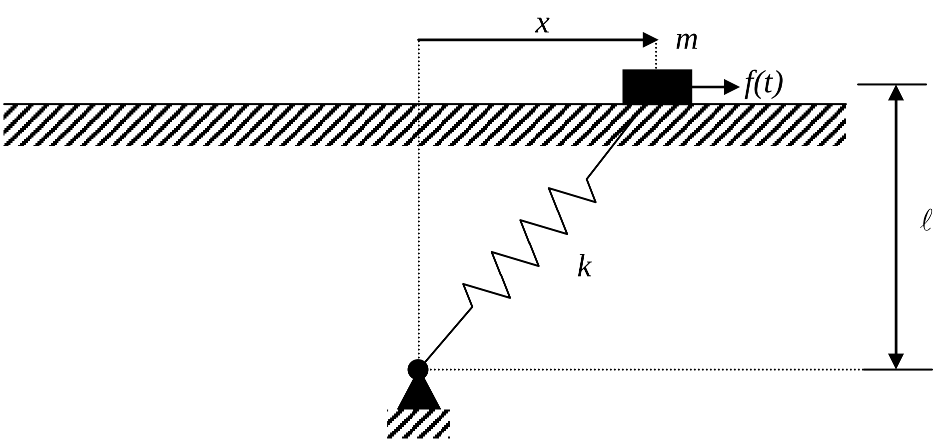

Fig. 3

4 A nonlinear vibratory system is described by the equation:

$$\ddot{x} - \mu\dot{x} - x\dot{x} - x + x^2 = 0$$

such that the real parameter $\mu < 0$.

(a) Identify the singular points of the system. [20%]

(b) Determine the type and stability of each singular point. Comment on the nature of the singular points as μ is varied. [40%]

(c) Sketch the behaviour of the system in the phase plane for $-1 < \mu < 0$. [40%]

END OF PAPER

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