**4C7 2009 Answers**

1. (a) $H_{y\eta}(\omega) = \alpha\omega - ib\beta/\omega$

(b) $\sigma_{\dot{y}}^2 = (S_0\alpha^2/7)(\omega_2^7 - \omega_1^7) + (S_0 b^2\beta^2/3)(\omega_2^3 - \omega_1^3)$

$\sigma_{\ddot{y}}^2 = (S_0\alpha^2/9)(\omega_2^9 - \omega_1^9) + (S_0 b^2\beta^2/5)(\omega_2^5 - \omega_1^5)$

(c) $P \approx 0.1$

2. (a) $S_{xx}(\omega) = \dfrac{(k_2/M)^2 S_{yy}(\omega)}{(\omega_n^2 - \omega^2)^2 + (2\beta\omega_n\omega)^2}, \quad \omega_n^2 = (k_1 + k_2)/M, \quad 2\beta\omega_n = C/M.$

(b) $\sigma_x^2 = \dfrac{\pi k_2^2 S_0}{C(k_1 + k_2)}, \quad \sigma_{\dot{x}}^2 = \dfrac{\pi k_2^2 S_0}{MC}.$

(d) $\mathrm{E}[P] = \dfrac{\pi k_2^2 S_0}{M}$

3. (b) There is a pitchfork bifurcation.

(c) The response has frequency components:
$\Omega_1, \Omega_2, 2\Omega_1 \pm \Omega_2, 2\Omega_2 \pm \Omega_1, 3\Omega_1, 3\Omega_2$.

4. (a),(b) Stable spiral at $x = \dot{x} = 0$.

(c) For $0 < \dot{x} < \pi$ the damping is positive and the amplitude decays; for $\pi < \dot{x} < 2\pi$ the damping is negative and the amplitude grows; for $2\pi < \dot{x} < 3\pi$ the damping is positive and the amplitude decays, etc. This gives an unstable limit cycle at radius $\pi$ around the origin; a stable limit cycle at $2\pi$ around the origin, etc.