# 4C7 2012 RANDOM AND NONLINEAR VIBRATIONS

## ANSWERS

1. (c) $\nu = \left(\frac{1}{2\pi}\right)\left(\frac{\sigma_a}{\sigma_b}\right)\left\{\exp\left[-\frac{(\mu N_0)^2}{2(1-\mu\gamma)^2\sigma_a^2}\right]+\exp\left[-\frac{(\mu N_0)^2}{2(1+\mu\gamma)^2\sigma_a^2}\right]\right\}$

2. (a) $S_{xx}(\omega) = \frac{S_0}{(\omega_n^2-\omega^2)^2+(2\beta_n\omega_n\omega)^2}$

(b) $S_{pp}(\omega) = \frac{\alpha^2\omega^4 S_{xx}(\omega)}{(\omega_a^2-\omega^2)^2+(2\beta_a\omega_a\omega)^2}$

(d) $\sigma_p^2 = \left(\frac{\pi\alpha^2\omega_n S_0}{4\beta_n}\right)\left[(\omega_n^2-\omega_a^2)^2+(2\beta_a\omega_a\omega_n)^2\right]^{-1}$

(e) $\sigma_p^2 = \left(\frac{\pi\alpha^2 S_0}{4}\right)\left(\frac{\omega_a}{\beta_a}+\frac{\omega_n}{\beta_n}\right)\left(\frac{1}{\omega_n^2-\omega_a^2}\right)^2$

3. (b) $DF = \frac{2s}{\pi}\left[\frac{\pi}{2}-\cos^{-1}\left(\frac{b}{\alpha}\right)+\left(\frac{b}{\alpha}\right)\sqrt{1-\frac{b^2}{\alpha^2}}\right]$ (for $\alpha > b$)

(c) $-m\omega^2\alpha+\frac{2s\alpha}{\pi}\left[\frac{\pi}{2}-\cos^{-1}\left(\frac{b}{\alpha}\right)+\left(\frac{b}{\alpha}\right)\sqrt{1-\frac{b^2}{\alpha^2}}\right] = f$ (for $\alpha > b$)

4. (a) $\dot{x} = x = 0$, and for $\alpha > 0$ the two additional points $\dot{x} = 0$, $x = \pm\sqrt{\alpha}$

(b) For the point $\dot{x} = x = 0$:

$\alpha > 0$ saddle point

$-\zeta^2 < \alpha < 0$ stable node

$\alpha < -\zeta^2$ stable focus

For the points $\dot{x} = 0$, $x = \pm\sqrt{\alpha}$ (note that $\alpha > 0$):

$\alpha < \zeta^2/2$ stable node

$\alpha > \zeta^2/2$ stable focus