1 a) i)

N2L for m_s

$$m_s \ddot{z}_s + c\dot{z}_s + kz_s = c\dot{z}_r + kz_r.$$

Laplace transform, $s = j\omega$.

$$\left\{ (j\omega)^2 m_s + (j\omega)c + k \right\} Z_s(j\omega) = \left\{ (j\omega)c + k \right\} Z_r(j\omega)$$

now
$$\frac{Z_s(j\omega) - Z_r(j\omega)}{\dot{Z}_r(j\omega)} = \frac{Z_s(j\omega) - Z_r(j\omega)}{j\omega \, Z_r(j\omega)}$$

$$= \frac{\cancel{(j\omega)c} + \cancel{k} - \left\{ (j\omega)^2 m_s + \cancel{(j\omega)c} + \cancel{k} \right\}}{\cancel{j\omega}\left\{ (j\omega)^2 m_s + (j\omega)c + k \right)}$$

$$= \frac{-(j\omega)m}{(j\omega)^2 m + (j\omega)c + k}$$

compare with $H_2(j\omega) = \dfrac{(j\omega)B_1 + B_0}{(j\omega)^2 A_2 + (j\omega)A_1 + A_0}$

$\therefore B_0 = 0$; $B_1 = -m$; $A_0 = k$; $A_1 = c$; $A_2 = m$

$$\therefore \int_{-\infty}^{\infty} |H_2(j\omega)|^2 d\omega = \frac{\pi(A_0 B_1^2 + A_2 B_0^2)}{A_0 A_1 A_2} = \frac{\pi \cancel{k} m^{\cancel{2}}}{\cancel{k} c \cancel{m}} = \frac{\pi m}{c}$$

$$\therefore E\left[(z_s - z_r)^2\right] = \frac{\pi m}{c} \cdot S_0 .$$

1 a) ii)

for sprung mass acceleration, transfer function

is $$\frac{\ddot{Z}_s(j\omega)}{\dot{Z}_r(j\omega)} = \frac{(j\omega)^2 Z_s(j\omega)}{(j\omega)\, Z_r(j\omega)} = \frac{(j\omega) Z_s(j\omega)}{Z_r(j\omega)}$$

$$= \frac{(j\omega)^2 c + (j\omega) k}{(j\omega)^2 m + (j\omega) c + k}$$

but $$\left|\frac{\ddot{Z}_2(j\omega)}{\dot{Z}_r(j\omega)}\right| \to \infty \quad \text{as } \omega \to \infty$$

Therefore mean square will be ∞ for white noise $\dot{Z}_r$

Including the unsprung mass ensures transfer function tends to zero as $\omega \to \infty$.

b) i) let $m_s + m_u = M$

$$E\left[\left(k_t(z_r - z_u)\right)^2\right] = \frac{\pi S_0 \left\{M\left[M^2k^2 + k_t\left(Mc^2 - 2k m_u m_s\right)\right] + m_u m_s^2 k_t^2\right\}}{m_s^2 c}$$

$$\frac{\partial E\left[\left(k_t(z_r - z_u)\right)^2\right]}{\partial k} = \frac{\pi S_0 \left\{M\left[2M^2k + k_t\left(-2m_u m_s\right)\right]\right\}}{m_s^2 c}$$

for turning point set this to zero

$$2M^2 k = 2k_t m_u m_s$$

$$k = \frac{k_t m_u m_s}{(m_u + m_s)^2} = \frac{200.10^3 . 45 . 400}{445^2}$$

$$\underline{\underline{k = 18.18 \text{ kN/m}}}$$

b) ii) find value of c for minimum RMS spring mass accn.

$$E\left[\ddot{z}_s^2\right] = \frac{\pi S_0 \left[Mk^2 + k_t c^2\right]}{m_s^2 c}$$

$$\frac{dE\left[\ddot{z}_s^2\right]}{dc} = \pi S_0 \left\{\frac{m_s^2 c . 2k_t c - \left(Mk^2 + k_t c^2\right) m_s^2}{\left(m_s^2 c\right)^2}\right\}$$

equate to zero.

$$2 \cancel{m_s^2} c^2 k_t = \left(Mk^2 + k_t c^2\right) \cancel{m_s^2}$$

$$2k_t c^2 = Mk^2 + \cancel{k_t c^2}$$

$$c^2 = (m_s + m_u)\frac{k^2}{k_t^2} = \frac{445 . \left(18.18 . 10^3\right)^2}{\left(200 . 10^3\right)}$$

$$\underline{\underline{c_{min} = 858 \text{ Ns/m}}}$$

There is no benefit in setting c lower than this: working space and spring mass accn. will increase

What is the maximum value of c?

In practice, for higher values of c the working space is dominated by steady state deflection of k due to step forces on the sprung mass arising from cornering or braking.

Use the 1 dof model (Fig 1a) and choose c to give damping ratio $\zeta = 1$. This will be the minimum value of c needed to prevent overshoot of the steady state displacement.

From mechanics data book:

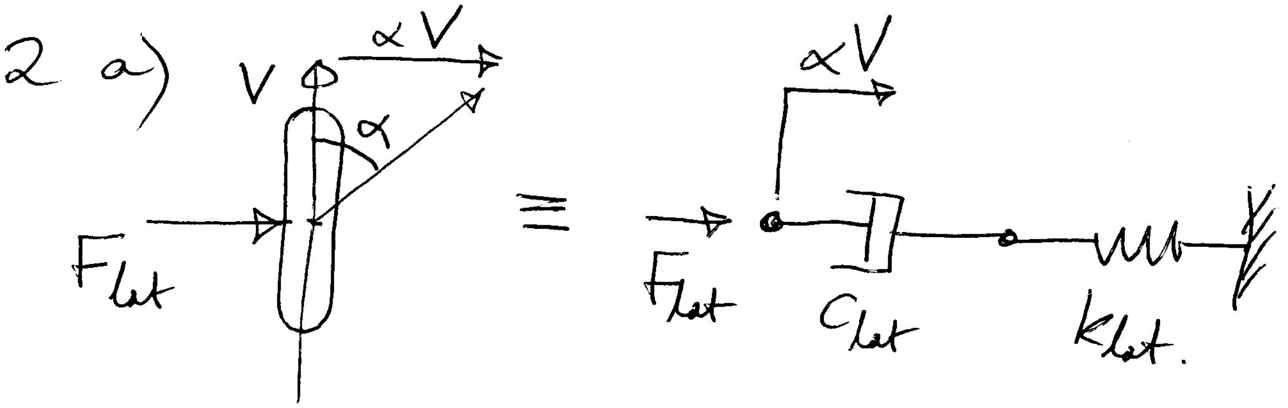
$$\frac{2\zeta}{\omega_n} = \frac{c}{k} \qquad \text{where } \omega_n = \sqrt{\frac{k}{m_s}}$$

$$\therefore c = 2\zeta\sqrt{k m_s}$$

$$= 2\sqrt{18{\cdot}18.10^3.400}$$

$$c_{max} = 5393 \text{ Ns/m}$$

2 a)



Steady state lateral tyre force $F_{lat} = \alpha C$

equivalent viscous damping $C_{lat} = \dfrac{F_{lat}}{\alpha V}$

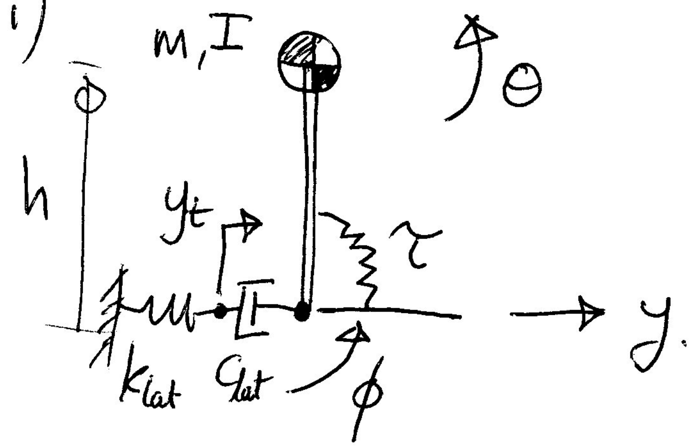
hence $C_{lat} = \dfrac{\alpha C}{\alpha V} = \dfrac{C}{V}$

b) The left and right road displacements Z_L and Z_R can be defined as the sum of the average vertical displacement Z_V and an <u>uncorrelated</u> roll displacement $\pm Z_\phi$.

The equations of motion of the roll-plane model can be decoupled into a 2 dof vertical response model (with Z_V input) and a 4 dof lateral-roll model (with Z_ϕ input).

Thus the vertical and lateral-roll responses are uncorrelated and can be calculated separately.

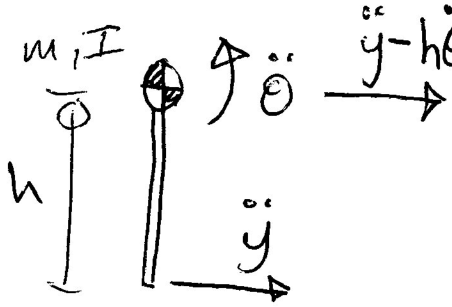
c) i)



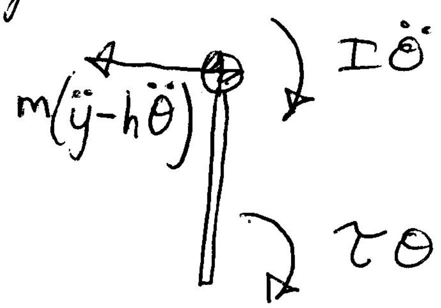
$m = m_s + m_u$

ii) very high speed: $c_{lat} \to 0$.

accns:



forces:



(neglect vertical forces)

Moments about centre of mass:

$$I\ddot{\theta} + \tau\theta = 0.$$

Sum horizontal forces:

$$m(\ddot{y} - h\ddot{\theta}) = 0$$

matrix form. free vibration.

$$\begin{bmatrix} I & 0 \\ -hm & m \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} \tau & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \theta \\ y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Laplace and $s = j\omega$.

$$\left\{ -\omega^2 \begin{bmatrix} I & 0 \\ -hm & m \end{bmatrix} + \begin{bmatrix} \tau & 0 \\ 0 & 0 \end{bmatrix} \right\} \begin{Bmatrix} \theta(j\omega) \\ y(j\omega) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

non-trivial solutions when $\begin{vmatrix} \tau - I\omega^2 & 0 \\ \omega^2 hm & -\omega^2 m \end{vmatrix} = 0.$

$$(\tau - \omega^2 I)(-\omega^2 m) = 0$$

$$-\omega^2 m \tau + \omega^4 I m = 0$$

$$\underline{\underline{\omega^4 I - \omega^2 \tau = 0}}$$

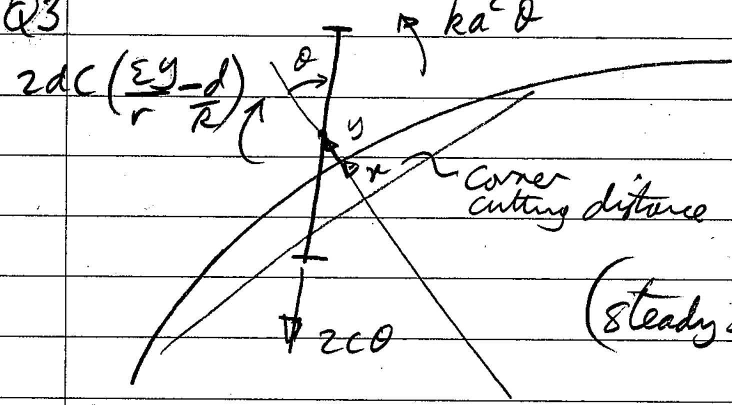
$\therefore \omega^2 = 0$ or $\omega = \sqrt{\frac{\tau}{I}}$.

(rigid body mode)

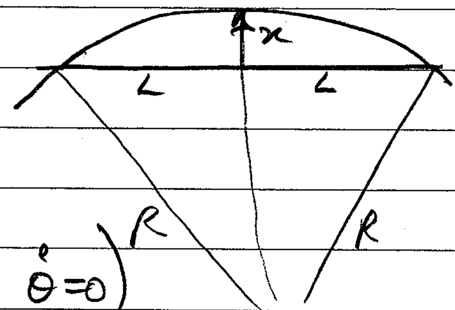
d)

- vertical vibration response (z_s, z_u) is uncoupled to lateral-roll motion (y, θ) hence unaffected by the no-roll aspect of the suspension.
- the effective damping provided by the tyre is low at low vehicle speeds ($C_{lat} \to \infty$) and low at high speeds ($C_{lat} \to 0$)
- in the likely frequency range of the lateral-roll modes of vibration, roll excitation and vertical excitation from the road will tend to contribute equally at low speeds (Z_L and Z_R uncorrelated). Vertical excitation will dominate at high speeds (Z_L and Z_R correlated)
- considering the combined effect of the tyre damping and road excitation dependencies on speed, it is likely that the no-roll suspension will be more satisfactory at high vehicle speeds than low speeds.

Q3



$\left(\text{steady state } \dot{\theta}=0\right)$



$$x = R-\sqrt{R^2-L^2} \approx \frac{L^2}{2R} \quad \text{—} (1)$$

(a)(i) $\sum F: \; k(x+y)+2c\theta = 0$ — (2)

$\circlearrowleft \sum M: \; ka^2\theta - 2dC\left(\frac{\varepsilon y}{r}-d/R\right)=0$ — (3)

Combine (1), (2) & (3) to eliminate θ:

$$k\left(L^2/2R+y\right)+2c\left[\frac{2dC}{ka^2}\left(\frac{\varepsilon y}{r}-d/R\right)\right]=0$$

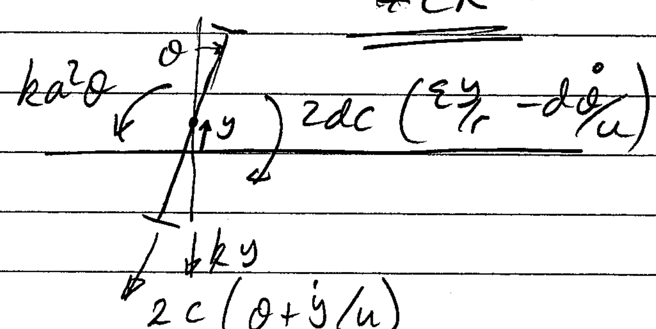
$$\therefore \; y\left(\frac{4dc^2\varepsilon}{rka^2}+k\right)=\frac{4d^2c^2}{ka^2R}-kL^2/2R$$

$\therefore \; y=0$ when RHS $=0$ ie $\underline{\underline{a=2\sqrt{2}\,dc/kL}}$ — (4)

(ii) (2) & (4) with $y=0$ and using (1) gives:

$$\underline{\underline{\theta = -\frac{kL^2}{4CR}}} \quad \text{—} (5)$$

(b)



$\sum F=0: \; 2c\left(\theta+\dot{y}/u\right)+ky=0$ — (6)

$\sum M=0: \; 2dc\left(\frac{\varepsilon y}{r}-\frac{d\dot{\theta}}{u}\right)-ka^2\theta=0$ — (7)

(6) $\rightarrow \; \theta = -\frac{ky}{2c}-\dot{y}/u$ — (8)

Q3 cont

⑧ into ⑦ gives

$$\frac{\varepsilon y}{r} - \frac{d}{u}\left(\frac{-k\dot{y}}{2c} - \frac{\ddot{y}}{u}\right) - \frac{ka^2}{2dc}\left(\frac{-ky}{2c} - \frac{\dot{y}}{u}\right) = 0$$

$$\text{ie } \ddot{y} + \dot{y}\left[\frac{uk}{2d^2c}(a^2+d^2)\right] + y\left[\frac{u^2}{d}\left(\frac{\varepsilon}{r} + \frac{a^2k^2}{4dc^2}\right)\right] = 0 \quad - ⑨$$

Hunting wavelength $\omega_n^2 = \frac{u^2}{d}\left(\frac{\varepsilon}{r} + \frac{a^2k^2}{4dc^2}\right)$

$$\lambda = \frac{2\pi u}{\omega_n} = \frac{2\pi \cancel{u}}{\cancel{u}\sqrt{\frac{\varepsilon}{dr} + \frac{a^2k^2}{4d^2c^2}}}$$

For zero suspension stiffness $k=0$, $\lambda = 2\pi\sqrt{\frac{dr}{\varepsilon}}$

as per free wheelset ✓

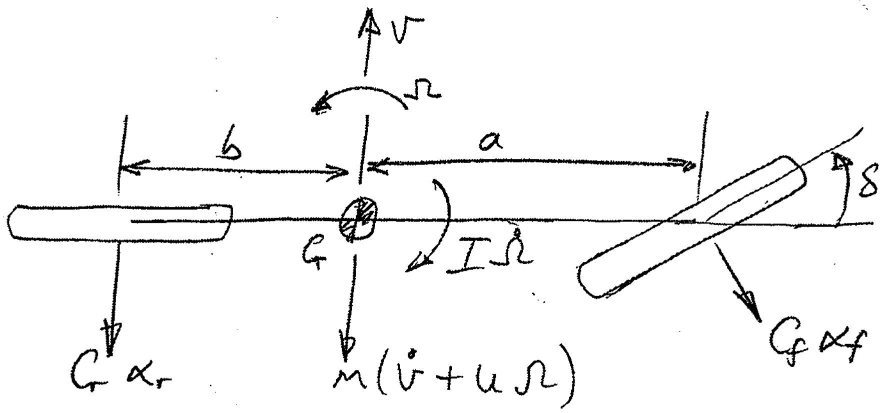
Using the value of a^2 from (a)(ii) $a = 2\sqrt{2}\, dc/kL$

$$\lambda = \frac{2\pi}{\sqrt{\frac{\varepsilon}{dr} + \frac{\cancel{2}8\cancel{d^2}\cancel{c^2}}{\cancel{k^2}L^2}\frac{\cancel{k^2}}{\cancel{4}\cancel{d^2}\cancel{c^2}}}} = \frac{2\pi}{\sqrt{\frac{\varepsilon}{dr} + \frac{2}{L^2}}} \quad //$$

(iii) Damping is proportional to the $\dot{y}$ term in ⑨
It is zero when $k=0$ as expected from simple wheelset ✓ then increases with k
Damping arises from energy dissipated by the creep forces.

Q4

~~Q4~~



(a) $\Sigma F: \; m(\dot v + u\Omega) + C_f\alpha_f + C_r\alpha_r = 0$

$\circlearrowleft + \Sigma M_G \quad I\dot\Omega + aC_f\alpha_f - bC_r\alpha_r = 0$ ①

Slip angles: $\alpha_f = \frac{v + a\Omega}{u} - \delta, \quad \alpha_r = \frac{v - b\Omega}{u}$ — ②

Combining ① & ②:

$$\left.\begin{aligned} m(\dot v + u\Omega) + (C_f + C_r)\frac{v}{u} + (aC_f - bC_r)\frac{\Omega}{u} &= C_f\delta \\ I\dot\Omega + (aC_f - bC_r)\frac{v}{u} + (a^2C_f + b^2C_r)\frac{\Omega}{u} &= aC_f\delta \end{aligned}\right\} \quad ③$$

__Assumptions__:
- All angles are small
- Neglect tyre realigning moment
- Tyres behave linearly
- δ is average of 2 steered front wheels
- Neglect motion of sprung mass on suspension

(b) Steady turning: $\dot\Omega = \dot v = 0$ & $\Omega = u/R$ ④

④ into ③ gives

$$\begin{bmatrix} c & cs + mu^2 \\ cs & cq^2 \end{bmatrix} \begin{Bmatrix} \beta \\ 1/R \end{Bmatrix} = C_f\delta \begin{Bmatrix} 1 \\ a \end{Bmatrix} \quad ⑤$$

where $c = C_f + C_r$, $l = a + b$, $s = \frac{aC_f - bC_r}{C_f + C_r}$, $q = \frac{a^2C_f + b^2C_r}{C_f + C_r}$, $\beta = v/u$

Solving ⑤ for $1/R$ gives

$$\frac{1/R}{\delta} = \frac{cC_f(a - s)}{C_fC_rl^2 - CsMu^2} = \frac{lC_fC_r}{C_fC_rl^2 - CsMu^2} \quad ⑥$$

Q4

~~Q2~~ Cont

ie $\delta = \frac{l}{R}\left(1 - \frac{CSmu^2}{l^2 C_f C_r}\right)$ // ——— (7)

Differentiate (7): $\frac{d\delta}{du} = -\frac{2l}{R}\left(\frac{CSm}{l^2 C_f C_r}\right)u$ ——— (8)

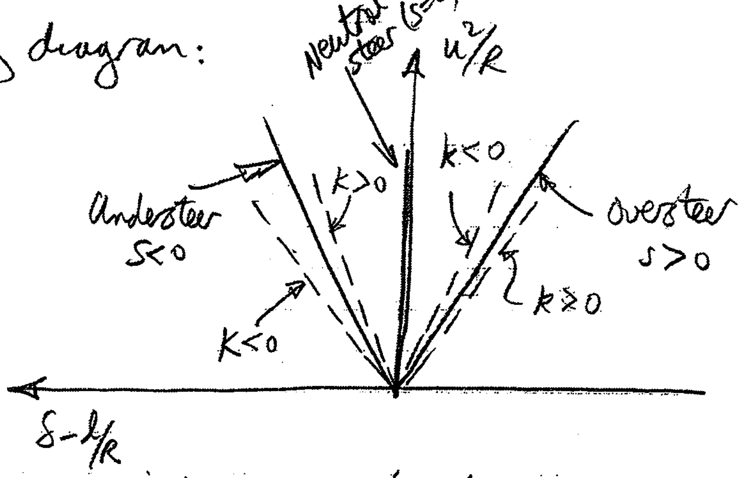
Neutral steer $(S=0) \rightarrow \delta = l/R$ & $d\delta/du = 0$

Understeer $(S<0) \rightarrow d\delta/du > 0$ all speeds

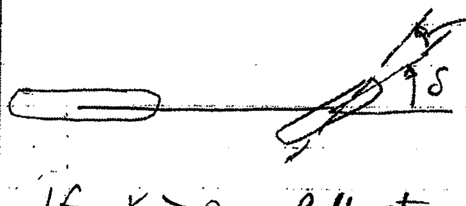
Oversteer $(S>0) \rightarrow d\delta/du < 0$ all speeds

Vehicle becomes unstable when $\delta \leq 0 \Rightarrow u \geq \sqrt{\frac{C_f C_r l^2}{CSm}}$

Handling diagram:



(c) Roll steer induces additional steer angle: Ku^2/R



$\left[\delta - l/R = -\frac{Ku^2}{R} - \frac{l}{R}\left(\frac{CSmu^2}{l^2 C_f C_r}\right)\right]$

If $K>0$, Roll steer generates <u>greater</u> steer angle than the vehicle without roll-steer.

⇒ For the understeering vehicle, this additional steer angle increases the lateral acceleration – ie it makes the vehicle <u>less</u> understeering (more oversteering)

⇒ For the oversteering vehicle, the additional steer angle reduces the lateral acceleration – ie it make the vehicle <u>more</u> oversteering