Version DC/3

EGT3
ENGINEERING TRIPOS PART IIB

Thursday 28 April 2022 9.30 to 11.10

Module 4C8

VEHICLE DYNAMICS

Answer not more than three questions.

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed

Attachment: 4C8 data sheet, 2017 (3 pages).

Engineering Data Book.

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Figure 1(a) shows a linear model with one degree of freedom, z_s, for predicting the vertical vibration of a vehicle travelling along a road surface with random vertical displacement z_r. The model has sprung mass m_s, suspension stiffness k and suspension damping c. The vertical velocity of the road surface has double-sided mean square spectral density S_0.

(i) Derive an expression for the mean square working space $\mathrm{E}[(z_s - z_r)^2]$.

You may use the following result for the definite integral of $H_2(j\omega)$:

$$H_2(j\omega) = \frac{(j\omega)B_1 + B_0}{(j\omega)^2 A_2 + (j\omega)A_1 + A_0}$$

$$\int_{-\infty}^{\infty} |H_2(j\omega)|^2 \mathrm{d}\omega = \frac{\pi(A_0 B_1^2 + A_2 B_0^2)}{A_0 A_1 A_2}.$$

[35%]

(ii) Explain why it is necessary to include the unsprung mass and its degree of freedom in the model before expressions for mean square sprung mass acceleration and mean square dynamic tyre force can be derived. [10%]

(b) Figure 1(b) shows a linear model with two degrees of freedom, z_s and z_u, for predicting the vertical vibration of a vehicle travelling along a rough road surface. In addition to the parameters shown in Fig.1(a) the model has unsprung mass m_u and tyre stiffness k_t. The mean square values of sprung mass acceleration, working space and dynamic tyre force due to vertical velocity input spectral density S_0 are given by:

$$\mathrm{E}[\ddot{z}_s^2] = \frac{\pi S_0[(m_s + m_u)k^2 + k_t c^2]}{m_s^2 c}$$

$$\mathrm{E}[(z_s - z_u)^2] = \frac{\pi S_0(m_s + m_u)}{c}$$

$$\mathrm{E}[(k_t(z_r - z_u))^2] = \frac{\pi S_0\{(m_s + m_u)[(m_s + m_u)^2 k^2 + k_t((m_s + m_u)c^2 - 2k m_u m_s)] + m_u m_s^2 k_t^2\}}{m_s^2 c}$$

(i) For the parameter values shown in Fig. 1(b) find the value of suspension stiffness k that minimises the mean square dynamic tyre force. [15%]

(ii) The suspension stiffness k is set to the value found in (b)(i). A variable damper is used to quickly adjust the trade-off between sprung mass acceleration and working space to suit varying operating conditions. In addition to travelling on a rough road surface, the sprung mass may also be subjected to external vertical step forces arising from cornering or braking manoeuvres. Calculate suitable maximum and minimum values of c. If needed, use $S_0 = 10^{-4}$ m^2 s^{-1} rad^{-1}. [40%]

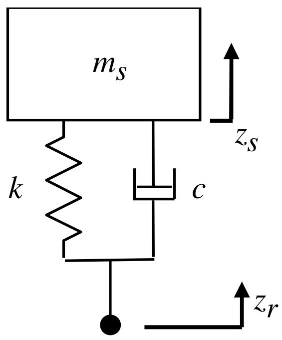

Fig. 1(a)

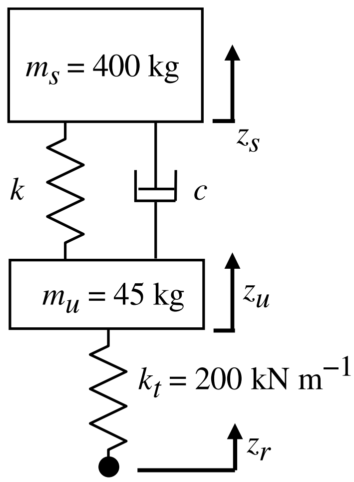

Fig. 1(b)

2 A roll-plane model of a vehicle is shown in Fig. 2. The vehicle has suspension with a slider mechanism that allows vertical displacement but no roll displacement between the sprung and unsprung masses. The model has five degrees of freedom: z_s, z_u, θ, y and y_t. The sprung mass m_s has roll inertia I_s. The unsprung mass m_u has roll inertia I_u. The height of the centre of the sprung mass above the road is h_s and that of the unsprung mass is h_u. The lateral tyre behaviour is represented by a series spring and damper, k_{lat} and c_{lat}. The vehicle travels at speed V along a randomly rough road surface which has vertical displacements z_L and z_R of the left and right tracks.

(a) Show that the damping coefficient of the lateral tyre model is given by:

$$c_{\text{lat}} = \frac{C}{V}$$

where C is the total tyre cornering stiffness. [20%]

(b) Explain why calculation of the vehicle vibration can be decoupled into separate calculations for vertical vibration and for lateral-roll vibration. [15%]

(c) The lateral-roll vibration of the vehicle is to be examined.

(i) Sketch a simplified model with three degrees of freedom that represents only the lateral-roll vibration of the vehicle in Fig. 2. [15%]

(ii) For the simplified model in (i) and the case of very high vehicle speed V, derive the equations of motion and hence show that the natural frequencies of vibration are given by the solutions to:

$$\omega^4 I - \omega^2 \tau = 0.$$

where I is the roll inertia and τ is the roll stiffness of the simplified model. It is not necessary to express I and τ in terms of the parameters given in Fig. 2. [30%]

(d) Discuss the effectiveness of a suspension with no roll displacement, commenting on: (i) vibration in vertical and lateral-roll directions; (ii) vibration response at low, medium and high V; (iii) effect of c_{lat}; (iv) effect of correlation between z_L and z_R. [20%]

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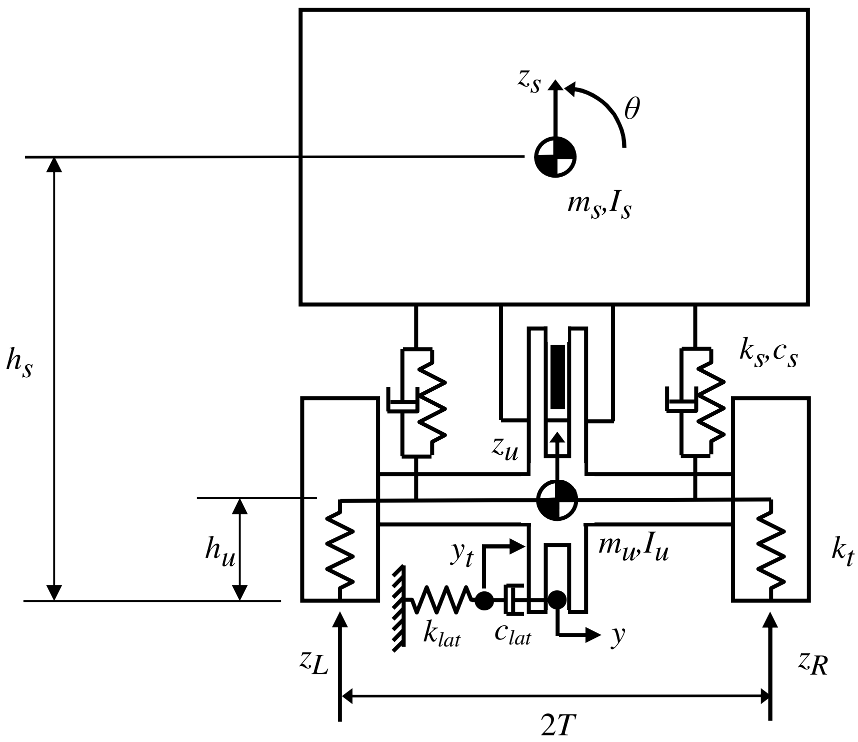

Fig. 2

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3 Figure 3 shows a special railway vehicle that is intended for measuring the vertical roughness of the track. A light central wheelset is located midway between the front and rear wheelsets of the vehicle, which are distance $2L$ apart. It is attached to the vehicle body through a suspension system which has lateral stiffness k (restoring force per unit displacement) and yaw stiffness ka^2 (restoring moment per unit of yaw angle), where a is constant. The front and rear wheelsets may be assumed to track perfectly, with zero lateral tracking error and zero yaw angle.

In a turn of constant radius R at steady speed u the creep forces on the centre wheelset result in a total lateral force Y of

$$Y = 2C\left(\frac{\dot{y}}{u} + \theta\right)$$

and a total yawing moment N of

$$N = 2dC\left(\frac{\varepsilon y}{r} - \frac{d\dot{\theta}}{u} - \frac{d}{R}\right)$$

where y is the lateral tracking error of the wheelset, θ is the yaw angle of the wheelset, $2d$ is the track gauge, r is the rolling radius of each wheel when, $y = \theta = 0$, ε is the effective conicity of the wheelset and C is the creep coefficient relating the creep velocities to the creep forces.

(a) The vehicle moves around a curve of radius R in steady motion ($\dot{y} = \dot{\theta} = 0$). It is desired to design the suspension system of the centre wheelset to minimise its lateral tracking error in curves.

(i) Show that the centre of the vehicle 'cuts' the corner by approximately $L^2/2R$ at point G and derive equations for the motion of the central wheelset.

(ii) Show that with a suitable choice of the value of a, the lateral tracking error of the centre wheelset y can be eliminated entirely.

(iii) What is the angle θ when a takes the value in part (a)(ii)? [50%]

(b) The vehicle now travels in a straight line with the front and rear wheelsets tracking perfectly, with zero lateral tracking error and zero yaw angle.

(i) Derive an equation for the motion of the central wheelset.

(ii) Find an expression for the wavelength of the hunting motion. Compare it with the hunting wavelength of a free wheelset when a takes the value from part (a)(ii).

(iii) Comment on the damping of the hunting mode and explain its origin. [50%]

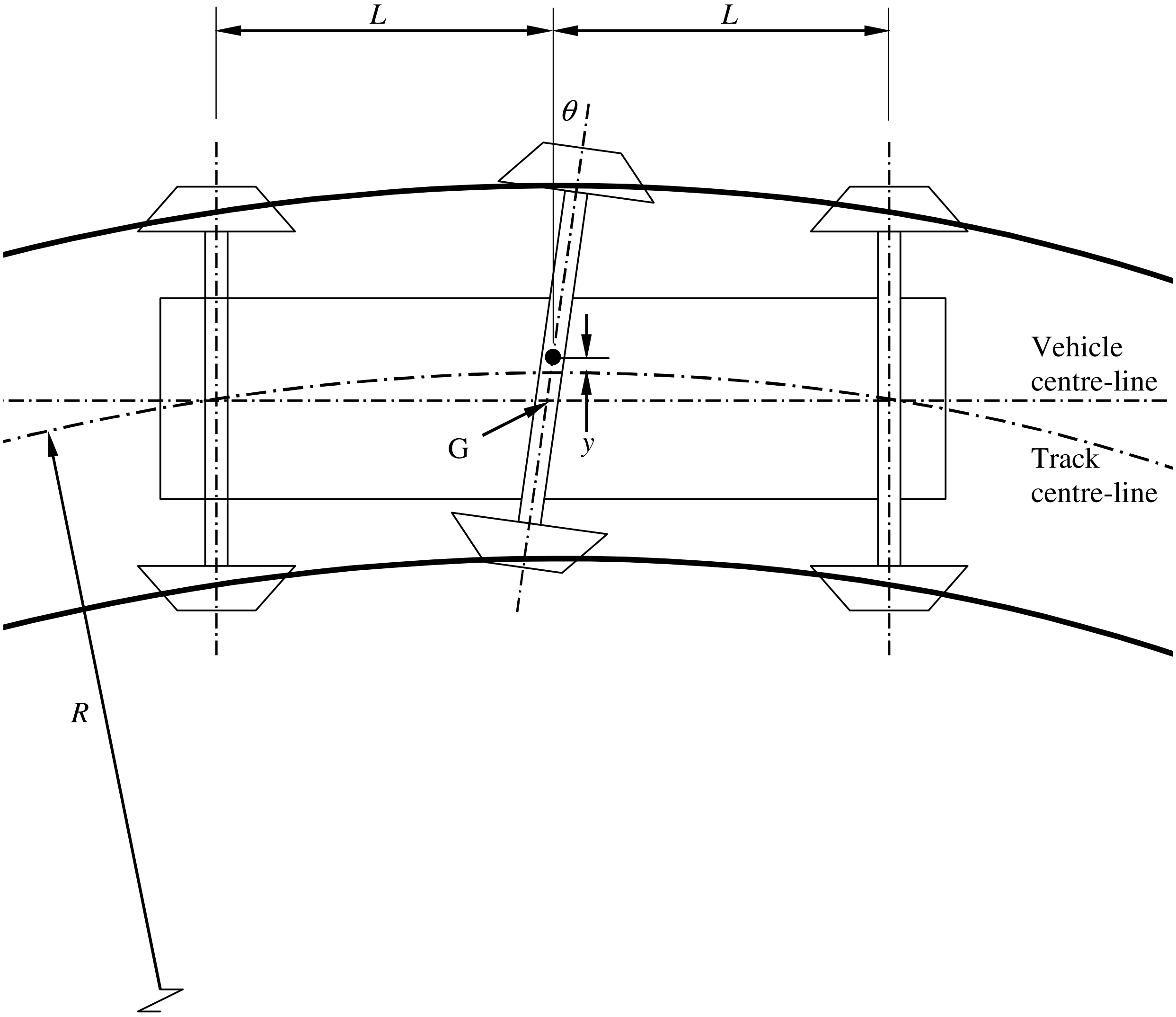

Fig. 3

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4 (a) A 'bicycle' model of a car, with freedom to sideslip with velocity v and yaw at rate Ω, is shown in Fig. 4. The car moves at steady forward speed u. It has mass m, yaw moment of inertia I, and lateral creep coefficients C_f and C_r at the front and rear tyres. The lengths a and b and the steering angle δ are defined in the figure. The equations of motion in a coordinate frame rotating with the vehicle are given by:

$$m(\dot{v} + u\Omega) + (C_f + C_r)\frac{v}{u} + (aC_f - bC_r)\frac{\Omega}{u} = C_f\delta$$

$$I\dot{\Omega} + (aC_f - bC_r)\frac{v}{u} + (a^2C_f + b^2C_r)\frac{\Omega}{u} = aC_f\delta$$

State the assumptions behind these equations. [15%]

(b) The steering angle is set to a constant value δ and the vehicle follows a steady circular path of radius R. Use the equations of motion to derive an expression for the steady state curvature response ($1/R$). Sketch a handling diagram, showing responses for under-steering, neutral steering and over-steering vehicles. Explain the behaviour with increasing speed and state the parameters that determine the type of response obtained. Explain which responses are unstable. [55%]

[Recall that a handling diagram is a graph of u^2/R vs ($\delta - L/R$), where $L = a + b$.]

(c) A particular vehicle develops a small additional steer angle at the front wheel due to 'roll-steer'. This additional small steer angle is proportional to the lateral acceleration at the centre of gravity, with constant of proportionality K. Sketch the effect of the roll steer on your handling diagram from part (b). Explain how the sign of K affects the behaviour of under-steering and over-steering vehicles. [30%]

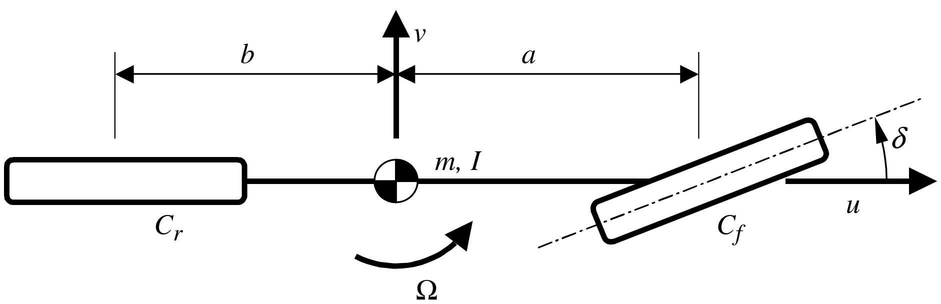

Fig. 4

END OF PAPER

Engineering Tripos Part IIB

Data sheet for Module 4C8: Vehicle Dynamics

DATA ON VEHICLE DYNAMICS

1. Creep Forces in Rolling Contact

1.1 Surface tractions

Longitudinal force $X = \iint_A \sigma_x \, dA$

Lateral force $Y = \iint_A \sigma_y \, dA$

Realigning Moment $N = \iint_A \left(x \, \sigma_y - y \, \sigma_x \right) dA$

where

σ_x, σ_y = longitudinal, lateral surface tractions

x, y = coordinates along, across contact patch

A = area of contact patch

1.2 Brush model

$\sigma_x = K_x \, q_x, \ \sigma_y = K_y \, q_y$ for $\sqrt{\sigma_x^2 + \sigma_y^2} \leq \mu p$

where

q_x, q_y = longitudinal, lateral displacements of 'bristles' relative to wheel rim

K_x, K_y = longitudinal, lateral stiffness per unit area

μ = coefficient of friction

p = local contact pressure

1.3 Linear creep equations

$$X = -C_{11}\xi$$

$$Y = -C_{22}\alpha - C_{23}\psi$$

$$N = C_{32}\alpha - C_{33}\psi$$

where X, Y, N, are defined as in 1.1 above.

C_{ij} = coefficients of linear creep

ξ = longitudinal creep ratio = longitudinal creep speed/forward speed

α = lateral creep ratio = (lateral speed /forward speed) - steer angle

ψ = spin creep ratio = spin angular velocity/forward speed

2. Plane Motion in a Moving Coordinate Frame

$$\ddot{\mathbf{R}}_{\mathbf{O}_1} = (\dot{u} - v\Omega)\mathbf{i} + (\dot{v} + u\Omega)\mathbf{j}$$

$(\mathbf{i}, \mathbf{j}, \mathbf{k})$ axis system fixed to body at point O_1

where

u = speed of point O_1 in $\mathbf{i}$ direction

v = speed of point O_1 in $\mathbf{j}$ direction

$\Omega\mathbf{k}$ = absolute angular velocity of body

3. Routh-Hurwitz stability criteria

$$\left(a_2 \frac{d^2}{dt^2} + a_1 \frac{d}{dt} + a_0\right) y = x(t)$$

Stable if all $a_i > 0$

$$\left(a_3 \frac{d^3}{dt^3} + a_2 \frac{d^2}{dt^2} + a_1 \frac{d}{dt} + a_0\right) y = x(t)$$

Stable if (i) all $a_i > 0$
and also (ii) $a_1 a_2 > a_0 a_3$

$$\left(a_4 \frac{d^4}{dt^4} + a_3 \frac{d^3}{dt^3} + a_2 \frac{d^2}{dt^2} + a_1 \frac{d}{dt} + a_0\right) y = x(t)$$

Stable if (i) all $a_i > 0$
and also (ii) $a_1 a_2 a_3 > a_0 a_3^2 + a_4 a_1^2$

DATA ON VEHICLE VIBRATION

Random Vibration

$$\mathrm{E}\left[x(t)^2\right] = \frac{1}{T}\int_{t=0}^{t=T} x^2(t)\,dt = \int_{\omega=-\infty}^{\omega=\infty} S_x(\omega)\,d\omega \qquad \left(\text{or } \int_{\omega=0}^{\omega=\infty} S_x(\omega)\,d\omega \quad \text{if } S_x(\omega) \text{ is single sided}\right)$$

$S_{\dot{x}}(\omega) = \omega^2 S_x(\omega)$ (Spectrum of time derivative of x is ω^2 times spectrum of x).

Single Input – Single Output

$$S_y(\omega) = \left|H_{yx}(\omega)\right|^2 S_x(\omega)$$

$$y(\omega) = H_{yx}(\omega)x(\omega)$$

Two Input – Two Output

$$\begin{Bmatrix} y_1(\omega) \\ y_2(\omega) \end{Bmatrix} = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix} \begin{Bmatrix} x_1(\omega) \\ x_2(\omega) \end{Bmatrix}$$

$$\begin{bmatrix} S_{11}^y(\omega) & S_{12}^y(\omega) \\ S_{21}^y(\omega) & S_{22}^y(\omega) \end{bmatrix} = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix}^* \begin{bmatrix} S_{11}^x(\omega) & S_{12}^x(\omega) \\ S_{21}^x(\omega) & S_{22}^x(\omega) \end{bmatrix} \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix}^{\mathrm{T}}$$

* means complex conjugate, T means transpose

If x_1 and x_2 are uncorrelated:

$$S_{(x_1+x_2)}(\omega) = S_{x_1}(\omega) + S_{x_2}(\omega)$$

$$S_{12}^x(\omega) = S_{21}^x(\omega) = 0$$

$$\mathrm{E}\left[(x_1(t) + x_2(t))^2\right] = \mathrm{E}\left[x_1(t)^2\right] + \mathrm{E}\left[x_2(t)^2\right]$$