EGT3
ENGINEERING TRIPOS PART IIB

Thursday 27 April 2023 9.30 to 11.10

Module 4C8

VEHICLE DYNAMICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS

Single-sided script paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed.

Attachment: 4C8 data sheet, 2023 (3 pages).

Engineering Data Book.

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 Figure 1 shows a quarter-car model of vehicle vibration. MS (mean square) sprung mass acceleration, A_{MS}, and MS suspension displacement, D_{MS}, are given by:

$$A_{\mathrm{MS}} = \mathrm{E}[\ddot{z}_s^2] = \frac{\pi S_0[(m_s + m_u)k^2 + k_t c^2]}{m_s^2 c}$$

$$D_{\mathrm{MS}} = \mathrm{E}[(z_s - z_u)^2] = \frac{\pi S_0(m_s + m_u)}{c}$$

where S_0 is the double-sided mean square spectral density of a white noise vertical velocity input, $\dot{z}_r$, from the road surface.

(a) Figure 1 also shows schematically the relationship between A_{MS} and D_{MS}. The two solid lines are contours of suspension stiffness k with varying suspension damping c, all other parameters are fixed.

(i) Find the relationship between A_{MS} and D_{MS} for k=0.

(ii) Find the gradient of the asymptote for a given k (dashed line in Fig. 1).

(iii) Show that the value of c that minimises A_{MS} for a given k is:

$$c_{\mathrm{optimum}} = k\sqrt{\frac{m_s+m_u}{k_t}}$$ [30%]

(b) Investigate and present in tabular form the effect on A_{MS} and D_{MS} of changing the sprung mass m_s (for example, when payload is added to the vehicle) when the suspension stiffness k:

(i) is held constant;

(ii) varies in proportion to m_s;

(iii) varies in proportion to m_s^2.

All other parameters including c are constant. Assume that $m_s \gg m_u$ and $(m_s + m_u)k^2 \gg k_t c^2$. [25%]

(c) Repeat the investigation in (b) but with suspension damping c adjusted to the value given in (a)(iii). Do not assume $(m_s + m_u)k^2 \gg k_t c^2$. [25%]

(d) With reference to the results of (b) and (c) discuss which is the best strategy for adapting suspension stiffness and damping to variation in sprung mass and outline technologies that could be used. [20%]

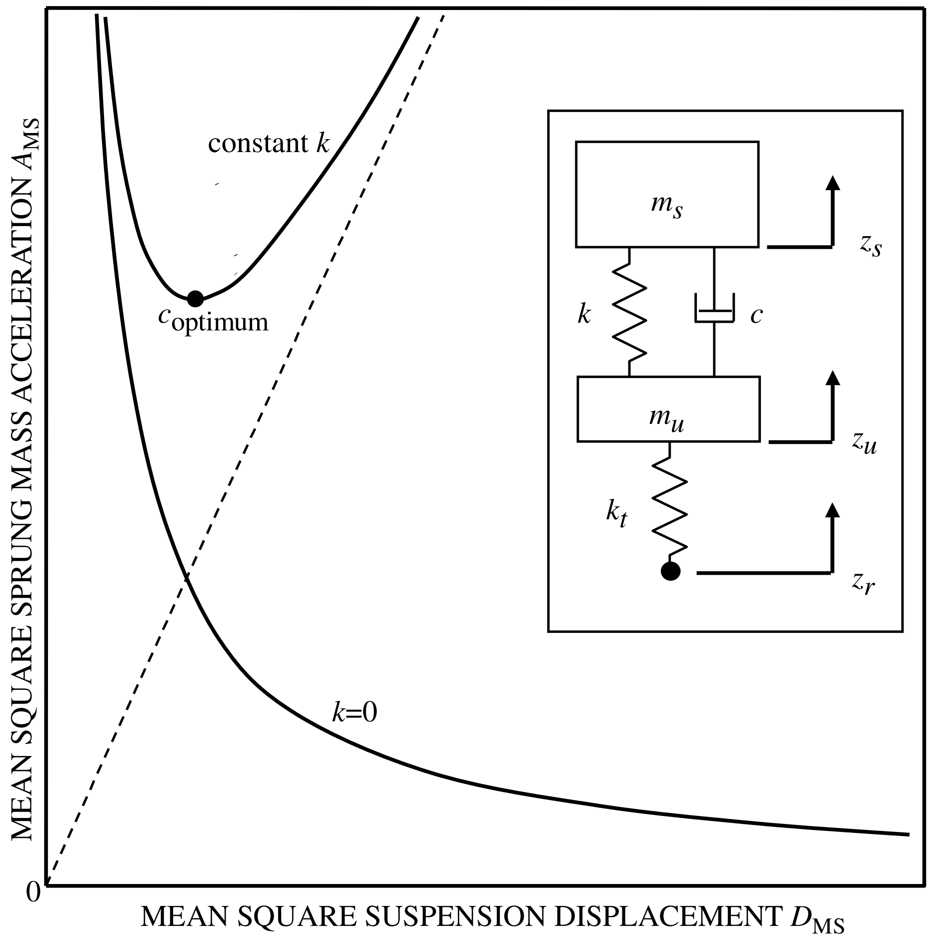
MEAN SQUARE SPRUNG MASS ACCELERATION A_{MS}
constant k
$c_{optimum}$
k=0
0
MEAN SQUARE SUSPENSION DISPLACEMENT D_{MS}
m_s
z_s
k
c
m_u
z_u
k_t
z_r

Fig. 1

2 Figure 2 shows a pitch-plane model of a vehicle with wheelbase $2a$, suspension stiffness k, damping λ, mass $2m$ and pitch inertia $I = 2ma^2$. The vehicle travels with constant speed U over a randomly rough road surface.

(a) Show that the frequency response functions relating the vertical and pitch displacement responses z and θ of the vehicle mass to the vertical displacement inputs z_{r1} and z_{r2} at the front and rear axles are given by

$$\begin{Bmatrix} z(\mathrm{j}\omega) \\ \theta(\mathrm{j}\omega) \end{Bmatrix} = \frac{1}{2}\mathrm{H}(\mathrm{j}\omega)\begin{bmatrix} 1 & 1 \\ -1/a & 1/a \end{bmatrix}\begin{Bmatrix} z_{r1}(\mathrm{j}\omega) \\ z_{r2}(\mathrm{j}\omega) \end{Bmatrix}$$

where $\mathrm{H}(\mathrm{j}\omega)$ is the frequency response of case(c) of a linear mass-damper-spring system in the Mechanics Data Book. [25%]

(b) By considering the time delay between the front axle and the rear axle passing over a given point on the road surface, derive the following expression: [35%]

$$\left|\frac{z(\mathrm{j}\omega)}{z_{r1}(\mathrm{j}\omega)}\right| = |\mathrm{H}(\mathrm{j}\omega)|\sqrt{\frac{1+\cos\left(\frac{2a\omega}{U}\right)}{2}}$$

(c) For $a = 1.3$ m, $k = 90$ kN m^{-1}, $\lambda = 5$ kN s m^{-1}, $m = 800$ kg, $U = 8$ m s^{-1} sketch the functions $|\mathrm{H}(\mathrm{j}\omega)|$, $\sqrt{\frac{1+\cos\left(\frac{2a\omega}{U}\right)}{2}}$ and $\left|\frac{z(\mathrm{j}\omega)}{z_{r1}(\mathrm{j}\omega)}\right|$ over the frequency range 0 rad s^{-1} to 40 rads^{-1}. Annotate the sketches with salient values of frequency and gain. [25%]

(d) Explain the significance of the functions sketched in Part (c). [15%]

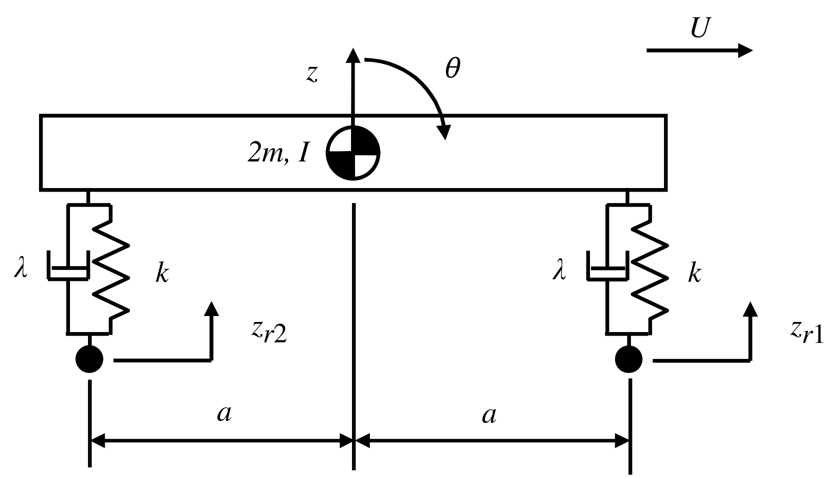

Fig. 2

3 The linear creep equations governing wheel forces in rolling contact are given by:

$$X = -C_{11}\xi$$
$$Y = -C_{22}\alpha + C_{23}\psi$$
$$N = C_{32}\alpha + C_{33}\psi$$

(All terms are defined on the data sheet).

(a) The contact area of a particular tyre has dimensions $2l$ x $2h$. The normal contact pressure distribution is triangular with a peak value of P_0, as shown in Fig. 3. The coefficient of friction is μ.

(i) Use the 'brush' model with lateral bristle stiffness K_y to determine the lateral force as a function of lateral slip α and sketch a graph of the function showing salient values. [40%]

(ii) Calculate the value of C_{22} and comment on your result. [10%]

(iii) Explain how you could calculate C_{32}. [10%]

(b) Without detailed calculations, explain the source of the C_{23} and C_{33} coefficients in the linear creep equations. Give examples of where they arise in vehicle dynamics. [20%]

(c) Explain which of the linear creep coefficients are needed to perform simple analyses of the stability of:

(i) cars;

(ii) railway bogies. [20%]

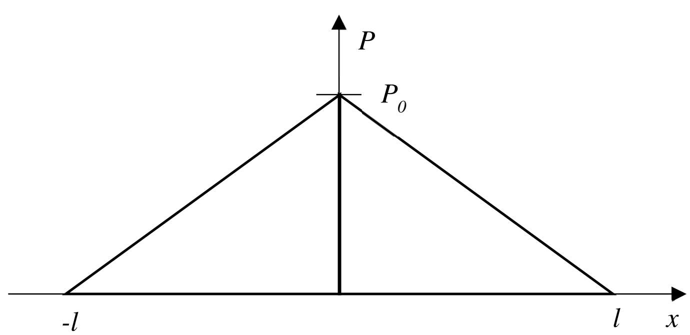

Fig. 3

4 (a) A 'bicycle' model of a car, with freedom to sideslip with velocity v and yaw at rate Ω, is shown in Fig. 4. The car moves at steady forward speed u. It has mass m, yaw moment of inertia about its centre of gravity I, and lateral creep coefficients C_f and C_r at the front and rear tyres. The lengths a and b and the steering angle δ are defined in the figure. The car is subject to a steady side wind of magnitude Y which acts a distance x forward of the centre of gravity. Show that the equations of motion in a coordinate frame rotating with the vehicle are given by:

$$m(\dot{v} + u\Omega) + (C_f + C_r)\frac{v}{u} + (aC_f - bC_r)\frac{\Omega}{u} = Y + C_f\delta$$
$$I\dot{\Omega} + (aC_f - bC_r)\frac{v}{u} + (a^2C_f + b^2C_r)\frac{\Omega}{u} = xY + aC_f\delta$$

State your assumptions. [30%]

(b) Determine an expression for the steer angle δ_{ss} needed to set the steady state yaw rate Ω_{ss} to zero. For low speeds (less than the 'critical speed'), explain how this steady-state steer angle varies with x and the vehicle parameters. [40%]

(c) Determine an expression for the steady state sideslip angle β_{ss} at the centre of gravity of the vehicle when $\Omega_{ss} = 0$. Explain how the motion of the vehicle varies with x and the vehicle parameters. [30%]

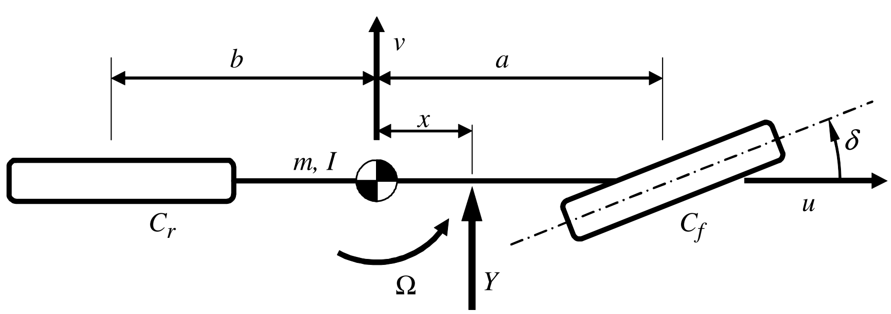

Fig. 4

END OF PAPER

Engineering Tripos Part IIB

Data sheet for Module 4C8: Applications of Dynamics

DATA ON VEHICLE DYNAMICS

1. Creep Forces In Rolling Contact

1.1 Surface tractions

Longitudinal force $X = \iint_A \sigma_x \, dA$

Lateral force $Y = \iint_A \sigma_y \, dA$

Realigning Moment $N = \iint_A \left(x\,\sigma_y - y\,\sigma_x\right) dA$

where

σ_x, σ_y = longitudinal, lateral surface tractions

x, y = coordinates along, across contact patch

A = area of contact patch

1.2 Brush model

$\sigma_x = K_x\, q_x,\ \sigma_y = K_y\, q_y$ for $\sqrt{\sigma_x^2 + \sigma_y^2} \le \mu p$

where

q_x, q_y = longitudinal, lateral displacements of 'bristles' relative to wheel rim

K_x, K_y = longitudinal, lateral stiffness per unit area

μ = coefficient of friction

p = local contact pressure

1.3 Linear creep equations

$X = -C_{11}\xi$

$Y = -C_{22}\alpha + C_{23}\psi$

$N = C_{32}\alpha + C_{33}\psi$

where X, Y, N, are defined as in 1.1 above.

C_{ij} = coefficients of linear creep

ξ = longitudinal creep ratio = longitudinal creep speed/forward speed

α = lateral creep ratio = (lateral speed /forward speed) - steer angle

ψ = spin creep ratio = spin angular velocity/forward speed

2. Plane Motion in a Moving Coordinate Frame

$$\ddot{\mathbf{R}}_{\mathbf{O}_1} = (\dot{u} - v\Omega)\mathbf{i} + (\dot{v} + u\Omega)\mathbf{j}$$

($\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$) axis system fixed to body at point O_1

where

u = speed of point O_1 in $\mathbf{i}$ direction

v = speed of point O_1 in $\mathbf{j}$ direction

$\Omega\mathbf{k}$ = absolute angular velocity of body

3. Routh-Hurwitz stability criteria

$$\left(a_2\frac{d^2}{dt^2} + a_1\frac{d}{dt} + a_0\right)y = x(t)$$

Stable if all $a_i > 0$

$$\left(a_3\frac{d^3}{dt^3} + a_2\frac{d^2}{dt^2} + a_1\frac{d}{dt} + a_0\right)y = x(t)$$

Stable if (i) all $a_i > 0$
and also (ii) $a_1 a_2 > a_0 a_3$

$$\left(a_4\frac{d^4}{dt^4} + a_3\frac{d^3}{dt^3} + a_2\frac{d^2}{dt^2} + a_1\frac{d}{dt} + a_0\right)y = x(t)$$

Stable if (i) all $a_i > 0$
and also (ii) $a_1 a_2 a_3 > a_0 a_3^2 + a_4 a_1^2$

DATA ON VEHICLE VIBRATION

Random Vibration

$\mathrm{E}\left[x(t)^2\right] = \frac{1}{T}\int_{t=0}^{t=T} x^2(t)\,dt = \int_{\omega=-\infty}^{\omega=\infty} S_x(\omega)\,d\omega$ (or $\int_{\omega=0}^{\omega=\infty} S_x(\omega)\,d\omega$ if $S_x(\omega)$ is single sided)

$S_{\dot{x}}(\omega) = \omega^2 S_x(\omega)$ (Spectrum of time derivative of x is ω^2 times spectrum of x).

Single Input – Single Output

$$S_y(\omega) = \left|H_{yx}(\omega)\right|^2 S_x(\omega)$$

$$y(\omega) = H_{yx}(\omega)x(\omega)$$

Two Input – Two Output

$$\begin{Bmatrix} y_1(\omega) \\ y_2(\omega) \end{Bmatrix} = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix} \begin{Bmatrix} x_1(\omega) \\ x_2(\omega) \end{Bmatrix}$$

$$\begin{bmatrix} S_{11}^y(\omega) & S_{12}^y(\omega) \\ S_{21}^y(\omega) & S_{22}^y(\omega) \end{bmatrix} = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix}^* \begin{bmatrix} S_{11}^x(\omega) & S_{12}^x(\omega) \\ S_{21}^x(\omega) & S_{22}^x(\omega) \end{bmatrix} \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix}^{\mathrm{T}}$$

* means complex conjugate, T means transpose

If x_1 and x_2 are uncorrelated:

$$S_{(x_1+x_2)}(\omega) = S_{x_1}(\omega) + S_{x_2}(\omega)$$

$$S_{12}^x(\omega) = S_{21}^x(\omega) = 0$$

$$\mathrm{E}\left[(x_1(t) + x_2(t))^2\right] = \mathrm{E}\left[x_1(t)^2\right] + \mathrm{E}\left[x_2(t)^2\right]$$