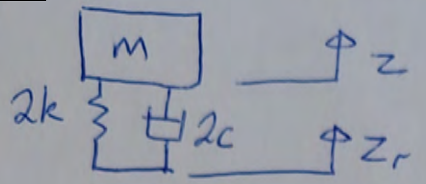
Cribs for Question 1

1(a) (i): 30%

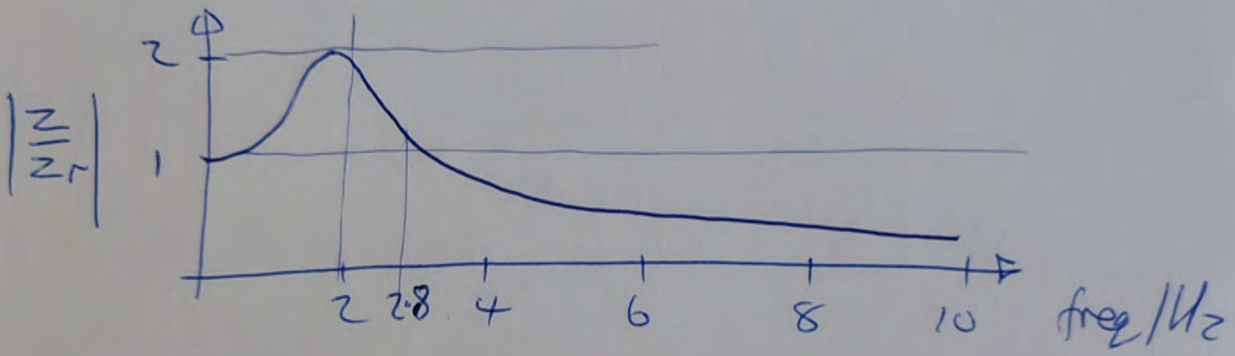
when $z_{r_1} = z_{r_2}$, idealise as:



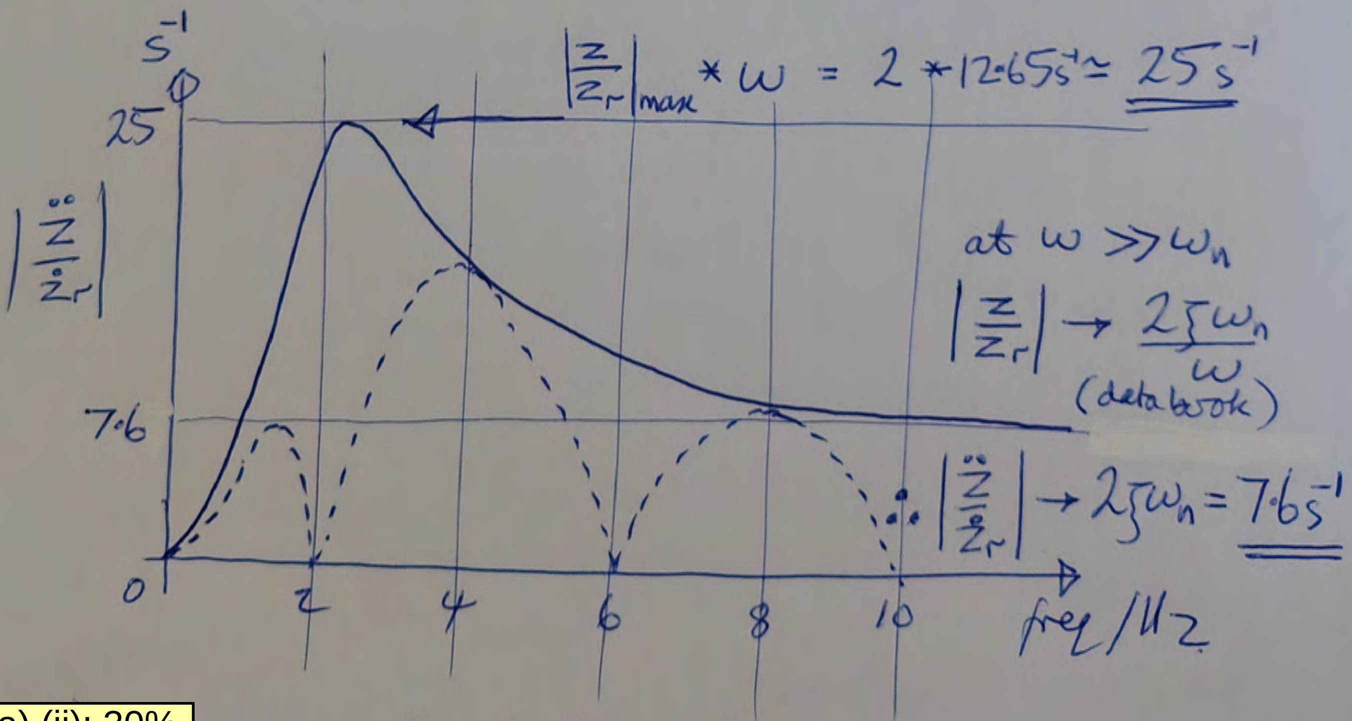
$$\omega_n = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2 \cdot 80 \cdot 10^3}{10^3}} = 12.65 \text{ rad/s} = 2.0 \text{ Hz}$$

$$\zeta = \frac{2c}{2\sqrt{2km}} = \frac{2 \cdot 3800}{2\sqrt{2 \cdot 80 \cdot 10^3 \cdot 10^3}} = 0.3$$

see mechanics data book case (c) for $\left|\frac{z}{z_r}\right|$:



multiply by ω to give $\left|\frac{\ddot{z}}{\dot{z}_r}\right|$:



1(a) (ii): 30% - - - - with wheelbase filtering

a) ii) $\left|\frac{\ddot{Z}}{\dot{Z}_r}\right|$ is zero when $L=(i+\frac{1}{2})\lambda$ i is integer, λ is wavelength

$$U=f\lambda \quad \therefore \quad f=\frac{U}{\lambda}=\frac{U}{L}\left(i+\frac{1}{2}\right)$$

$\therefore$ zeros when: $i=0$, $f=\frac{10}{2.5}\cdot\frac{1}{2}=2\text{Hz}$

$i=1$, $f=\frac{10}{2.5}\cdot\frac{3}{2}=6\text{Hz}$

$i=2$, $f=\frac{10}{2.5}\cdot\frac{5}{2}=10\text{Hz}$

see dashed line on graph.

1(b): 20%

pitch excitation zero when $L=i\lambda$, i is integer.

$$U=f_{pitch}\cdot\lambda=f_{pitch}\cdot\frac{L}{i} \quad \text{where } f_{pitch}=\frac{1}{2\pi}\sqrt{\frac{2ka^2}{I}}$$

$$U=\frac{1}{2\pi}\sqrt{\frac{2ka^2}{I}}\cdot\frac{L}{i}$$

$$k=2\left(\frac{\pi U i}{aL}\right)^2 I \quad \therefore \quad k\propto U^2$$

choose $i=1$ to avoid large k $\quad\therefore\quad k=2\left(\frac{\pi}{aL}\right)^2 I U^2$

1(c): 20%

bounce excitation zero when $L=(i+\frac{1}{2})\lambda$ (see part (a)(ii))

$$U=f_{bounce}\frac{L}{i+\frac{1}{2}} \quad \text{where } f_{bounce}=\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$

$$\therefore\quad k=2m\left(\frac{\pi}{L}\right)^2\left(i+\frac{1}{2}\right)^2 U^2$$

equate to expression for k in part (b)

$$2\left(\frac{\pi U}{L}\right)^2\frac{I}{a^2}=2m\left(\frac{\pi U}{L}\right)^2\left(i+\frac{1}{2}\right)^2$$

$$\therefore\quad I=\left(i+\frac{1}{2}\right)^2 ma^2 \qquad i \text{ is integer.}$$

if $i=0$, $I=\frac{1}{4}ma^2$

if $i=1$, $I=\frac{9}{4}ma^2$

$I=0.9ma^2$ for typical car so seems unrealistic to achieve.

Cribs for Question 2

2(a): 15%

$$z_R = z_v + z_\phi$$

z_v & z_ϕ uncorrelated so

$$S_z(n) = S_{z_R}(n) = S_{z_v}(n) + S_{z_\phi}(n)$$

$$= S_{z_v}(n)\left(1 + |G(n)|^2\right)$$

$$= S_{z_v}(n)\left(1 + \frac{n^2}{n_c^2 + n^2}\right)$$

$$S_z(n) = S_{z_v}(n)\left(\frac{n_c^2 + 2n^2}{n_c^2 + n^2}\right)$$

$$S_{z_v}(n) = S_z(n)\left(\frac{n_c^2 + n^2}{n_c^2 + 2n^2}\right)$$

2(b): 20%

$S_z(n) = K\, n^{-2}$ but $\omega = 2\pi U n$ (rad/s; rad/cycle; m/s; cycle/m)

so $S_z(n) = K\left(\frac{\omega}{2\pi U}\right)^{-2}$

must ensure that

$$\int_{n=0}^{n=\infty} S_z(n)\, dn = \int_{\omega=0}^{\omega=\infty} S_z(\omega)\, d\omega$$

but $d\omega = 2\pi U\, dn$

so
$$\int_{\omega=0}^{\omega=\infty} K\left(\frac{\omega}{2\pi U}\right)^{-2} \frac{d\omega}{2\pi U} = \int_{\omega=0}^{\omega=\infty} S_z(\omega)\, d\omega$$

$$\therefore \quad S_z(\omega) = \frac{K}{2\pi U}\left(\frac{\omega}{2\pi U}\right)^{-2}$$

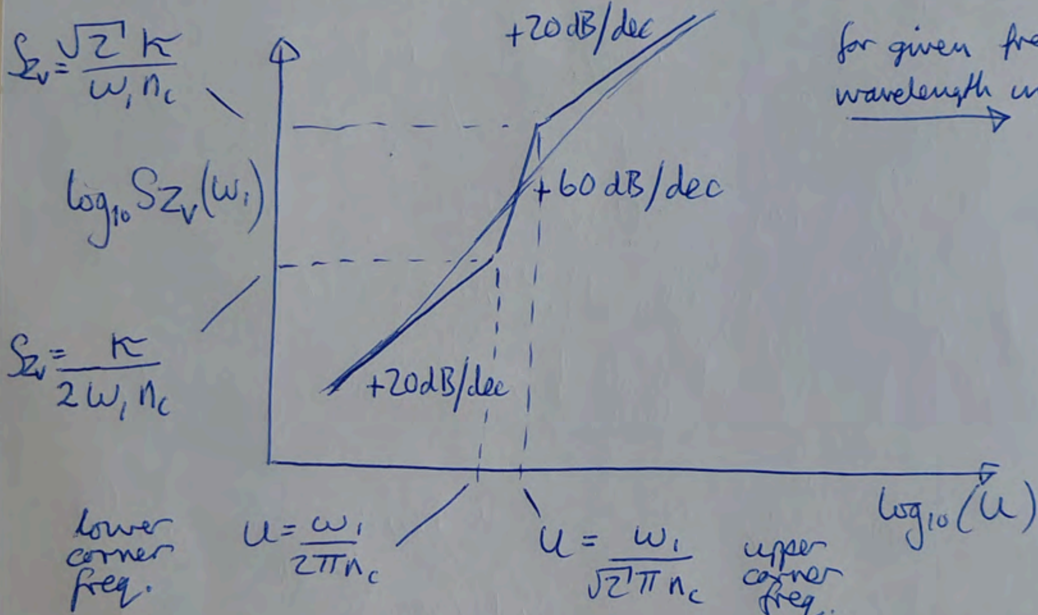
$$S_z(\omega) = 2\pi U\, K\, \omega^{-2}$$

2(c): 40%

from (a) $$S_{z_v}(n) = S_z(n)\left(\frac{n_c^2 + n^2}{n_c^2 + 2n^2}\right)$$

$$S_{z_v}(\omega) = S_z(\omega)\left(\frac{(2\pi U)^2 n_c^2 + \omega^2}{(2\pi U)^2 n_c^2 + 2\omega^2}\right)$$

$$S_{z_v}(\omega) = \frac{2\pi U \kappa}{\omega^2} \cdot \frac{(2\pi U)^2 n_c^2 + \omega^2}{(2\pi U)^2 n_c^2 + 2\omega^2}$$



for given frequency ω, wavelength increases with speed U.

2(d): 25%

+20 dB/dec gradient due to higher speed U requiring longer wavelength (of larger amplitude) to give constant frequency ω.

for $\omega_1 = 5$ rad/s, $U_{lower} = 8$ m/s, $U_{upper} = 11.3$ m/s
$n_c = 0.1$ cycles/m

+60 dB/dec between these two speeds corresponds to increasing correlation of left and right tracks. There is a corresponding reduction in $S_{z\phi}$. Thus spring mass modes are increasingly excited in bounce and less in roll as speed U increases.

[Cribs for Question 3]

3(a): 30%

The key assumptions involve:

- The vehicle is assumed to be a rigid body.
- The centre of mass is on the ground, meaning that, the height of the centre of mass is zero. As a result, the vehicle cannot roll or roll over with respect to the road.
- There is no lateral or longitudinal load transfer.
- The lateral creep coefficient at the front axle of the bicycle model C_f is approximately twice of the lateral creep coefficient of a single tyre on a wheel of a car (for the case where the car has two front wheels and one tyre on each wheel). This assumption is also applied to the rear axle.
- The tyres maintain ground contact.
- The front wheel steer angle is assumed to be small.

Following lecture notes, the equations of motion of the bicycle model is given by:

$$m(\dot{v}+u\Omega)+(C_f+C_r)\frac{v}{u}+(aC_f-bC_r)\frac{\Omega}{u}=C_f\delta \tag{1.1}$$

$$I\dot{\Omega}+(aC_f-bC_r)\frac{v}{u}+(a^2C_f+b^2C_r)\frac{\Omega}{u}=aC_f\delta \tag{1.2}$$

Notes: For part (a), students are expected to derive (1.1) and (1.2) following the procedure taught in the lecture, or otherwise.

3(b): 30%

Applying Laplace transforms to the equations of motion (1.1) and (1.2) gives

$$mu[sv(s)+u\Omega(s)]+(C_f+C_r)v(s)+(aC_f-bC_r)\Omega(s)=uC_f\delta(s) \tag{2.1}$$

$$uIs\Omega(s)+(aC_f-bC_r)v(s)+(a^2C_f+b^2C_r)\Omega(s)=uaC_f\delta(s) \tag{2.2}$$

Dividing (2.1) and (2.2) by $\delta(s)$ then gives

$$mus\frac{v(s)}{\delta(s)}+mu^2\frac{\Omega(s)}{\delta(s)}+(C_f+C_r)\frac{v(s)}{\delta(s)}+(aC_f-bC_r)\frac{\Omega(s)}{\delta(s)}=uC_f \tag{3.1}$$

$$uIs\frac{\Omega(s)}{\delta(s)}+(aC_f-bC_r)\frac{v(s)}{\delta(s)}+(a^2C_f+b^2C_r)\frac{\Omega(s)}{\delta(s)}=uaC_f \tag{3.2}$$

Combining (3.1) and (3.2) gives:

$$\begin{bmatrix} mus+C_f+C_r \\ aC_f-bC_r \end{bmatrix}\frac{v(s)}{\delta(s)}+\begin{bmatrix} mu^2+aC_f-bC_r \\ uIs+a^2C_f+b^2C_r \end{bmatrix}\frac{\Omega(s)}{\delta(s)}=uC_f\begin{bmatrix} 1 \\ a \end{bmatrix} \tag{4}$$

as requested.

For a neutral steer car, aC_f-bC_r is zero. Substituting this condition into the transfer functions given by the question, i.e. (4) yields

$$\begin{bmatrix} mus+C_f+C_r \\ 0 \end{bmatrix}\frac{v(s)}{\delta(s)}+\begin{bmatrix} mu^2 \\ uIs+a^2C_f+b^2C_r \end{bmatrix}\frac{\Omega(s)}{\delta(s)}=uC_f\begin{bmatrix} 1 \\ a \end{bmatrix} \tag{5}$$

The yaw rate transfer function can be directly obtained from (5):

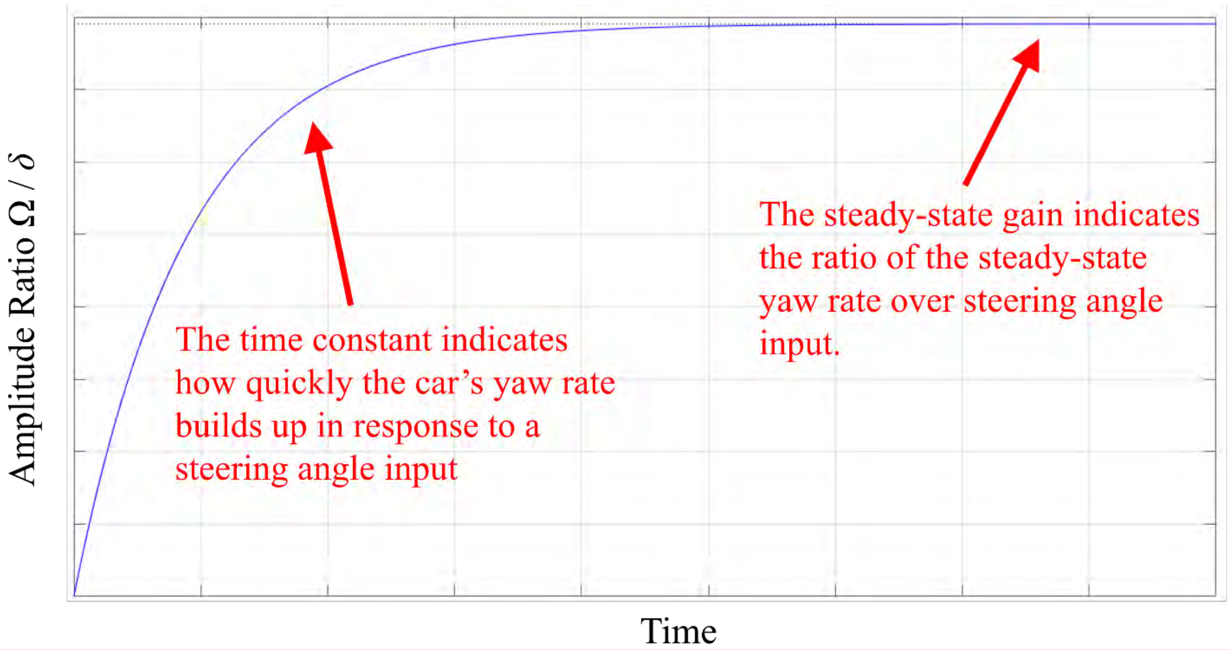
$$\frac{\Omega(s)}{\delta(s)}=\frac{uaC_f}{uIs+a^2C_f+b^2C_r}\text{, or equivalently}$$

$$\frac{\Omega(s)}{\delta(s)}=\frac{uaC_f}{a^2C_f+b^2C_r}\cdot\frac{1}{\frac{uI}{a^2C_f+b^2C_r}s+1} \tag{6}$$

It can be seen that (6) is a first-order transfer function. The two key features are:

steady-state gain: $\dfrac{uaC_f}{a^2C_f + b^2C_r}$, and time constant: $\dfrac{uI}{a^2C_f + b^2C_r}$.

The yaw rate response to a step steering angle input can be sketched as below, where the two key features shall be marked.



Notes: For part (b), students are expected to use the definition of neutral steer to derive (6), and then sketch the graph above and identify two key features: 1) time constant and 2) steady-state gain.

3(c): 40%

When the vehicle is cornering steadily, $\dot{v} = 0$ and $\dot{\Omega} = 0$. Substituting these two equations, along with $\delta = \delta_0$ into equations (1.1) and (1.2) gives

$$mu\Omega + (C_f + C_r)\frac{v}{u} + (aC_f - bC_r)\frac{\Omega}{u} = C_f\delta_0 \quad (7.1)$$

$$(aC_f - bC_r)\frac{v}{u} + (a^2C_f + b^2C_r)\frac{\Omega}{u} = aC_f\delta_0 \quad (7.2)$$

(7.1) and (7.2) can be written as

$$mu^2\Omega + (C_f + C_r)v + (aC_f - bC_r)\Omega = uC_f\delta_0 \quad (8.1)$$

$$(aC_f - bC_r)v + (a^2C_f + b^2C_r)\Omega = uaC_f\delta_0 \quad (8.2)$$

Note that (8.1) and (8.2) can also be obtained by ignoring the terms associated with the Laplace transformer s in the transfer functions provided by the question, i.e. (4) (this means zeroing the terms associated with time differentiation) and replacing $\delta(s)$ with δ_0.

Dividing (8.1) and (8.2) respectively by $C_f + C_r$ gives

$$\frac{mu^2}{C_f + C_r}\Omega + v + \frac{aC_f - bC_r}{C_f + C_r}\Omega = \frac{uC_f\delta_0}{C_f + C_r} \quad (9.1)$$

$$\frac{aC_f - bC_r}{C_f + C_r}v + \frac{a^2C_f + b^2C_r}{C_f + C_r}\Omega = \frac{uaC_f\delta_0}{C_f + C_r} \quad (9.2)$$

Now defining three intermediate variables C, S and Q as suggested in the lecture notes:

$$C = C_f + C_r \quad (10.1)$$

$$S = \frac{aC_f - bC_r}{C_f + C_r} \quad (10.2)$$

$$Q = \frac{a^2C_f + b^2C_r}{C_f + C_r} \quad (10.3)$$

Substituting (10.1), (10.2) and (10.3) into (9.1) and then (9.2) then gives

$$\frac{mu^2}{C}\Omega + v + S\Omega = \frac{uC_f\delta_0}{C} \quad (11.1)$$

$$Sv + Q\Omega = \frac{uaC_f\delta_0}{C} \quad (11.2)$$

Substituting (11.2) into (11.1) to solve for yaw rate Ω:

$$\Omega = \frac{uC_f\delta_0(a-S)}{CQ - CS^2 - mu^2 S} \quad (12)$$

The steady-state cornering radius R is defined as

$$R = \frac{u}{\Omega} \quad (13)$$

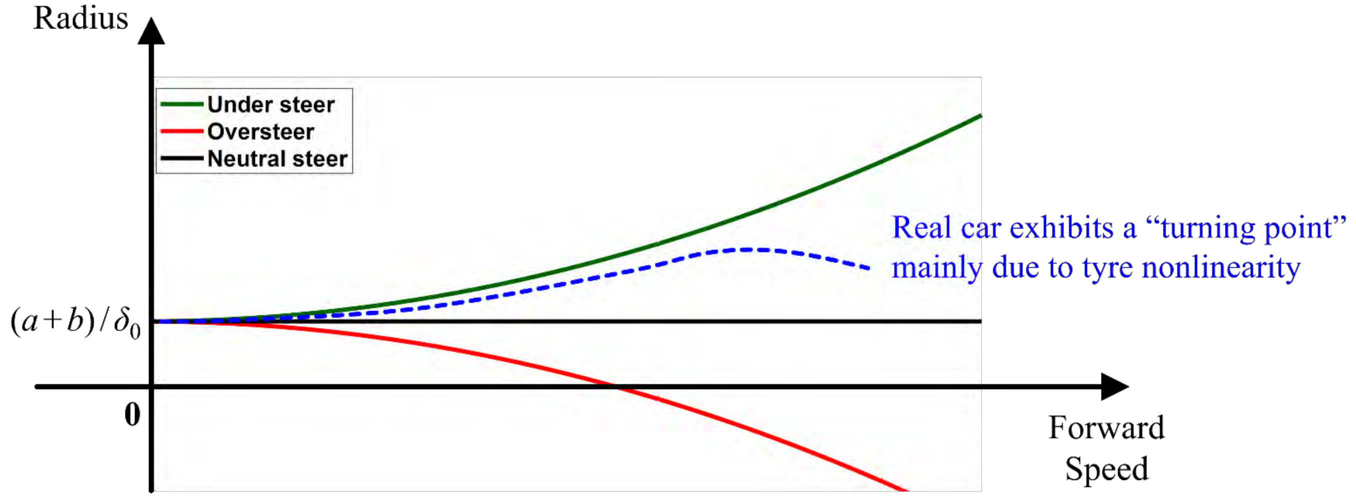
Substituting (12) into (13) gives

$$R = \frac{CQ - CS^2 - mu^2 S}{C_f\delta_0(a-S)} \quad (14)$$

Substituting (10.1), (10.2) and (10.3) into (14) to eliminate intermediate variables C, S and Q. This gives

$$R = \frac{C_f C_r (a+b)^2 - mu^2(aC_f - bC_r)}{(a+b)C_f C_r \delta_0} \quad (15)$$

The variations of steady-state cornering radius R with forward speed u is sketched as below:



[Cribs for Question 4]

4(a): 30%

Following the lecture notes, the instantaneous rolling radius of the right wheel (denoted as 'wheel #1') r_1 and that of the left wheel (denoted as 'wheel #2') r_2 are respectively

$$r_1 = r - y\tan\varepsilon \tag{1.1}$$

$$r_2 = r + y\tan\varepsilon \tag{1.2}$$

r_1 and r_2 are indicated in my sketch Fig. S1. In Fig. S1, the centre of the entire wheelset is marked as point C, the centre of the wheel #1 that corresponds to instantaneous rolling radius is marked as A1, and the centre of the wheel #2 that corresponds to instantaneous rolling radius is marked as A2.

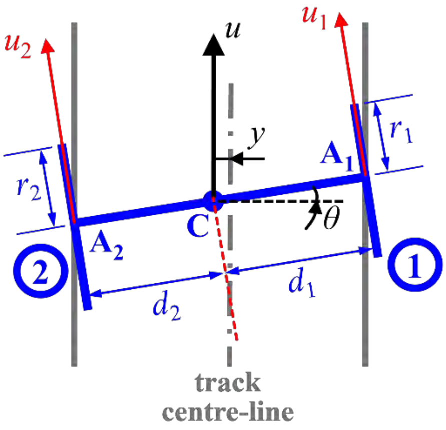


Fig. S1

Continuing to follow lecture notes, let's introduce ω to represent the rotation speed (angular speed) of the wheelset. Assuming there is 'no-slip' between the wheels and track, u_1, wheel #1's velocity at A_1, and u_2, wheel #2's velocity at A_2, centre can be expressed as:

$$u_1 = \omega r_1 \tag{2.1}$$

$$u_2 = \omega r_2 \tag{2.2}$$

u_1 and u_2 are schematically indicated Fig. S1. Substituting (1.1) and (2) into (3.1), and substituting (1.2) and (2) into (3.2) respectively yield

$$u_1 = \omega\left(r - y\tan\varepsilon\right) \tag{3.1}$$

$$u_2 = \omega\left(r + y\tan\varepsilon\right) \tag{3.2}$$

Assuming the wheelset is a rigid body, then u_1 and u_2 can be related via the wheelset's yaw rate $\dot{\theta}$ (Recall the IA Mechanics lectures on rigid body kinematics):

$$u_1 - u_2 = \left(d_1 + d_2\right)\dot{\theta} \tag{4}$$

It can be observed from Fig. S1 that

$$d_2 + d_1 = d / \cos\theta \tag{5}$$

Substituting (5) into (4) gives

$$u_1 - u_2 = \frac{2d}{\cos\theta}\dot{\theta} \tag{6}$$

Assuming that θ is small. This suggests that $\sin\theta = \theta$ and $\cos\theta = 1$. Therefore, (6) can be written as

$$u_1 - u_2 = 2d\dot{\theta} \tag{13}$$

Substituting (3.1) and (3.2) into (6) to eliminate u_1 and u_2 yields

$$\dot{\theta} = -\frac{\omega\tan\varepsilon}{d}y \tag{14}$$

Again by employing the kinematics analysis for a rigid body, $u_1 \sin\theta$ and $u_2 \sin\theta$ can be respectively related to $\dot{y}$, the lateral velocity of wheelset centre C via $\dot{\theta}$:

$$\begin{cases} u_1 \sin\theta = \dot{y} + d_1\dot{\theta}\sin\theta \\ u_2 \sin\theta = \dot{y} - d_2\dot{\theta}\sin\theta \end{cases} \tag{15}$$

Noting that $d_2 \neq d_1$. Specifically,

$$\begin{cases} d_1 = \dfrac{d+y}{\cos\theta} \\ d_2 = \dfrac{d-y}{\cos\theta} \end{cases} \tag{16}$$

However, when assuming $y \ll d$, $d_1 = d_2$ can be approximated. Consequently, adding the two equations involved in (15) together gives

$$(u_1 + u_2)\sin\theta = 2\dot{y} \tag{17}$$

Substituting (3.1) and (3.2) into (17) and assuming small angle θ yields

$$\dot{y} = \omega r\theta \tag{18}$$

Differentiating (18) and substituting the outcome into (14) to eliminate $\dot{\theta}$ gives

$$\ddot{y} + \frac{\omega^2 r \tan\varepsilon}{d} y = 0 \tag{19}$$

Noting that the rotation speed (angular speed) of the wheelset ω in (19) was early introduced as an intermediate variable. Using the relation between ω can be related to wheel central position radius r and wheelset forward u (both are given by the question) using

$$\omega = \frac{u}{r} \tag{20}$$

Substituting (20) into (19), and using $\tan\varepsilon = \varepsilon$ (for typical railway wheelset, $\tan\varepsilon \approx 1/20$) gives

$$\ddot{y} + \frac{u^2\varepsilon}{rd} y = 0 \tag{21}$$

(21) describes a simple harmonic motion associated with y, with the natural angular frequency being

$\omega_{\mathrm{n}} = u\sqrt{\dfrac{\varepsilon}{rd}}$, as requested, and the wavelength $\lambda = \dfrac{u}{\omega_{\mathrm{n}}/(2\pi)} = 2\pi\sqrt{\dfrac{dr}{\varepsilon}}$.

Below is a summary of the key assumptions made for deriving the natural angular frequency:

(a1) The wheelset is a rigid body. This was used to derive (4) and (15).

(a2) Small angle θ. This was used to obtain (13) and (18).

(a3) $y \ll d$. As a result of this assumption, $d_1 = d_2$ can be approximated. This was used to obtain

(a4) No slip between wheel and track. This was used to derive (3.1) and (3.2).

For part (a), students are expected to present at least (3.1), (3.2), (6), (14), (17), (18), and (21).

4(b): 35%

Following the lecture notes, the wheel #1's longitudinal creep velocities v_{X1} and wheel #2's longitudinal creep velocity v_{X2} can be respectively expressed as

$$v_{\mathrm{X1}} = u - d\left(\dot{\theta} + \frac{u}{R}\right) - r_1\frac{u}{r} \tag{22.1}$$

$$v_{\mathrm{X2}} = u + d\left(\dot{\theta} + \frac{u}{R}\right) - r_2\frac{u}{r} \tag{22.2}$$

The two wheels' lateral creep velocities v_{Y1} and v_{Y2} are the same:

$$v_{\mathrm{Y1}} = \dot{y} + u\theta \tag{23.1}$$

$$v_{Y2} = \dot{y} + u\theta \tag{23.2}$$

Directions of v_{X1}, v_{X2}, v_{Y1} and v_{Y2} are shown in Fig. S2.

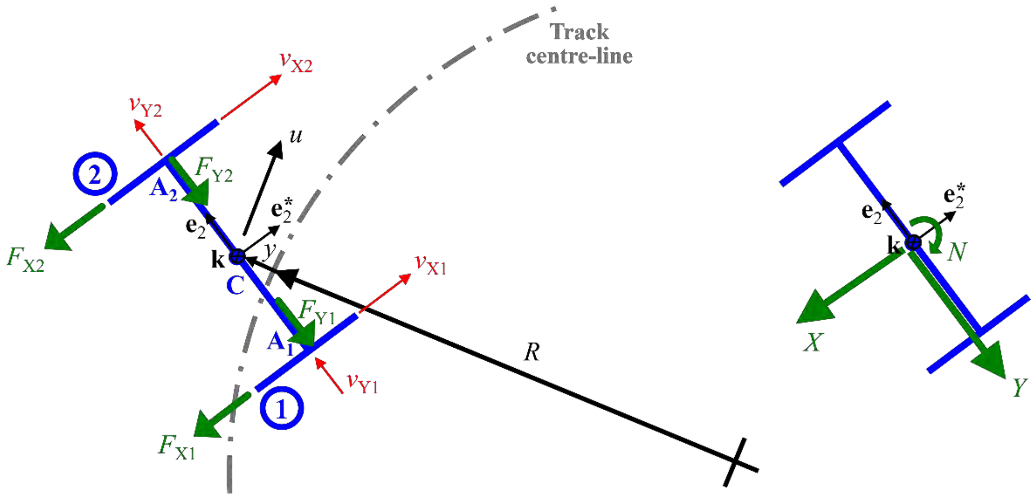


Fig. S2

Continuing to follow lecture notes, the longitudinal creep forces can be determined as:

$$F_{X1} = C\frac{v_{X1}}{u} = C\left[1 - \frac{d}{u}\left(\dot{\theta} + \frac{u}{R}\right) - \frac{r_1}{r}\right] \tag{24.1}$$

$$F_{X2} = C\frac{v_{X2}}{u} = C\left[1 + \frac{d}{u}\left(\dot{\theta} + \frac{u}{R}\right) - \frac{r_2}{r}\right] \tag{24.2}$$

The lateral creep forces can be determined as

$$F_{Y1} = C\frac{v_{Y1}}{u} = C\left(\frac{\dot{y}}{u} + \theta\right) \tag{25.1}$$

$$F_{Y2} = C\frac{v_{Y2}}{u} = C\left(\frac{\dot{y}}{u} + \theta\right) \tag{25.2}$$

Directions of F_{X1}, F_{X2}, F_{Y1} and F_{Y2} are shown in Fig. S2. Note that the direction of the creep forces is opposite to corresponding creep velocities.

The total longitudinal force X is therefore

$$X = F_{X1} + F_{X2} \tag{26}$$

Substituting (24.1), (24.2), (1.1) and (1.2) into (26) gives

$$X = 0 \tag{27}$$

as requested.

The total lateral force Y is therefore

$$Y = F_{Y1} + F_{Y2} \tag{28}$$

Substituting (25.1) and (25.2) into (28) gives

$$Y = 2C\left(\frac{\dot{y}}{u} + \theta\right) \tag{29}$$

as requested.

The total yaw moment can be calculated as

$$N = F_{X1}\,d - F_{X2}\,d \tag{30}$$

Substituting (24.1), (24.2), (1.1) and (1.2) into (30), and using $\tan\varepsilon = \varepsilon$ gives

$$N = 2dC\left(\frac{y\varepsilon}{r} - \frac{d\dot{\theta}}{u} - \frac{d}{R}\right) \tag{31}$$

as requested.

Directions of X, Y and N are shown in Fig. S2

Below is a summary of the key assumptions made for deriving the natural angular frequency:

(b1) The wheelset is a rigid body.

(b2) Small angle θ.

(b3) $y \ll d$. As a result, $d_1 = d_2$ can be approximated.

(b4) $\dot{y}\theta = 0$.

Comparison of assumptions made in parts (a) and (b): In part (a), the 'no-slip condition' assumption (a4) was made. This is not the case for part (b), where the resultant creep velocities are needed in order to work out the creep forces acting on each wheel and the net forces and moment acting on the entire wheelset. In addition, the assumption $\dot{y}\theta = 0$ was made in part (b) to simplify the derivation of creep velocities. This is a valid assumption because $\dot{y}\theta$ essentially represents a second-order small quantity (the product of two small quantities), which can be reasonably neglected.

For part (b), students are expected to present at least (22.1), (22.2), (23.1), (23.2), (24.1), (24.2), (25.1), (25.2) to demonstrate that they can reach the three equations provided in the question.

4(c)(i): 10%

A cant angle (bank angle) is created by elevating the outside rail comparing to the inside rail. It allows the train to be tilted, thus using a component of the train's weight to generate a centripetal force. This centripetal force enables the train to negotiating a curve, with little lateral force applied by the rail to the wheel of the train. Below are some brief explanations showing how a zero lateral force F between the rail and the wheel can be achieved using a cant angle.

When the outside and the inside rail are set to have the same elevation, a lateral force $F_a = Mu^2/R$, where u is the forward speed of the bogie and R is the radius of the curvature is applied to the wagon by the bogie to provide a centripetal force allowing the bogie to negotiate the circular path. This is illustrated in Fig. S3 (a).

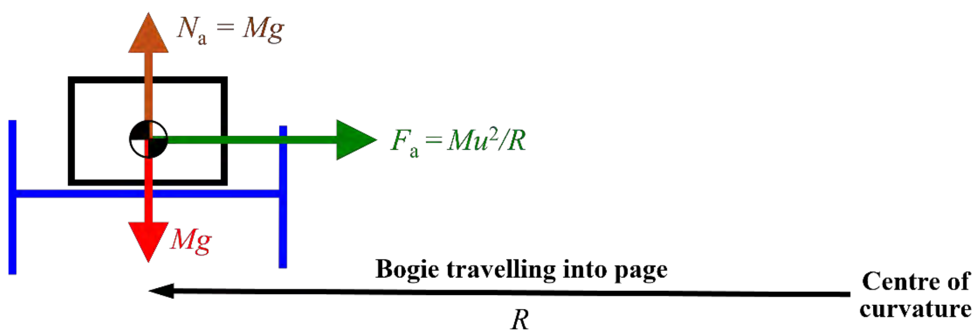


Fig. S3 (a)

When the outside rail is elevated with respect to the inside rail to form a cant angle (bank angle) β, as illustrated in Fig. S3 (b), the lateral force applied by the bogie to the wagon, F_b, will reduce to zero if $Mg \tan\beta$, the 'centripetal' component of the wagon's weight fully provide the centripetal force required. That is, $F_b = 0$ if $\tan\beta = u^2/(Rg)$.

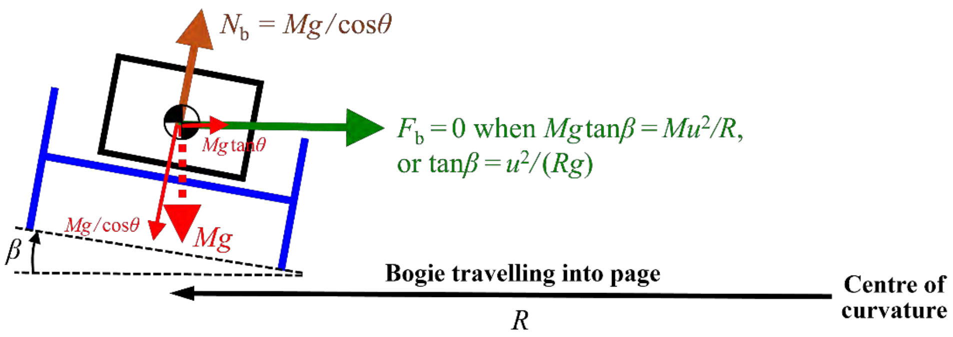


Fig. S3 (b)

4(c)(ii): 25%

Fig. S4 shows the lateral forces and yaw moments acting on the two wheelsets of the bogie when the bogie is running along a straight track at a forward speed u. y represents the lateral deviation of the bogie's centre of the gravity G with respect to track centre-line (i.e. the bogie's lateral tracking error at G). y_1 and y_2 respectively represents the lateral deviation at the front and rear wheelset's centre. θ is the yaw angle of the bogie. It can be seen from Fig. S4 that

$$\begin{cases} y_1 = y - a\theta \\ y_2 = y + a\theta \end{cases} \tag{32}$$

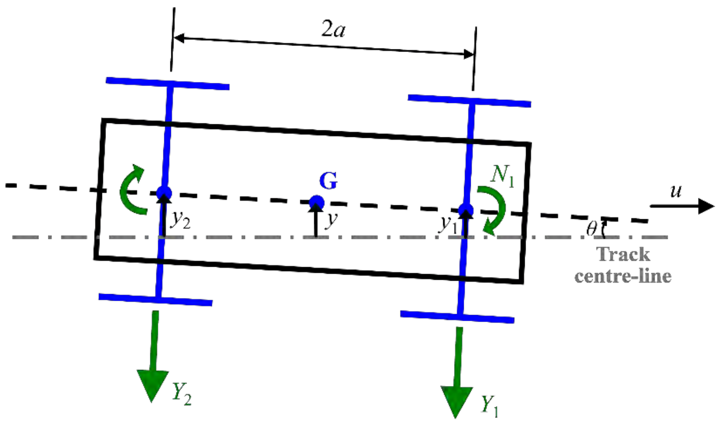


Fig. S4

The front wheelset's net yaw moment N_1 (about its own centre, due to creep forces) can be determined using (31), with the radius of curvature $R \to \infty$:

$$N_1 = 2dC\left(\frac{y_1\varepsilon}{r} - \frac{d\dot{\theta}}{u}\right) \tag{33.1}$$

(31) is given in part (b) of the question. By the same token, the rear wheelset's net yaw moment N_2 is:

$$N_2 = 2dC\left(\frac{y_2\varepsilon}{r} - \frac{d\dot{\theta}}{u}\right) \tag{33.2}$$

Similarly, the front and rear wheelsets' lateral forces Y_1 and Y_2 can be determined using (29):

$$Y_1 = 2C\left(\frac{\dot{y}_1}{u} + \theta\right) \tag{34.1}$$

$$Y_2 = 2C\left(\frac{\dot{y}_2}{u} + \theta\right) \tag{34.2}$$

For hunting motion analysis, mass and inertia of the bogie are assumed to be zero. Applying the 2nd Law of Newton to the lateral and yaw motion of the bogie gives:

$$0 = N_1 + N_2 + aY_1 - aY_2 \tag{35}$$

$$0 = Y_1 + Y_2 \tag{36}$$

Substituting (32), (33.1), (33.2), (34.1) and (34.2) into (35) gives

$$\dot{\theta} = \frac{\varepsilon du}{r\left(d^2 + a^2\right)} y \tag{37}$$

Substituting (32), (34.1) and (34.2) into (36) gives

$$\dot{y} = -u\theta \tag{38}$$

Combining (37) and (38) gives

$$\ddot{y} + \frac{\varepsilon du^2}{r\left(d^2 + a^2\right)} y = 0 \tag{39}$$

(39) describes a simple harmonic motion associated with y for the bogie, with the natural angular frequency being

$$\omega_{\mathrm{n}}^{\mathrm{Bogie}} = u\sqrt{\frac{\varepsilon d}{r\left(d^2 + a^2\right)}} .$$

The wavelength is therefore

$$\lambda^{\mathrm{Bogie}} = \frac{u}{\omega_{\mathrm{n}}^{\mathrm{Bogie}} / (2\pi)} = 2\pi\sqrt{\frac{dr}{\varepsilon}}\sqrt{1 + \frac{a^2}{d^2}} .$$

Comparing the expression for bogie wavelength λ^{Bogie} with a single wheelset's wavelength λ obtained from part (a) of the question, it can be found that λ^{Bogie} and λ can be related using the following expression:

$$\lambda^{\mathrm{Bogie}} = \lambda\sqrt{1 + \frac{a^2}{d^2}} .$$