Version DC/2

EGT3
ENGINEERING TRIPOS PART IIB

Thursday 29 April 2021 1.30 to 3.10

Module 4C8

VEHICLE DYNAMICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet and at the top of each answer sheet.*

STATIONERY REQUIREMENTS
Write on single-sided script paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachment: 4C8 data sheet, 2017 (3 pages).
You are allowed access to the electronic version of the Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.

The time taken for scanning/uploading answers is 15 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

1 Figure 1 shows a roll-plane vehicle model. The model is used to calculate the vertical acceleration $\ddot{z}_s$ and roll acceleration $\ddot{\theta}_s$ at the centre of the sprung mass, and the vertical acceleration $\ddot{z}_p$ at a distance p from the centre of mass. The inputs from a randomly rough road surface are the vertical displacements z_L and z_R at the left and right tyres. In addition to the parameter values given in the figure, k_{lat} = 178 kN m^{-1} and c_{lat} varies in inverse proportion to the speed of the vehicle.

Figure 2 shows the mean square spectral densities of the sprung mass acceleration responses for a vehicle speed of 30 m s^{-1} and c_{lat} = 3 kN s m^{-1}. Figure 3 shows the responses for a vehicle speed of 1 m s^{-1} and c_{lat} = 90 kN s m^{-1}. The vertical velocity input from the road surface is white noise and is the same for both speeds.

The vehicle is modified by adding an anti-roll bar to the suspension (not shown), which provides infinite roll stiffness between the sprung and unsprung masses.

(a) Sketch a model with three degrees of freedom suitable for calculating the lateral-roll vibration response of the modified vehicle and determine suitable values for all the parameters of the model. [30%]

(b) By making appropriate simplifications of the lateral tyre model, derive equations of motion and determine the natural frequencies of the model in part (a) for:

(i) very high vehicle speed; [10%]

(ii) very low vehicle speed. [40%]

(c) Explain how the addition of the anti-roll bar changes the frequency and magnitude of resonances appearing in the spectral densities shown in Fig. 2 and Fig. 3. Use your results from part (b) to aid your explanation. [20%]

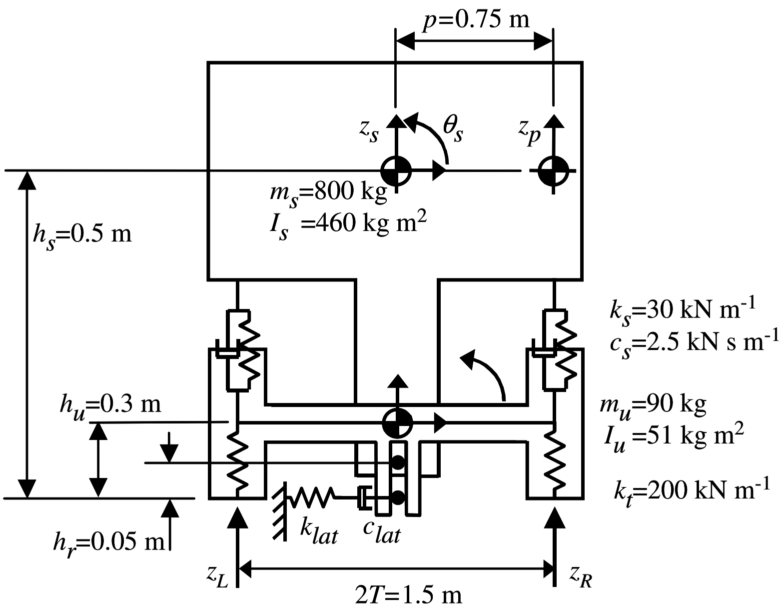

Fig. 1

(cont.

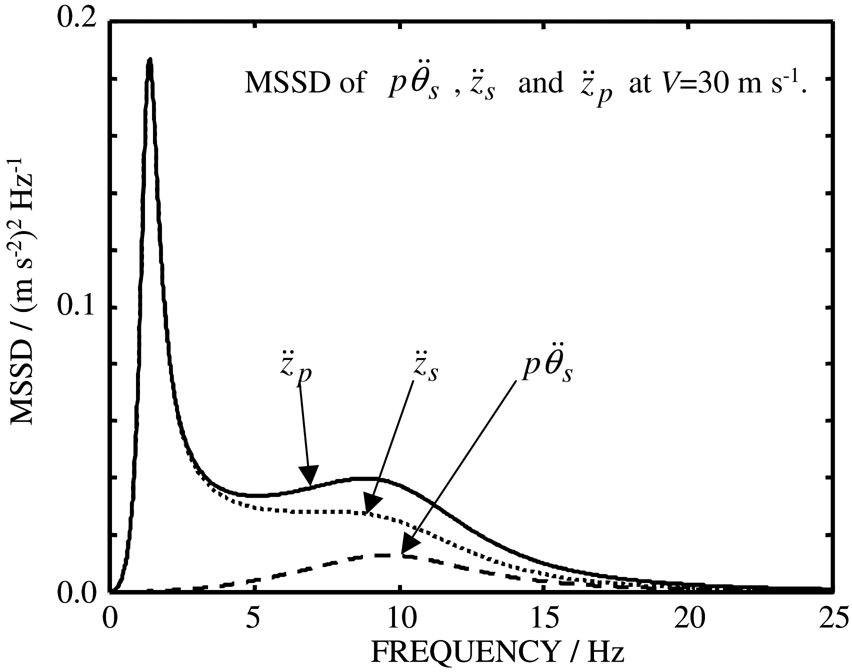
MSSD of $p\ddot{\theta}_s$, $\ddot{z}_s$ and $\ddot{z}_p$ at V=30 m s^{-1}.
MSSD / (m s^{-2})^2 Hz^{-1}
FREQUENCY / Hz
$\ddot{z}_p$
$\ddot{z}_s$
$p\ddot{\theta}_s$
0.2
0.1
0.0
0
5
10
15
20
25

Fig. 2

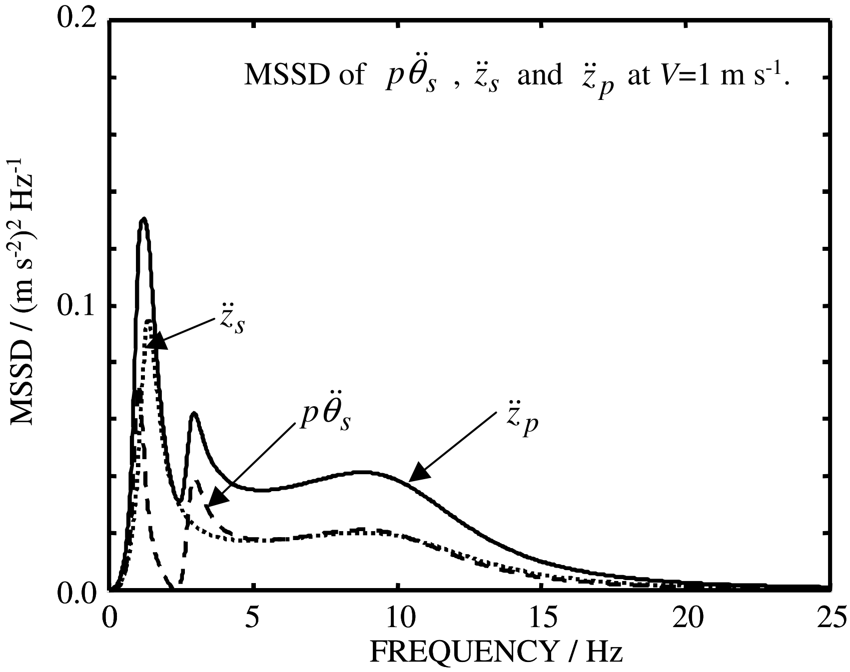
MSSD of $p\ddot{\theta}_s$, $\ddot{z}_s$ and $\ddot{z}_p$ at V=1 m s^{-1}.
MSSD / (m s^{-2})^2 Hz^{-1}
FREQUENCY / Hz
$\ddot{z}_s$
$p\ddot{\theta}_s$
$\ddot{z}_p$
0.2
0.1
0.0
0
5
10
15
20
25

Fig. 3

2 Figure 4 shows a concept of a three-axle off-road vehicle. The front axle is located at the front of the vehicle's body. The middle axle and rear axle are connected by a beam. The centre of the beam is located near the rear of the vehicle's body; the beam can pitch relative to the body. The arrangement is intended to allow all three wheels to remain in contact with a very rough surface.

(a) Figure 4 does not show any details of the suspension stiffness and damping elements that would be required in a practical vehicle. Sketch a suitable arrangement of springs and dampers, showing where the springs and dampers attach to the body, axles and beam. Give reasons for your chosen arrangement. [20%]

(b) Referring to Fig. 4 and neglecting any mass or suspension, show that the magnitude of the transfer function from road displacement input at the front axle (z_1) to vertical displacement of the body (z) is given by:

$$\left|\frac{z(j\omega)}{z_1(j\omega)}\right| = \sqrt{\frac{1 + 8\cos^2\left(\frac{1}{2}\frac{\omega a}{V}\right)\cos\left(\frac{\omega a}{V}\right)}{9}}$$

[50%]

(c) Find the values of $\omega a/V$ which give unity magnitude of the transfer function in part (b). Interpret your answer in terms of the wavelengths of z_1 that give unity magnitude. [15%]

(d) Find the values of $\omega a/V$ which give zero magnitude of the transfer function. [15%]

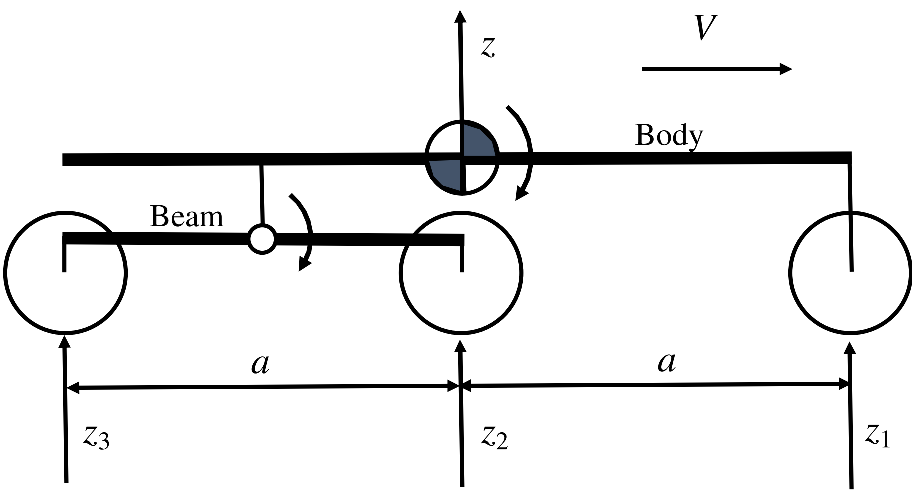

Fig. 4

3 (a) Explain why longitudinal creep forces are important in curving of railway bogies but not in the handling of cars. [15%]

(b) Explain how the lateral creep forces in a tyre-road contact depend on the motion of the wheel. [15%]

(c) The castered landing wheel of an aircraft is idealised as shown in plan view in Fig. 5. A flexible strut OP, with lateral stiffness k is fixed to the aircraft at O. A massless yoke PQ, of length a is free to rotate about a vertical axis in the strut at point P. A wheel of mass m, yaw moment of inertia I and lateral creep coefficient C is carried by an axle at G which is attached to the yoke. Point O moves in a straight line at forward speed u. Point P has the same longitudinal coordinate as O and can move laterally with displacement x. The yoke can yaw relative to the path of P by angle θ. The runway surface is smooth and level.

(i) Derive equations for small lateral oscillations of the system in the horizontal plane. Neglect gyroscopic effects. [35%]

(ii) Find the conditions for which the motion of the system is stable. [35%]

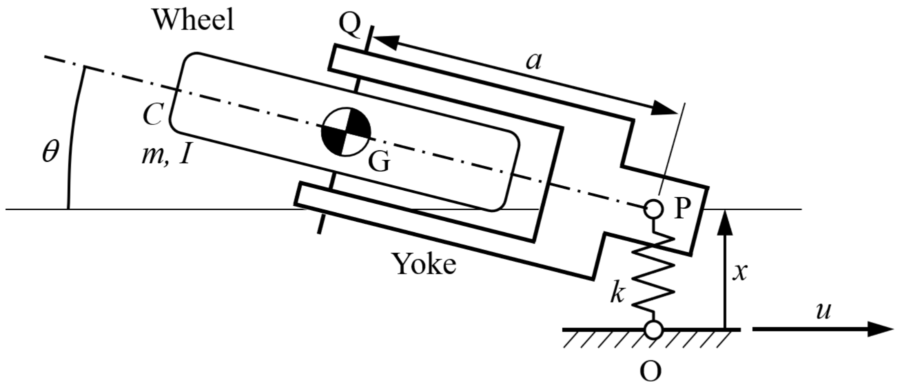

Fig. 5

4 It has been suggested that the performance of a car can be improved if the suspension geometry causes the rear wheels to adopt a steering angle ε proportional to the combined lateral force Y_r on the rear tyres. A 'bicycle' model of the car, as shown in Fig. 5, is used to investigate the effects of this passive rear wheel steering. In the model, which has the usual notation, the front wheels are steered at angle δ and the rear wheels adopt angle ε, where $\varepsilon = \gamma Y_r$ and γ is a constant. All angles should be assumed to be small.

(a) Assuming a lateral creep coefficient C for the combined response of the tyres on each axle, write an expression for the lateral force Y_f generated by the front axle and show that the lateral force Y_r generated by the rear axle is given by

$$Y_r = \frac{C}{u}\left(\frac{v - b\Omega}{1 + \gamma C}\right)$$ [30%]

(b) Derive equations of motion for the response of the vehicle to side force Y applied to the centre of mass and steer angle δ. Put these equations into matrix form

$$[M]\dot{\mathbf{y}} + [K]\mathbf{y} = \mathbf{F}, \quad \text{where} \quad \mathbf{y} = [v \quad \Omega]^{\mathrm{T}}$$

Define $[M]$, $[K]$ and $\mathbf{F}$. [20%]

(c) Without solving the equations of motion, explain how you would determine the following vehicle responses:

(i) the conditions for stable motion in a straight line; [10%]

(ii) the steady-state cornering behaviour, when a constant steer angle δ is applied and held; [10%]

(iii) the transfer function relating the side-slip response $\beta = v/u$ to a sinusoidal steer angle $\delta = \Delta \sin \omega t$. [10%]

(d) Derive an expression for the steady-state yaw rate Ω of the vehicle when it is subjected to a lateral force Y at the centre of mass. What value of γ will ensure that the vehicle is neutral steer? [20%]

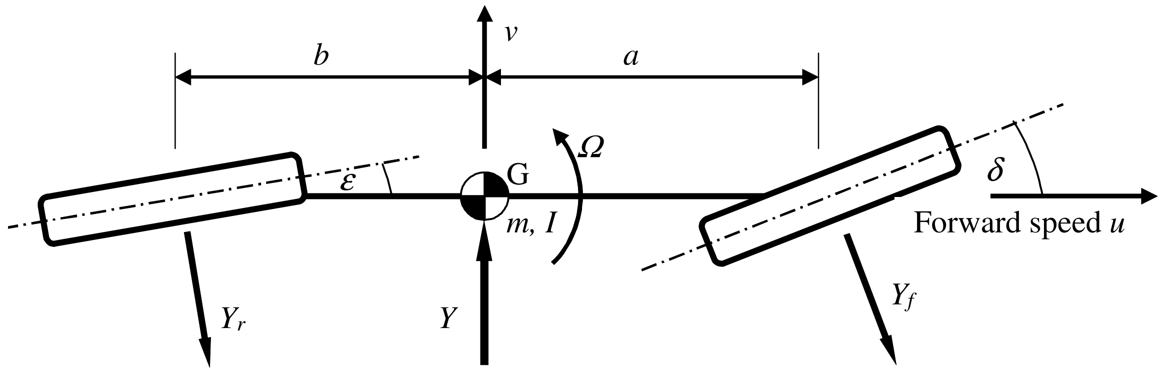

Fig. 5

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Engineering Tripos Part IIB

Data sheet for Module 4C8: Vehicle Dynamics

DATA ON VEHICLE DYNAMICS

1. Creep Forces in Rolling Contact

1.1 Surface tractions

Longitudinal force $X = \iint_A \sigma_x \, dA$

Lateral force $Y = \iint_A \sigma_y \, dA$

Realigning Moment $N = \iint_A \left(x\,\sigma_y - y\,\sigma_x\right) dA$

where

σ_x, σ_y = longitudinal, lateral surface tractions

x, y = coordinates along, across contact patch

A = area of contact patch

1.2 Brush model

$\sigma_x = K_x q_x, \; \sigma_y = K_y q_y$ for $\sqrt{\sigma_x^2 + \sigma_y^2} \le \mu p$

where

q_x, q_y = longitudinal, lateral displacements of 'bristles' relative to wheel rim

K_x, K_y = longitudinal, lateral stiffness per unit area

μ = coefficient of friction

p = local contact pressure

1.3 Linear creep equations

$$X = -C_{11}\xi$$

$$Y = -C_{22}\alpha - C_{23}\psi$$

$$N = C_{32}\alpha - C_{33}\psi$$

where X, Y, N, are defined as in 1.1 above.

C_{ij} = coefficients of linear creep

ξ = longitudinal creep ratio = longitudinal creep speed/forward speed

α = lateral creep ratio = (lateral speed /forward speed) - steer angle

ψ = spin creep ratio = spin angular velocity/forward speed

2. Plane Motion in a Moving Coordinate Frame

$$\ddot{\mathbf{R}}_{\mathbf{O}_1} = (\dot{u} - v\Omega)\mathbf{i} + (\dot{v} + u\Omega)\mathbf{j}$$

$(\mathbf{i}, \mathbf{j}, \mathbf{k})$ axis system fixed to body at point O_1

where

u = speed of point O_1 in $\mathbf{i}$ direction

v = speed of point O_1 in $\mathbf{j}$ direction

$\Omega\mathbf{k}$ = absolute angular velocity of body

3. Routh-Hurwitz stability criteria

$$\left(a_2 \frac{d^2}{dt^2} + a_1 \frac{d}{dt} + a_0\right) y = x(t)$$

Stable if all $a_i > 0$

$$\left(a_3 \frac{d^3}{dt^3} + a_2 \frac{d^2}{dt^2} + a_1 \frac{d}{dt} + a_0\right) y = x(t)$$

Stable if (i) all $a_i > 0$
and also (ii) $a_1 a_2 > a_0 a_3$

$$\left(a_4 \frac{d^4}{dt^4} + a_3 \frac{d^3}{dt^3} + a_2 \frac{d^2}{dt^2} + a_1 \frac{d}{dt} + a_0\right) y = x(t)$$

Stable if (i) all $a_i > 0$
and also (ii) $a_1 a_2 a_3 > a_0 a_3^2 + a_4 a_1^2$

DATA ON VEHICLE VIBRATION

Random Vibration

$$\mathrm{E}\left[x(t)^2\right]=\frac{1}{T}\int_{t=0}^{t=T} x^2(t)\,dt=\int_{\omega=-\infty}^{\omega=\infty} S_x(\omega)\,d\omega \qquad \text{(or } \int_{\omega=0}^{\omega=\infty} S_x(\omega)\,d\omega \quad \text{if } S_x(\omega) \text{ is single sided)}$$

$S_{\dot{x}}(\omega)=\omega^2 S_x(\omega)$ (Spectrum of time derivative of x is ω^2 times spectrum of x).

Single Input – Single Output

$$S_y(\omega)=\left|H_{yx}(\omega)\right|^2 S_x(\omega)$$

$$y(\omega)=H_{yx}(\omega)x(\omega)$$

Two Input – Two Output

$$\begin{Bmatrix} y_1(\omega) \\ y_2(\omega) \end{Bmatrix}=\begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix}\begin{Bmatrix} x_1(\omega) \\ x_2(\omega) \end{Bmatrix}$$

$$\begin{bmatrix} S_{11}^y(\omega) & S_{12}^y(\omega) \\ S_{21}^y(\omega) & S_{22}^y(\omega) \end{bmatrix}=\begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix}^*\begin{bmatrix} S_{11}^x(\omega) & S_{12}^x(\omega) \\ S_{21}^x(\omega) & S_{22}^x(\omega) \end{bmatrix}\begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix}^{\mathrm{T}}$$

* means complex conjugate, T means transpose

If x_1 and x_2 are uncorrelated:

$$S_{(x_1+x_2)}(\omega)=S_{x_1}(\omega)+S_{x_2}(\omega)$$

$$S_{12}^x(\omega)=S_{21}^x(\omega)=0$$

$$\mathrm{E}\left[(x_1(t)+x_2(t))^2\right]=\mathrm{E}\left[x_1(t)^2\right]+\mathrm{E}\left[x_2(t)^2\right]$$