Version NAF/2

EGT3
ENGINEERING TRIPOS PART IIB

Friday 25 April 2014 9.30 to 11

Module 4C9

CONTINUUM MECHANICS

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C9 Continuum Mechanics data sheet (6 pages).
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Evaluate the following (where δ_{ij} is the usual Kronecker delta and ε_{ijk} is the permutation symbol):

(i) $\delta_{ij}\delta_{ik}\delta_{jk}$

(ii) $\varepsilon_{ijk}a_j a_k$

(iii) $\varepsilon_{pqs}\varepsilon_{sqr}$ [30%]

(b) If B_{ij} is a skew-symmetric Cartesian tensor for which the vector $b_i = \frac{1}{2}\varepsilon_{ijk}B_{jk}$ show that $B_{pq} = \varepsilon_{pqi}b_i$. [20%]

(c) Using indicial notation show that $\tilde{a}\times(\tilde{b}\times\tilde{c}) = (\tilde{a}\cdot\tilde{c})\tilde{b} - (\tilde{a}\cdot\tilde{b})\tilde{c}$. [20%]

(d) For the deformation $x_1 = X_1$, $x_2 = X_2 + AX_3$ and $x_3 = X_3 + AX_2$, where A is a constant, compute the deformation gradient tensor, the finite Lagrangian strain tensor and the infinitesimal strain tensor. [30%]

2 (a) Use $\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\delta_{ij}\sigma_{kk}$ to derive the bulk modulus for uniform pressure $\sigma_{ij} = -p\delta_{ij}$, where the variables have their usual meaning. [30%]

(b) What is the Boussinesq solution and why is it useful? [20%]

(c) A plane strain, metal extrusion operation is shown in Figure 1, with the ratio of *h/w* between 1 and 2. The metal is rigid, ideally plastic with a shear yield strength *k*, and the dies are frictionless. Use the upper bound method to estimate the extrusion force *P*. Discuss how your answer would change when all interfaces are sticking. [50%]

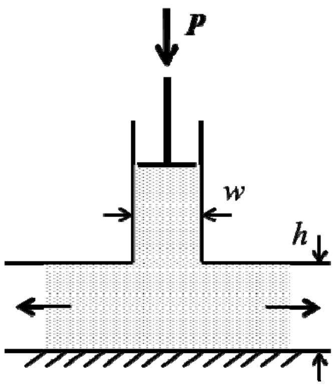

Fig. 1

3 (a) Explain the differences between the upper bound theorem, lower bound theorem and slip line field theory in plasticity theory. [20%]

(b) A material has a uniaxial stress versus plastic strain relation that is accurately described by the power-law relation $\sigma = A\varepsilon^N$ where A and N are material constants. It can be assumed that the material does not deform elastically, is incompressible, and yields according to the von Mises criterion, with the flow law

$$\dot{\varepsilon}_{ij}^{\mathrm{p}} = \frac{3}{2}\frac{s_{ij}}{\sigma_{\mathrm{e}}}\dot{\varepsilon}_{\mathrm{e}}^{\mathrm{p}}$$

where s_{ij} is the deviatoric stress, $\sigma_{\mathrm{e}} = \sqrt{(3/2)s_{ij}s_{ij}}$ is the effective stress and $\dot{\varepsilon}_{\mathrm{e}}^{\mathrm{p}} = \sqrt{(2/3)\dot{\varepsilon}_{ij}^{\mathrm{p}}\dot{\varepsilon}_{ij}^{\mathrm{p}}}$ is the effective strain rate. Show that this flow law satisfies normality. [20%]

(c) A thin-walled circular cylinder, with closed ends, is made from this material. The cylinder has initial radius R_0 and wall thickness t_0 and is subjected to an internal pressure p. Obtain an expression for the components of s_{ij} in terms of p, and thereby the relationship between the components of $\dot{\varepsilon}_{ij}^{\mathrm{p}}$ and $\dot{\varepsilon}_{\mathrm{e}}^{\mathrm{p}}$. [40%]

(d) Show that the wall thickness of the cylinder changes with pressure loading according to the relation

$$t = t_0 \exp\left(-\frac{\sqrt{3}}{2}\varepsilon_{\mathrm{e}}^{\mathrm{p}}\right)$$

and hence obtain the relationship between wall thickness t and pressure p. [20%]

END OF PAPER

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