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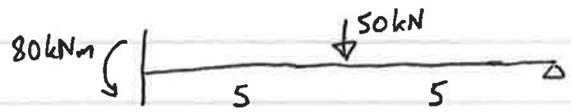
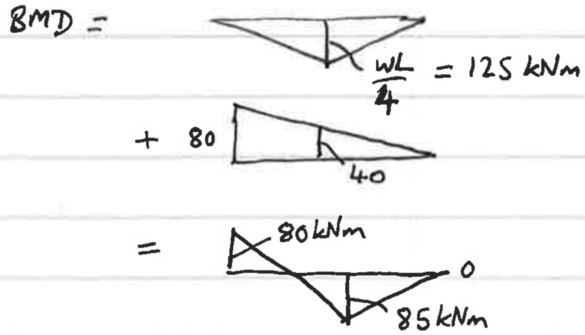
4D10 STRUCTURAL STEELWORK, 2014

Q1 a) Elastic-plastic interaction is taken account of by Perry-Robertson by largely avoiding it - i.e. first yield of any point on any cross-section is taken as the design "failure" criterion. An initial imperfection is assumed which grows as axial load is increased leading to bending and axial stress combinations → then check for axial load P which causes first yield.

b) The Perry-Robertson approach to LTB is complicated by the fact that there are <u>two</u> imperfections - a lateral one and a torsional one. One can pursue a Perry-Robertson analysis assuming two such imperfections and looking for first yield, but it gets complicated, and you need to make assumptions. Essentially EC3 handles this by just doing a curve fit to test data. - Also it rather cleverly uses $\sqrt{\text{Plastic}/\text{Elastic}}$ as the slenderness parameter such that column buckling curves can be used for LTB even though LTB does not fall away as $1/L^2$ as Euler buckling does. Residual stresses that arise due to differential cooling are treated similarly - i.e. it is just a curve fit, and there is no real attempt to actually look for first yield at any point.

c) 305 x 127 UB42, S275




∴ Critical span is RH span, as it is closer to "equal and opposite* end moments" case.

4D10 Q1 (c) i) cont'd

Compact?

305 x 127 UB42

S275

flange $\lambda = \left(\frac{124.3-8}{12.1}\right)/2 \sqrt{\frac{275}{355}} = 4.2 < 8$ OK

web (bending) $\lambda = \left(\frac{307.2 - 2(12.1)}{8}\right) \sqrt{\frac{275}{355}} = 31.1 < 56$ OK.

PLASTIC

$$M_{pl} = 275 \times 10^6 \text{ N/m}^2 \times 614 \times 10^{-6} \text{ m}^3 = \underline{\underline{168.9 \text{ kNm}}}$$

ELASTIC

$$M_{LTB} = \frac{\pi}{L} \sqrt{GJEI_{min}} \left[1 + \frac{\pi^2}{L^2}\frac{E\Gamma}{GJ}\right]^{1/2}$$

$J = 21.1 \text{ cm}^4$

$I_{min} = 389 \text{ cm}^4$

$G = 81 \text{ GPa}$

$E = 210 \text{ GPa}$

$$M_{basic} = \frac{\pi}{L}\sqrt{GJEI_{min}} = \frac{\pi}{5}(210 \times 10^9)\sqrt{\frac{81}{210} J.I_y}$$

$$= \frac{\pi}{5}(210 \times 10^9)\sqrt{\frac{81}{210}(21.1)(389) \times 10^{-8}}$$

$$= \underline{\underline{74.2 \text{ kNm}}}$$

$\Gamma = ID^2/4$ , D = dist. between flange centroids $= 307.2 - 12.1 = 295.1$ mm

$$\frac{\Gamma}{J} = \left(\frac{389}{21.1}\right)\frac{(0.2951)^2}{4} \text{ m}^2 = 0.401 \text{ m}^2$$

$$\frac{\pi^2}{L^2}\frac{E}{G}\frac{\Gamma}{J} = \frac{\pi^2}{25 \text{ m}^2} \cdot \frac{210}{81} \cdot 0.401 \text{ m}^2 = 0.4108$$

$$\sqrt{1 + \frac{\pi^2}{L^2}\frac{E\Gamma}{GJ}} = \sqrt{1 + 0.4108} = 1.1878$$

$$\therefore \quad M_{LTB} = (74.2 \text{ kNm}) \times 1.1878 = \underline{\underline{88.1 \text{ kNm}}}$$

$C_{unequal} = 0.6$ for $\psi = 0$ (DS3)

$$\therefore \quad M_{cr} = \frac{M_{LTB}}{C_{unequal}} = \frac{88.1}{0.6} = \underline{\underline{146.9 \text{ kNm}}}$$

4D10 Q1(c) cont'd.

$$\lambda = \sqrt{\frac{\text{Plastic}}{\text{Elastic}}} = \sqrt{\frac{168.9}{146.9}} = 1.072$$

Which curve? Rolled, $h/b = \frac{307.2}{124.3} = 2.47 > 2$

$\therefore$ curve (b) DS3

From DS1 $\chi \approx 0.55$ for $\lambda = 1.072$

$$\therefore M_{DESIGN} = \chi M_{PL} = 0.55(168.9) = \underline{\underline{93\ kNm}}$$

$\therefore$ OK for 85 kNm

c ii) But if end moment removed, central moment = 125 kNm all other parameters remain the same $\therefore$ 93 kNm strength is <u>NOT</u> adequate.

c iii) There are many conservative assumptions throughout, but in particular here we could argue that the LH ~~beam~~ half provides some torsional restraint to the RH half across the central point. This adds stiffness, so LTB moment is probably a bit higher.

4D10 2014

Q2 457 x 191 UB 82 beam column, S355, 10 m long

Assume compact.

a) $M_{PL(MAJ)} = 355 \times 10^6 \, N/m^2 \times 1831 \times 10^{-6} m^3 = 650 \, kNm$

$N_{PL} = 355 \times 10^6 \, N/m^2 \times 104 \times 10^{-4} m^2 = 3692 \, kN$

Webfraction: Total area $= 104 \, cm^2$

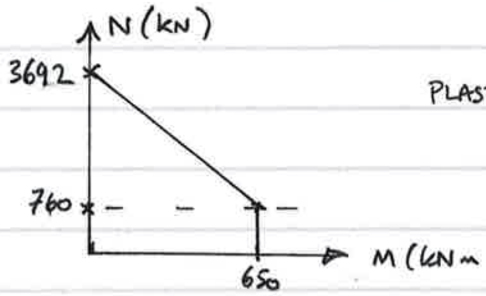
Flange area $= 2 \times 191.3 \times 16 \, mm^2 = 6122 \, mm^2 = 61.2 \, cm^2$

$\therefore$ web area $= 104 - 61.2 = 42.8 \, cm^2$

web fraction $a = \frac{42.8}{104} = 0.411$

half web fraction $= 0.206$

$\therefore$ kink at $0.206 \times 3692 = 760 \, kN$.




PLASTIC ENVELOPE.

b)


$I_{MAJ} = 37050 \times 10^{-8} m^4$

$I_{MIN} = 1871 \times 10^{-8} m^4$

$$N_{Euler, MAJ} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (210 \times 10^9)(37050 \times 10^{-8})}{10^2} = \underline{\underline{7679 \, kN}}$$

$$N_{Euler, MIN} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (210 \times 10^9)(1871 \times 10^{-8})}{5^2} = \underline{\underline{1551 \, kN}}$$

Slenderness: MAJOR: $\lambda = \sqrt{\frac{\text{Plastic}}{\text{Elastic}}} = \sqrt{\frac{3692}{7679}} = 0.6934$

MINOR: $\lambda = \sqrt{\frac{\text{Plastic}}{\text{Elastic}}} = \sqrt{\frac{3692}{1551}} = 1.5429$

4D10 2014, Q2 cont'd.

DS2: $h/b = 460/191.3 = 2.4 > 1.2$, flange $T = 16\text{mm} < 40\text{mm}$

S355. Major – about y-y → curve (a)

Minor – about z-z → curve (b)

DS1. Major: $\lambda = 0.6934 \rightarrow \chi \simeq 0.85$ curve (a)

Minor: $\lambda = 1.5429 \rightarrow \chi \simeq 0.33$ curve (b)

$$\therefore N_{cr,maj} = 0.85\,(3692) = \underline{\underline{3138\text{ kN}}}$$

$$N_{cr,min} = 0.33\,(3692) = \underline{\underline{1218\text{ kN}}}$$

**Moments:** PLASTIC $M_{PL,MAJ} = 650\text{ kNm}$ (earlier)

ELASTIC $M_{basic} = \frac{\pi}{L}\sqrt{GJEI_{min}}$

$J = 69.2\text{ cm}^4$

$I_{minor} = 1871\text{ cm}^4$

$L = 5\text{m}$ (LTB eff. leng)

$G = 81\text{ GPa}$

$E = 210\text{ GPa}$

$$= \frac{\pi}{5}(210\times10^9)\sqrt{\frac{81}{210}\,.\,69.2\,.\,1871} \times 10^{-8}$$

$$= \frac{\pi}{5}(2100)(223.5) = \underline{\underline{294.9\text{ kNm}}}$$

$$\Gamma = \frac{ID^2}{4}, \quad D = 460 - 16 = 444\text{mm}$$

$$I_{minor} = 1871\text{ cm}^4$$

$$\frac{\Gamma}{J} = \frac{1871\text{ cm}^4}{69.2\text{ cm}^4}\,\frac{(0.444)^2\text{ m}^2}{4} = 1.333\text{ m}^2$$

$$\frac{\pi^2}{L^2}\frac{E}{G}\frac{\Gamma}{J} = \frac{\pi^2}{25\text{ m}^2}\,\frac{210}{81}\,1.333 = 1.3639$$

$$\sqrt{1 + \frac{\pi^2}{L^2}\frac{E}{G}\frac{\Gamma}{J}} = \sqrt{1 + 1.3639} = 1.5375$$

$$M_{LTB} = M_{basic} \times \sqrt{\phantom{x}} = 294.9\text{ kNm} \times 1.5375 = \underline{\underline{453.4\text{ kNm}}}$$

$C_{unequal} = 1$ (equal + opp end moments)

$$\therefore \underset{\text{ELASTIC}}{M_{cr}} = M_{LTB} = \underline{\underline{453.4\text{ kNm}}}$$

4D10 2014

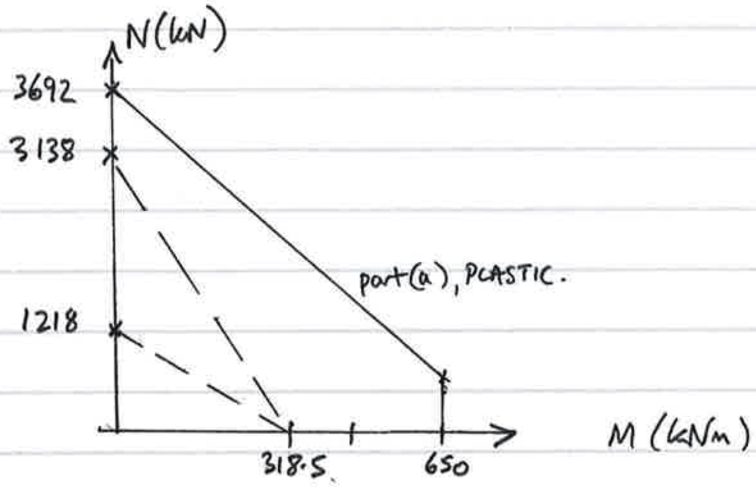
Q2 (cont'd).

Which curve?

Rolled I $\quad \frac{h}{b} = 2.4 > 2 \rightarrow$ curve (b) DS3

$$\chi \approx 0.49 \quad (DS1)$$

$$M_{cr} = 0.49(650) = \underline{\underline{318.5\ kNm}}$$



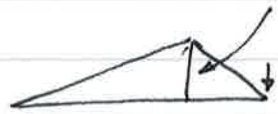
4D10 Structural Steelwork 2014

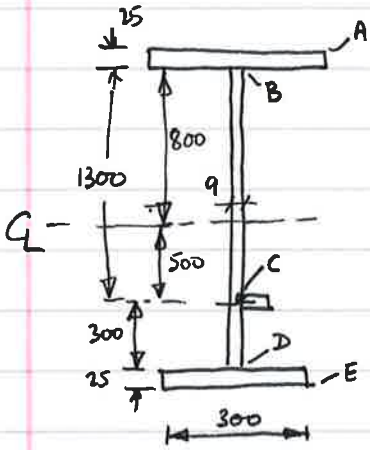
Q3 a)


$$3.2(1200) = 9F \Rightarrow F = \frac{3.2(1200)}{9} = 426.7\ \text{kN}$$

$$R = F + 1200 = \underline{1627\ \text{kN}}$$

$$\underset{\text{at support}}{BM} = 1200\ \text{kN} \times 3.2\ \text{m} = 3840\ \text{kNm}$$






$$I = \frac{1}{12}\left(1650^3(300) - (1600)^3(291)\right) = \underline{12.98 \times 10^9\ \text{mm}^4}$$

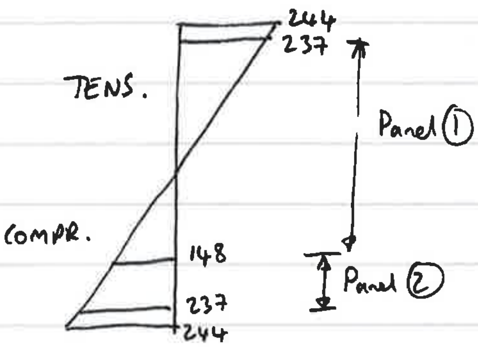
$$\sigma = \frac{My}{I} = \frac{(3840 \times 10^6\ \text{Nmm})\,y}{12.98 \times 10^9\ \text{mm}^4} = 0.2958\,y$$

($y$ in mm, $\sigma$ in MPa)

At A, E; $y = 825$ mm $\sigma = 244$ MPa

B, D; $y = 800$ mm $\sigma = 237$ MPa

C; $y = 500$ mm $\sigma = 148$ MPa




Shear stress $\tau = \frac{1200 \times 10^3\ \text{N}}{1600 \times 9} = 83.3\ \text{N/mm}^2$.

Bottom flange in compression.

Is it compact?

$$\lambda = \frac{b}{t}\sqrt{\frac{\sigma_y}{355}} = \frac{150 - 4.5}{25} = 5.82 < 8 \quad \therefore \text{OK}$$

and $\sigma < \sigma_y$ $\therefore$ OK.

Check local panel strength at B and D:

$\sigma \le \sqrt{\sigma_y^2 - 3\tau^2}$ ? (DS4) $\sigma^2 + 3\tau^2 \le \sigma_y^2$ ?

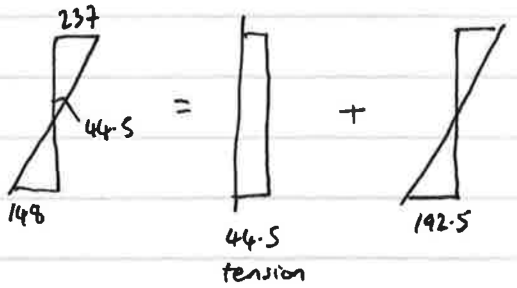
$$\left(\frac{\sigma}{\sigma_y}\right)^2 + 3\left(\frac{\tau}{\sigma_y}\right)^2 \le 1 \ ? \qquad \left(\frac{237}{355}\right)^2 + 3\left(\frac{83.3}{355}\right)^2$$

$$= 0.4457 + 3(0.055) = 0.61 < 1 \quad \therefore \text{OK}$$

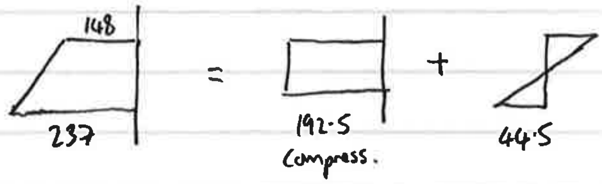
✓

4D10 Q3 cont'd.

Panel (1)




Panel (2)




$\lambda = \frac{b}{t}\sqrt{\frac{\sigma_y}{355}}$

(1) $\lambda = \frac{1300}{9} = 144$ $(> 56)$

(2) $\lambda = \frac{300}{9} = 33.3$ $\left(\begin{matrix} >24 \text{ but} \\ <56 \end{matrix}\right)$

| | $K_c$ | $K_b$ | $K_q$ |
|---|---|---|---|
| (1) | $\boxed{0^* \ (n/a)}$ | 0·91 | 0·36 |
| (2) | 0·82 | 1·21 | 1·0 |

DS3

Stability: $\frac{\sigma_c}{K_c \sigma_y} + \left(\frac{\sigma_b}{K_b \sigma_y}\right)^2 + \left(\frac{\tau}{K_q \sigma_y/\sqrt{3}}\right)^2$

(1) $= 0^* + \left(\frac{192.5}{0.91(355)}\right)^2 + \left(\frac{83.3\sqrt{3}}{0.36(355)}\right)^2$

$= 0^* + 0.355 + 1.27 = \underline{\underline{1.63 > 1}}$

$\underline{\underline{\text{FAIL.}}}$

(2) $= \frac{192.5}{0.82(355)} + \left(\frac{44.5}{1.21(355)}\right)^2 + \left(\frac{83.3\sqrt{3}}{1.0(355)}\right)^2$

$= 0.66 + 0.01 + 0.16 = \underline{0.83}$ $\therefore$ OK

So panel (1) fails. SUBTLETY: Panel (1) is in TENSION + BENDING so $K_c$ factors inappropriate – so best set first term in stability eqn to zero, rather than trying to use $K_c$ factors with $\sigma_c$ = negative.

To strengthen, add another stiffener into panel (1)

[Could also try moving the original stiffener up a bit, since panel (2) has a bit of spare capacity – but adding a stiffener is preferred]

4D10 Q3 cont'd.

Stress at stiffener = 148 MPa (from earlier)

Longitudinal fillet weld with transverse crack = Class D (DSS)

| $m$ | $K_2$ | $\sigma_0$ |
|---|---|---|
| 3 | $1.52 \times 10^{12}$ | 53 |

$\sigma_r > \sigma_0$

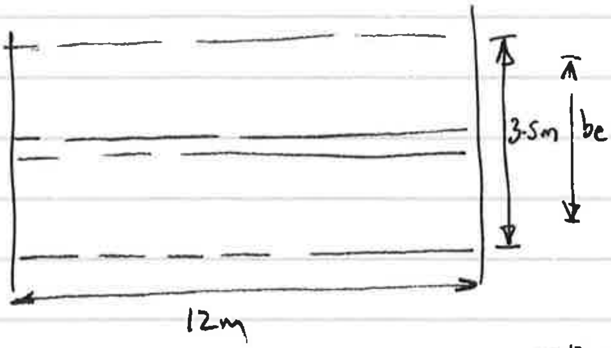
$$N \sigma_r^m = K_2$$

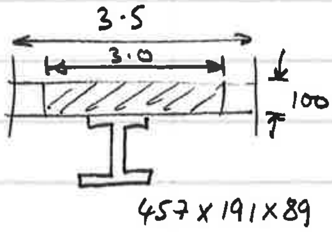
$$N = \frac{1.52 \times 10^{12}}{(148)^3} = 0.469 \times 10^6 \text{ cycles}$$

( ≈ ½ a million ).

4D10 Steel 2014

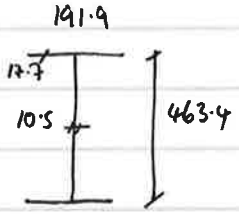
Q4. a)




$$\text{effective slab width} = \min\left(3.5, \frac{\text{span}}{4}\right) = \underline{\underline{3.0m}}$$

(span = 12m)




$$\lambda_{flange} = \frac{b}{t}\sqrt{\frac{355}{355}} = \frac{191.9 - 10.5}{(17.7)(2)} = 5.1 < 8 \quad \checkmark \text{OK}$$

$$\lambda_{web} = \frac{b}{t}\sqrt{1} = \frac{463.4 - 2(17.7)}{10.5} = 40.8 > 25$$

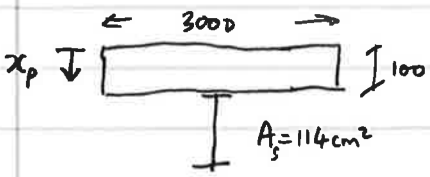
but < 56

∴ web not compact in compression
but compact in bending, so OK here.

| | | | |
|---|---|---|---|
| Permanent Load: Concrete | $24 kN/m^3 \times 0.1m \times 3.5m$ | = | $8.4\ kN/m$ |
| Steel | $89.3\ kg/m$ | = | $0.88\ kN/m$ |
| Finishes | $1 kN/m^3 \times 3.5m$ | = | $3.5\ kN/m$ |
| | | | $12.78\ kN/m$ |
| Imposed Load | $6 kN/m^2 \times 3.5m$ | = | $21\ kN/m$ |

$$\text{Design load} = 1.35(12.78) + 1.5(21) = 17.25 + 31.5 = \underline{\underline{48.8\ kN/m}}$$

$$BM = \frac{wL^2}{8} = \frac{48.8 \times 12^2}{8} = \underline{\underline{878.4\ kNm}}$$




Assume N/A in slab:

$$x_p = \frac{A_s f_{sy}}{0.6\, b_e f_{cd}} = \frac{114 \times 10^2 \times 355}{0.6(3000)(30)} = \underline{74.9\ mm}$$

∴ in slab ✓

$$M_{ult} = (114 \times 10^2)\,mm^2 \times 355 \left[\frac{463.4}{2} + 100 - \frac{74.9}{2}\right] mm \times N/mm^2$$

$$= \underline{\underline{1191\ kNm}} > 878.4\ kNm \quad \therefore \underline{\underline{OK}}$$

4D1 Steel 2014

Q4 b) Studs. For full strength need studs in each half span to be capable of carrying the UB axial load ($A_s\sigma_y$)

$$A_s\sigma_y = (114\times10^2)(355) \qquad mm^2 \quad N/mm^2$$
$$= 4047 \text{ kN}$$

Studs: 13mm dia, 65mm high → 47kN each DS6

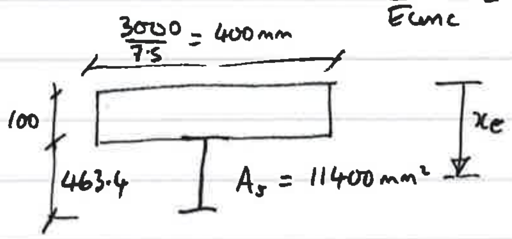
$\therefore$ No. req'd for whole beam $= \dfrac{2\times 4047}{47} = 172$

Spacing $= \dfrac{12000}{172} = 70$mm. singly

(or place (more) in pairs at 140mm × 0.8 = 112 say $\underline{\underline{110\text{mm}}}$)
80% efficient

c) Deflection. $E_{conc} = 28$ kN/mm² (short term) DS6

$$\frac{E_{steel}}{E_{conc}} = \frac{210}{28} = 7.5$$




centroid $x_c = \int x\,dA \Big/ \int dA$

$$= \frac{(400)(100)(50) + (11400)(100 + 463.4/2)}{40000 + 11400}$$

$$= \underline{112.5\text{mm}}$$

$$I_s = \underset{\text{steel}}{41020\times10^4} + \underset{\text{steel // axis}}{11400\left(\frac{463.4}{2} - 12.5\right)^2} + \underset{\text{conc}}{\frac{(400)(100)^3}{12}} + \underset{\text{conc // axis}}{400(100)(112.5-50)^2}$$

$$= 1147.6\times10^6 \text{ mm}^4.$$

$$\delta = \frac{5}{384}\,\frac{(21\times12)(12)^3\times10^9}{(210)(1147\times10^6)} = 23.5\text{mm}$$

$$\left(\text{cf } \frac{\text{span}}{250} = \frac{12000}{250} = 48, \text{ so deflection OK}\right).$$