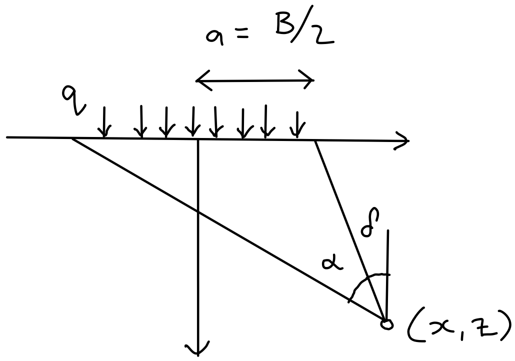
1). a. $z = 10\,m$; $B = 10\,m$



$$\tan\delta = \frac{x - 0.5B}{z} = \frac{10 - 0.5\times10}{10} = 0.5$$

$\therefore\ \delta = 0.46$ rad.

$$\tan(\alpha+\delta) = \frac{x + 0.5B}{z} = \frac{10 + 0.5\times10}{20} = 1.5$$

$\alpha + \delta = 0.98$ rad.

$\therefore\ \alpha = 0.52$ rad.

$$\sigma_v = \frac{q}{\pi}\left[\alpha + \sin\alpha\cos(\alpha + 2\delta)\right]$$

$$= \frac{100}{\pi}\left[0.52 + \sin 0.52 \cos\left(0.52 + (2\times 0.46)\right)\right]$$

$$= 18.6\ \text{kPa}$$

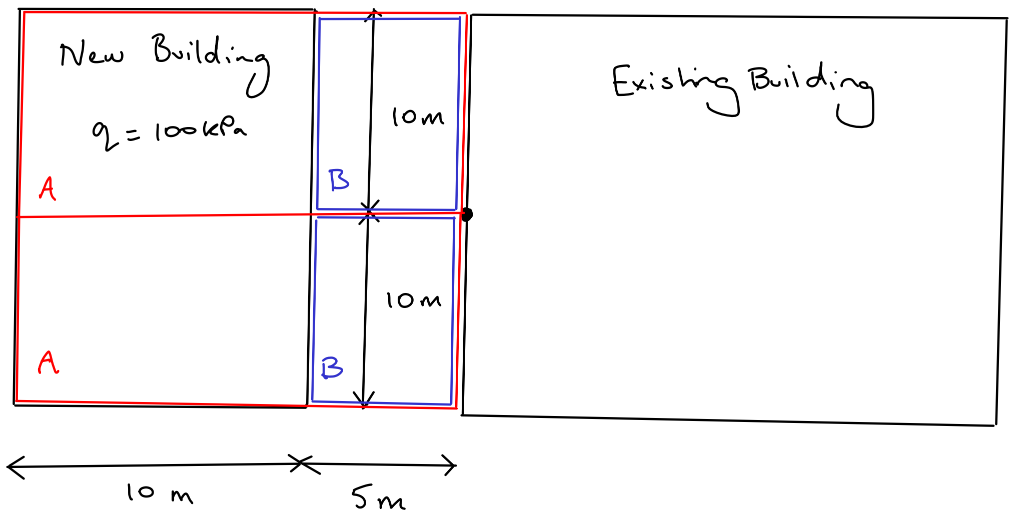
$$\sigma_h = \frac{q}{\pi}\left[\alpha - \sin\alpha\cos(\alpha + 2\delta)\right]$$

$$= \frac{100}{\pi}\left[0.52 - \sin 0.52 \cos\left(0.52 + (2\times 0.46)\right)\right]$$

$$= 14.5\ \text{kPa}$$

[20%]

b.



$\sigma_v = 2\,I_{RA} - 2\,I_{RB}$ (by superposition)

for area A: $m = \frac{L}{z} = \frac{15}{10} = 1.5$; $n = \frac{B}{z} = \frac{10}{10} = 1$

for area B: $m = \frac{L}{z} = \frac{10}{10} = 1$; $n = \frac{B}{z} = \frac{5}{10} = \frac{1}{2}$

from Fadum chart: $I_{RA} = 0.19$ and $I_{RB} = 0.12$

$\therefore \sigma_v = 2 \times 100 \times 0.19 - 2 \times 100 \times 0.12$

$= 14$ kPa.

[20%]

c. The stress derived via Fadum's chart is lower because the shape of the foundation is accounted for explicitly via superposition. The lower the L/B ratio for the foundation, the lower the vertical stress at any given location relative to the strip solution where $L/B = \infty$.

[10%]

d. $\nu = 0.3$; $E = 50$ MPa

$$w_{corner} = \frac{(1-\nu)}{G}\frac{qB}{2} I_{rect}$$

$$G = \frac{E}{2(1+\nu)} = \frac{50{,}000}{2(1+0.3)} = 19{,}230 \text{ kPa}$$

where: $\frac{L}{B} = \frac{20}{10} = 2 \quad \therefore I_{rect} = 0.766$

$$\therefore w_{corner} = \frac{(1-0.3)}{19{,}230}\frac{100 \times 10}{2} \times 0.766$$

$$= 0.0139 \text{ m}$$

$$= 13.9 \text{ mm}$$

$$w_{center} = 4 \times \frac{1}{2} \times w_{corner} = 27.8 \text{ mm}$$

[20%]

e. $$w_r = \frac{(1-\nu)}{G}\frac{q_{avg}\sqrt{BL}}{2} I_{rgd}$$

<u>where</u> $0.7 \leq I_{rgd} \leq 0.9$ <u>for</u> $10 \geq L/B \geq 1$

$\therefore I_{rgd} \approx 0.88$ (by linear interpolation)

$$w_r = \frac{(1-0.3)}{19{,}230} \times \frac{100 \times \sqrt{20 \times 10}}{2} \times 0.88 = 0.0226 \text{ m}$$

$$= 22.65 \text{ mm}$$

[20%]

f. $w_{corner} < w_r < w_{center}$ as expected because a rigid foundation will generate stress concentrations at the periphery that will moderate the average settlement somewhat.

[10%]

This was a popular question that was answered by most students with little difficulty. The most common mistake was in the trigonometry in part a, or in students using degrees instead of radians.

2) a. $B = 200\ mm$; $L = 400 mm$; $\phi = 35°$; $z = 100\ mm$; $\gamma = 20\ kN/m^3$

$\gamma' \simeq \gamma - 10 = 20 - 10 = 10\ kN/m^3$

$q_f = s_q N_q \sigma'_{vo} + s_\gamma N_\gamma \frac{\gamma' B}{2}$

$s_q = 1 + (B \sin\phi)/L$

$= 1 + (200 \sin 35)/400$

$= 1.28$

$N_q = \tan^2(\pi/4 + \phi/2) e^{(\pi \tan\phi)}$

$= \tan^2\left(\pi/4 + \frac{\pi}{180} \cdot \frac{35}{2}\right) e^{\left(\pi \tan \frac{\pi}{180} \cdot 35\right)}$

$= 33.3$

$\sigma'_{vo} = \gamma' z = 10 \times 0.1 = 1\ kPa$

$s_\gamma = 1 - 0.3\ B/L$

$= 1 - 0.3 \times 200/400$

$= 0.78$

$N_\gamma = 2(N_q - 1) \tan\phi$

$= 2(33.3 - 1) \tan\left(\frac{\pi}{180} \cdot 35\right)$

$= 46.4$

$\therefore q_f = 1.28 \times 33.3 \times 1 + 0.78 \times 46.4 \times \frac{10 \times 200}{2 \times 1000}$

$= 78.81\ kPa$

$\therefore V_{ult} = q_f B L = 78.81 \times 0.2 \times 0.4 = 6.30\ kN$ [40%]

b. $$\left[\frac{H/V_{ult}}{t_h}\right]^2 = \left[\frac{V}{V_{ult}}\left(1-\frac{V}{V_{ult}}\right)\right]^2$$ where $t_h = 0.5$

$$V = \frac{25}{4} = 6.25 \text{ kN}$$

$$\frac{H/V_{ult}}{t_h} = \frac{V}{V_{ult}}\left(1-\frac{V}{V_{ult}}\right)$$

$$\therefore \quad H = V\left(1-\frac{V}{V_{ult}}\right)t_h = 6.25\left(1-\frac{6.25}{6.30}\right)\times 0.5$$

$$= 0.025 \text{ kN} \rightarrow \Sigma H = 4\times 0.025 = 0.1 \text{ kN}$$

[20 %]

c. $H = V\left(1-\frac{V}{V_{ult}}\right)t_h$ is a maxima when $V/V_{ult} = 0.5$

$$\therefore \quad V = 0.5 V_{ult} = 0.5\times 7.44 = 3.72 \text{ kN}$$

$$H = 3.72\left(1-\frac{3.72}{7.44}\right)0.5 = 0.93 \text{ kN}$$

$$\Sigma H = 4\times 0.93 = 3.72 \text{ kN}$$

Mass to remove:

$$M = 4\times(6.25-3.72)\cdot\frac{1000}{9.81} = 1032 \text{ kg}$$

[20 %]

d. Contact patch size could be increased by deflating the tyres, or the surcharge could be increasing by heaping sand either side of the wheel (but not in front or behind it!).
Alternatively, if the tide is falling, could wait for the groundwater table to fall, which would elevate the effective stress in the soil beneath the wheel.
All of the above would result in an increase in Vult, which would increase the H that could be mobilised.

[20%]

This question was tackled by a majority of students and generally answered fairly well. Some appeared confused about how to calculate the maximum horizontal capacity, but the written answers in parts c and d demonstrated a good understanding of the physics of the problem at hand in most cases.

QUESTION 3 :

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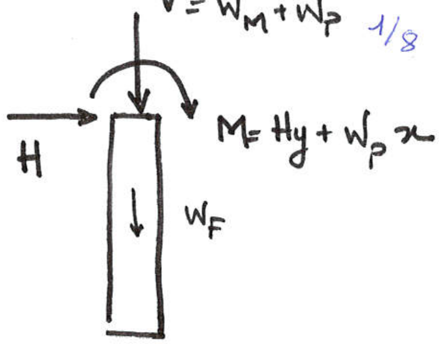
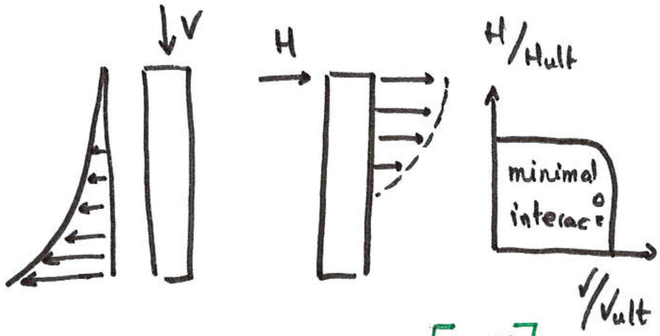


(a) • forces acting on foundation :

- Vertical load : $W_M + W_P \, (+W_F) = 51\text{kN}$
- a horizontal load $H = 2.3$ kN
- a moment load $M = Hy + W_p x = 150$ kNm

• for deep foundations (here, $\frac{L}{D} = 6.6 > 5 \rightarrow$ OK), the interaction between combined vertical and horizontal load does not significantly affect the vertical capacity and vice-versa. This is because of the low interaction between the soil resistance in the vertical direction and that in the horizontal direction, as shown in the diagram:

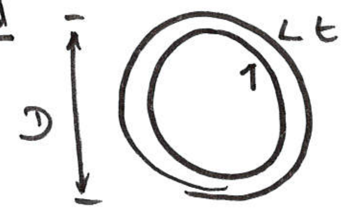


[10%]

(b) First, establish whether to use rigid or compressible pile solution:

$$\lambda = \frac{E_p}{G_L} \quad \text{with:} \quad G_L = G_{base} = 6\,000 \text{ kPa} = 300 \times \underbrace{20}_{Su} \text{ kPa.}$$

and



$$t \ll D \Rightarrow E_p = \frac{E_{steel} \, \pi D t}{\pi D^2/4}$$

$$E_p = 36.8 \times 10^8 \text{ MPa}$$

hence, $\lambda = 6\,140$

Most students took $\lambda = \frac{E_{steel}}{G_L} = 33.3 \times 10^3$
$\rightarrow$ Gives the same result with regards to pile behaviour.

And therefore: $\frac{\sqrt{\lambda}}{4} = 19.6$ vs. $\frac{L}{D} = 6.6 \Rightarrow \frac{L}{D} < \frac{\sqrt{\lambda}}{4}$

the pile is effectively rigid and the behaviour can be approximated by the rigid pile solution.

And therefore, applying the rigid pile solution, we have:

$$\frac{V}{w_{head}} = \frac{4 R_{base} G_{base}}{1-\nu} + \frac{2\pi L G_{avg}}{\zeta}$$

with $G_{avg} = 300 \times \bar{s}_u = 6000$ kPa.

and $\zeta = \ln\left(\left(\left(\cancel{0.5} + 5\rho(1-\nu) - \cancel{0.5}\right)\xi\right)\frac{L}{D}\right)$

$$\left.\begin{array}{l} \rho = \dfrac{G_{avg}}{G_L} = 1 \\ \\ \xi = \dfrac{G_L}{G_{base}} = 1 \end{array}\right\} \zeta = 2.8$$

And therefore: $\dfrac{V}{W_{head}} = \dfrac{4 \times 0.38 \times 6000}{1-0.5} + \dfrac{2\pi \times 5 \times 6000}{2.8}$

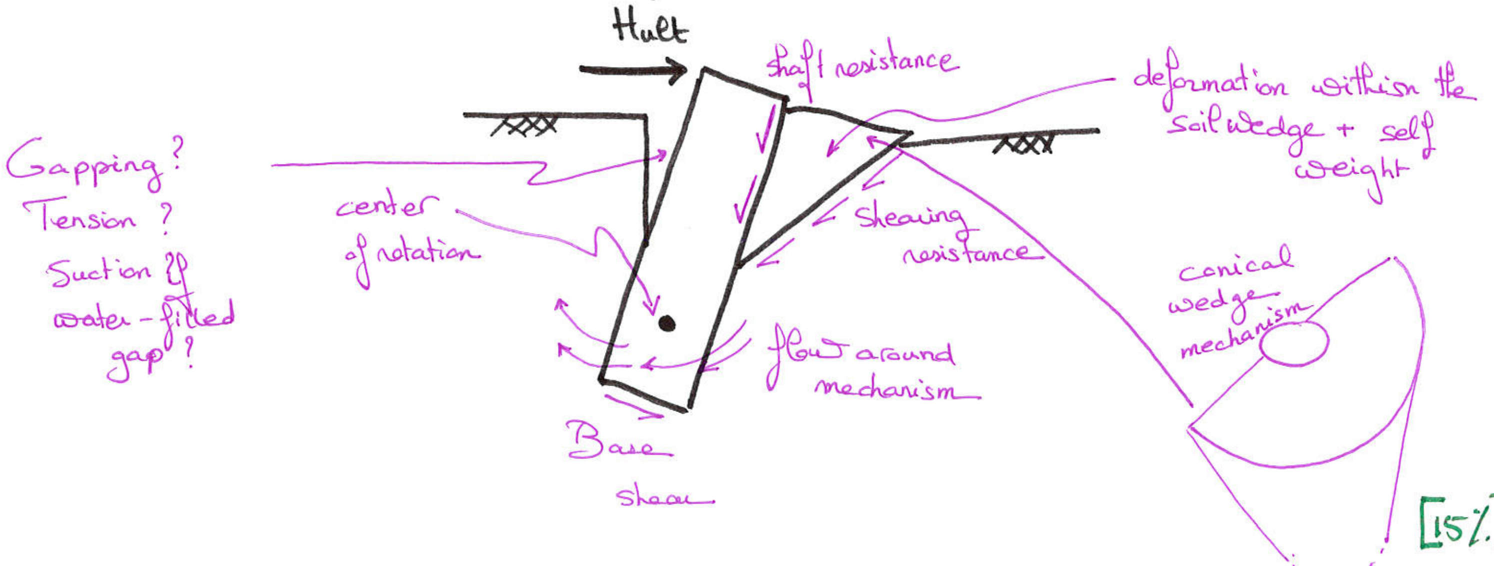
$$= 85.6 \text{ MN/m}$$

With an operational load $V = 51$ kN, this gives: $w_{head} = 0.6$ mm

$\Rightarrow w_{head} = 0.08\%$ of $D < 0.1\% \rightarrow$ OK $\therefore$

[40%]

(c) According to question (b), the pile is effectively rigid, and very likely to fail by rotation under horizontal load $\rightarrow$ short pile failure mechanism. The shallow failure mechanism is as follows:



[15%]

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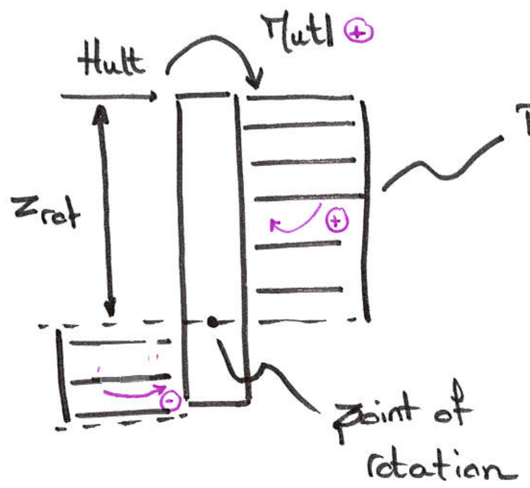
(d) Assume pile fails as rigid body under combined horizontal and moment load: For Conservatism purposes; assume:



$P_f = 9 D \bar{s}_u$

from question (a):

$$\frac{M_{ult}}{H_{ult}} = \frac{M_{op}}{H_{op}} = \frac{150}{2.3} = h_e$$

$$\Rightarrow \underline{h_e = 65.2\ m}$$

* Horizontal equilibrium: $H_{ult} = p_f z_{rot} - p_f (L - z_{rot}) = p_f (2z_{rot} - L)$

* Moment equilibrium: $M_{ult} + p_f z_{rot} \frac{z_{rot}}{2} = p_f (L - z_{rot}) \left(\frac{L + z_{rot}}{2} \right)$

$$\Rightarrow H_{ult} \cdot h_e = p_f \left(\frac{L^2}{2} + \cancel{\frac{L z_{rot}}{2}} - \cancel{\frac{L z_{rot}}{2}} - \frac{z_{rot}^2}{2} - \frac{z_{rot}^2}{2} \right)$$

$$\begin{cases} H_{ult} \cdot h_e = p_f \left(\frac{L^2}{2} - z_{rot}^2 \right) \\ H_{ult} = p_f (2z_{rot} - L) \quad \times h_e \quad (-) \end{cases}$$

$$\cancel{p_f} \left(-\frac{L^2}{2} + z_{rot}^2 \right) + \cancel{p_f} (2 z_{rot} - L) h_e = 0$$

$$\Rightarrow z_{rot}^2 + 2 z_{rot} h_e - \left(\frac{L^2}{2} + L h_e \right) = 0$$

$$\rightarrow x^2 + 130.4 x - 338.5 = 0$$

$$\Rightarrow \begin{cases} x = -132.9 & \rightarrow \text{impossible} \\ x = 2.55 < 5 & \rightarrow \text{OK} \end{cases}$$

$$\Rightarrow z_{rot} = 2.55\ m = 51\%\ \text{of } L$$

<u>This then gives</u> : $H_{ult} = P_f \, (2 \times z_{rot} - L)$

$$= \underbrace{9 S_u D}\,(2 \times z_{rot} - L)$$

$$P_f = 136 \cdot 8 \text{ kN/m}$$

$$H_{ult} = 13 \cdot 68 \text{ kN} \;\therefore$$

$$\Rightarrow \quad M_{ult} = H_{ult} \times h_e = 892 \text{ kNm} \;\therefore$$

And $FoS = \dfrac{H_{ult}}{H_{op}} = \dfrac{13 \cdot 68}{2 \cdot 3} = 5 \cdot 9 \rightarrow$ OK $\therefore$

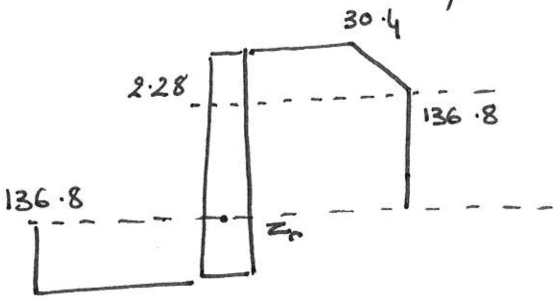
(technically, $FoS > 5 \cdot 9 > 3$)

<u>AND</u> Check for $M_p = D^2 t \sigma_y = 7$ MN m $\gg 892$ kN m

→ OK for failure as rigid body.

[35%]

<u>Note</u> : ① Some Students did the full calculation with:



<u>and obtained</u> :

$\dfrac{H_{ult}}{H} = 6 \cdot 7 = FoS$

<u>with</u>: $\begin{cases} z_r = 3 \cdot 0 \text{ m} \\ H_{ult} = 15 \cdot 5 \text{ kN} \\ M_{ult} = 1011 \text{ kNm} \end{cases}$

→ full marks were given in this case too.
but the method suggested in this CRIB is faster

② <u>Design Charts</u>:

Students should find that $\dfrac{e}{D} = 85 \cdot 5$
which is too large for use of the Design Charts in the Data Book.

Examiner's Comment : This question was attempted by most students, with different levels of success. Marks range from 11% to 90%. The most common mistake was to use the compressible pile solution in Q. (b), which often leads to algebraic mistakes and is more chronophage. A number of students also attempted to use design charts for question (d), with no success. A note has been added above as to why this does not work.

QUESTION 4

(a) Internal shaft resistance different from external shaft resistance because of "vertical arching" - or "reverse silo" effect inside the pile that can lead to "plugging" of the pile in sand.
The calculation of the internal shaft resistance $Q_{sf,i}$ is complicated by the confinement of the soil within the pile, and therefore, the vertical stress in the soil column is not simply $\sigma'_{v0} = \gamma' z$. because the downwards shaft resistance applied by the pile to the soil adds further vertical stress. This in turn increases the shaft resis--tance, which amplifies the vertical stress further. This feedback leads to an exponential increase in stress down the plug.

[15%]

(b) $\begin{cases} \phi = 31° \\ \delta = 15° \end{cases}$

* in this example, the wall thickness was not provided.

from "reverse silo" analysis in data book:

⚠ in example paper, there is no difference between D_i and D_0* but calculations should be with D_i not D_0 (see handout 7) $\underline{D_i = 0.75\,m}$

- $\sin\Delta = \dfrac{\sin\delta}{\sin\phi} = 0.5 \Rightarrow \Delta = 30.16°$

- $\beta = \dfrac{\sin\phi \sin(\Delta - \delta)}{1 + \sin\phi \cos(\Delta - \delta)} = 0.09$

$\Rightarrow \lambda = 4\beta \dfrac{h_p}{D_i}$ with h_p = height of soil plug $= 0.84L = 24.5\,m$

$\Rightarrow \lambda = 11.8$

And therefore $q_{bf,\,plug} = \gamma' h_p \left(\dfrac{e^{\lambda} - 1}{\lambda}\right)$

$= 10 \times 24.5 \times \left(\dfrac{e^{11.8} - 1}{11.8}\right) = 2\,670\ MPa$

$\mapsto$ This gives a "stress amplification" due to vertical arching of:

$$\frac{q_{bf,\,plug}}{\gamma' h_p} \approx 10.9 \times 10^3$$

And $Q_{bf,plug} = q_{bf,plug} \times \frac{\pi D_i^2}{4} = 1180$ MN

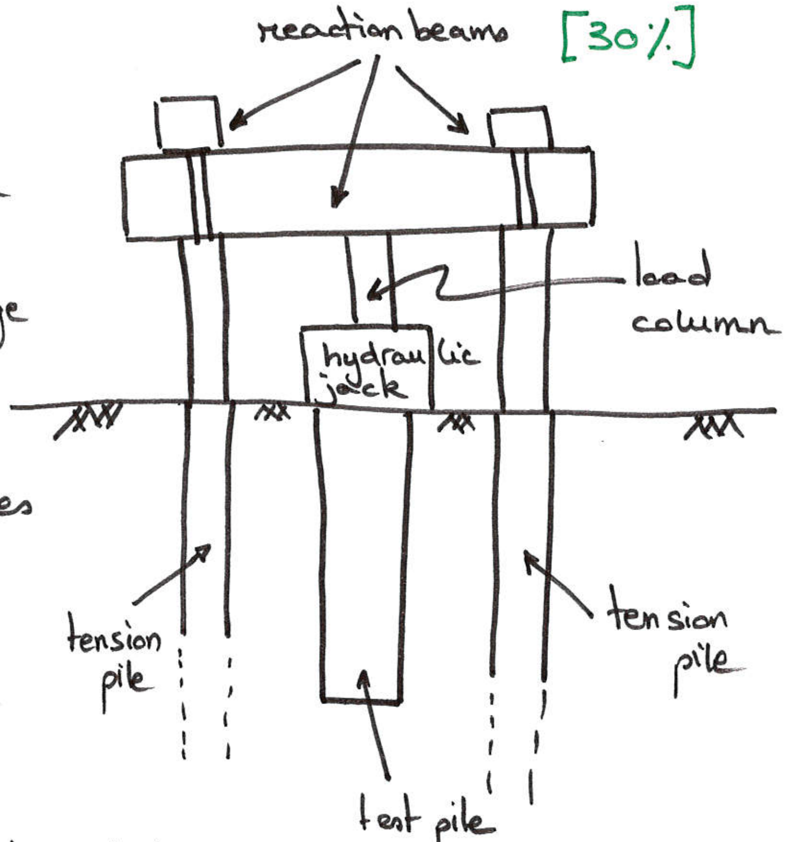
→ Weight of soil plug : $W_p = \pi \frac{D_i^2}{4} \gamma' h_p = 108$ kN → negligible

$\Rightarrow Q_{sf;i} = 1180$ MN ∴

And $\underbrace{q_{bf,plug}}_{2670 \text{ MPa}} >> \underbrace{q_c}_{10 \text{ MPa}}$ → failure occurs in plugged manner (i.e. closed-ended) after the soil plug has compressed to mobilise the resistance $Q_{sf,i}$ ∴

(c) Maintained Load Test :

[30%]



- A constant force is applied via a jack to the test pile head. Reaction is provided by kentledge or from nearby tension piles via a reaction beam
  The load is increased in stages & paused at each increment to allow for creep rate to subside and the settlement of the pile is measured.

- Maintained Load Test is the closest test to working conditions
  ⊕ ⇒ increased confidence in inferred capacity and no need for a back-analysis

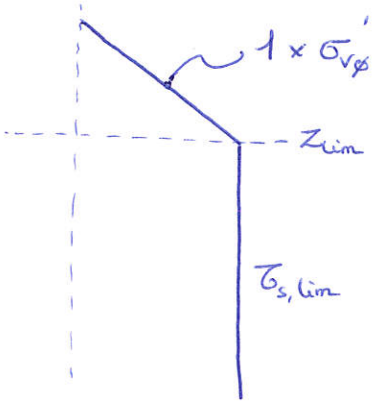
  ⊖ But tests are pricey and require reaction from either kentledge or beam ⊕ piles (can be difficult and costly to bring to site)
  And tests take a long time : need to wait for creep stages.

[15%]

(d) Pile fails in closed-ended manner

$$\Rightarrow Q_c = Q_s + Q_b = \bar{\tau}_s (\pi D_\phi) L + q_b \pi \frac{D_\phi^2}{4}$$

① τ_s such that:



From API:

$$K \sigma'_{v\phi} \tan\delta \leq \tau_{s,lim}$$

K: 1 (closed-ended); $\sigma'_{v\phi}$: $\gamma' \frac{L}{2}$; δ: $15°$ for loose silty sand.; $\tau_{s,lim}$: $50\,kN/m^2$

$$\Rightarrow \gamma' z_{lim} \tan\delta = \tau_{s,lim} \Leftrightarrow z_{lim} = \frac{\tau_{s,lim}}{\gamma' \tan\delta} \Rightarrow z_{lim} = 18.66\,m. < 29.2\,m$$

This then gives: $$Q = \pi D \int_0^L \tau_{sf}\, dz = \pi D \left[\int_0^{z_{lim}} K \tan\delta\, \gamma' z\, dz + \tau_{s,lim} (L - z_{lim}) \right]$$

$$= \pi D \left(K \tan\delta\, \gamma' \frac{z_{lim}^2}{2} + \tau_{s,lim} (L - z_{lim}) \right)$$

$$Q_s = 2.5\,MN \therefore$$

② Base Resistance: $$q_{bf} = \underbrace{\underset{\uparrow\, 8}{N_q}\ \sigma'_{v\phi}}_{2.3\,MN} \leq \underset{\uparrow\, 1.9\,MN}{q_{bf,lim}} = 1.9\,MN \therefore$$

$$\Rightarrow Q_{bf} = q_{bf} \frac{\pi D_\phi^2}{4} = 0.96\,MN \therefore$$

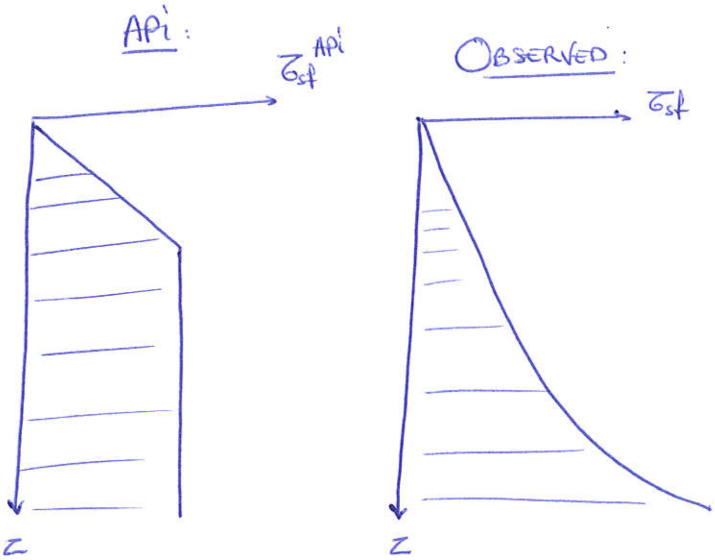
And Therefore: $$Q_c^{API} = Q_{sf} + Q_{bf} = 2.5 + 0.96 = \underline{3.45\,MN} \therefore$$

Note: Some students considered an open-ended pile despite the fact that it is plugged (and therefore, the correct value for K is $K = 1$). They used $K = 0.8$ instead and obtained:

$$\begin{cases} z_{lim} = 23.33\,m \\ Q_s = 2.203\,MN \\ Q_c^{API} = 3.16\,MN \end{cases}$$

- Measured ultimate Capacity: $Q_m = 5.27\,MN \Rightarrow \dfrac{Q_c^{API}}{Q_m} = 0.7$

API is conservative and under-estimating the capacity. This is because the API method assumes that local shaft (and base) resistance grows in proportion with the free field vertical stress σ'_{v0} and are relatively insensitive to changes in the sand state with depth. This is not the case. The only modification offered is that of an upper limit to both τ_{sf} and q_{bf}. It has been observed that $\tau_{sf} >> \tau_{s\,lim}$ close to the base of piles and that friction fatigue reduces the shaft resistance on the upper part of the pile during installation.



$\Rightarrow$ The observed trend of unit shaft resistance is not increasing proportionally with depth $\rightarrow$ the API method can therefore be unreliable in cases that are beyond the original data-base.

Examiner's Comment: This was the least popular question but was usually well succeeded when attempted. Marks range from 15% to 88%. The most common mistake was to use $K = 0.8$ instead of $K = 1$ despite having proven that the pile is plugged.