

1. a) Shaft stiffness = $\frac{2\pi LG}{\zeta} \sim \frac{\pi LG}{2} \sim 1.6 LG$

Base stiffness = $\frac{4RG}{1-\nu} \sim 8RG$

If soil is uniform and $\frac{L}{R}$ typically ~ 100

$\frac{\text{base}}{\text{shaft}} \sim \frac{8R}{1.6L} = \frac{5}{\frac{L}{R}} = \underline{5\%}$ small

disturbance of soil around base during boring will reduce base stiffness more.

b) $\frac{V}{\omega} = \frac{2\pi L G_{avg}}{\zeta}$ $\zeta = \ln \left\{ 5(1-\nu) \frac{L}{D} \right\}$ For assumed $\zeta=4$ see answers in red

$\nu = 1/2$ $\frac{L}{D} = \frac{20}{0.3}$ $\zeta = 5.11$

$\frac{V}{\omega} = \frac{2\pi \times 20 \times 10^7}{5.11} = 246 \text{ MN/m}$

If $\zeta=4$ 314 MN/m

c) Individual pile stiffness is relative to stationary soil. As pile is loaded, the surrounding soil is dragged down which will increase settlement of other piles, reducing their stiffness.

If all piles settle identically loads are not shared evenly.

In general this question was very well answered and almost all candidates had a good idea of how to proceed. The crib calculated magic radius from the pile length whereas many candidates assumed $\zeta=4$ which gave different answers (by quite a surprising amount..) I didn't penalize candidates for this sensible assumption.

d) Soil settlement around single pile

$$\tau = \tau_s \frac{R}{r}$$

$$V = \tau_s 2\pi RL$$

$$\delta = -\frac{\partial \omega}{\partial r}$$

$$\frac{\partial \tau}{\partial r} = G$$

$$\tau_s \frac{R}{r} = -G \frac{\partial \omega}{\partial r}$$

$$\int_r^R \frac{\partial \tau}{r} = -\frac{G}{\tau_s R} \int_0^{\omega} \partial \omega$$

$$\ln \left[\frac{r_m}{R} \right] = +\frac{G}{\tau_s R} \omega$$

$$\begin{aligned} \omega &= -\frac{\tau_s R}{G} \left[\ln \frac{r}{r_m} \right] = -\frac{\tau_s R}{G} \left[\ln \frac{r}{R} + \ln \frac{R}{r_m} \right] \\ &= \frac{\tau_s R}{G} \left[-\ln \frac{r}{R} + \zeta \right] \end{aligned}$$

Single pile $\omega_p = \frac{\tau_s R}{G} [\zeta] = \frac{V}{2\pi LG} \zeta$

Group $\omega = \frac{V}{2\pi LG} [\zeta - \ln \frac{r}{R}]$
 $= \omega_p \left[\frac{\zeta - \ln \frac{r}{R}}{\zeta} \right]$

(A)

(A)

(B)

(B)

(A)

(A)

A has

A @ 2

A @ 4

A @ $\sqrt{20}$

B @ 2

B @ $2\sqrt{2}$

B has

B @ 2

2A @ 2

2A @ $2\sqrt{2}$

dist	2	$2\sqrt{2}$	4	$\sqrt{20}$
$\frac{r}{R}$	13.33	18.85	26.67	29.81
$\zeta - \ln \frac{r}{R}$	2.52	2.17	1.83	1.72
$\frac{\zeta - \ln \frac{r}{R}}{\zeta}$	0.49	0.43	0.36	0.34

If zeta = 4

0.35

0.27

0.18

0.15

$$\omega_A = \frac{1}{246} \left[V_A [1 + 0.49 + 0.36 + 0.34] + V_B [0.49 + 0.43] \right]$$

$$= \frac{1}{246} [V_A (2.19) + 0.92 V_B]$$

$$\omega_B = \frac{1}{246} [V_A (2 \times 0.49 + 2 \times 0.43) + V_B (1 + 0.49)]$$

$$= \frac{1}{246} [1.84 V_A + 1.49 V_B]$$

$$\omega_A = \omega_B$$

$$1.84 V_A + 1.49 V_B = 2.19 V_A + 0.92 V_B$$

$$\circlearrowleft \quad 0.57 V_B = 0.35 V_A$$

$$\frac{V_B}{V_A} = \frac{0.35}{0.57} = 0.614$$

0.595 for zeta=4

$$4 V_A + 2 V_B = V$$

$$5.228 V_A = V$$

$$V_A = \frac{V}{5.228}$$

$$\omega = \frac{1}{246} \left(1.84 \frac{V}{5.228} + 1.49 \times 0.614 \frac{V}{5.228} \right)$$

$$= \frac{0.527 V}{246}$$

$$\frac{V}{\omega} = 467 \text{ MN/m} \quad \text{797 MN/m if zeta=4}$$

2 a) As pile approaches soil element vertical stress increases, forcing an increase in σ_H .

As pile tip passes soil is forced outwards, σ_H remains high and σ_V reduces.

Cyclic shearing of the soil around the pile on subsequent blows leads to the soil densifying and σ_H reducing leading to a reduction in interface friction.

b) API

$$\tau_s = K \sigma'_v \tan \delta$$

closed ended 1, open ended 0.8

closed piles lead to greater σ_h

In API $\tan \delta = f(\text{density})$

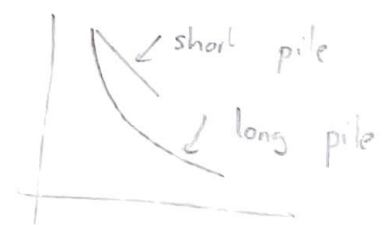
in reality K is $f(\text{density})$ denser soils cannot contract as much so give higher σ_h

Low High up on pile shear reduces due to friction fatigue, so shear capacity isn't $\propto L^2$

API uses



reality is



The explanation of stress path in sand was generally well described, but the explanation of how this leads to API approach was sometimes a bit thin as there were 5 marks available. Lateral loading was generally well dealt with, though some candidates used the wrong curves on the graphs and didn't think about the pile head conditions in a group.

One pile

Dense sand

$$\delta = 30^\circ$$

$$\tau_{s, \text{lim}} = 100 \text{ kPa}$$

$$N_q = 40$$

$$q_{b, \text{lim}} = 9.6 \text{ MPa}$$

Open ended $\rightarrow K = 0.8$

τ

$$\sigma_{vo}' = 19 \text{ z}$$

$$\tau = 0.8 \times 19 \times \tan 30$$

$$= 8.78 \text{ z kPa}$$

Limiting @ 11.4 m

$$q_b = 40 \times 19 \times 20 = 15.2 \text{ MPa}$$

Limits @ 9.6 MPa

Plugged or not?

$$\begin{aligned} \text{To core} \quad 9.6 \text{ MPa} \times \pi \times 0.25^2 &> \pi \times 0.5 \times 20 \times 190 \tan 30 \quad \left. \begin{array}{l} \text{friction} \\ + \\ 19 \times 20 \times 0.25^2 \text{ weight} \end{array} \right\} \\ &1885 > 3446 + 23 \end{aligned}$$

x - plugs

$$V = \underbrace{9.6 \text{ MPa} \times \pi \times 0.25^2}_{1885} + \underbrace{100 \times \pi \times 0.5 \times 8.6}_{\text{lower shaft}} + \underbrace{50 \times \pi \times 0.5 \times 11.4}_{\text{upper shaft}}$$

$$2246$$

$$V = 4131 \text{ kN}$$

Group $V = \underline{\underline{16.5 \text{ MN}}}$

$$\text{ii) } Z_p = \frac{d_e - d_i}{6} \quad K_p = 3.69$$

$$= 0.0024 \text{ m}^3$$

$$M_p = 0.84 \text{ MNm}$$

$$n = \delta' K_p^2 = 258.7 \text{ kN/m}^3$$

$$\frac{e}{D} = 4 \quad \frac{L}{D} = 40$$

$$\frac{M_p}{nD^4} = 52$$

$$\text{Long pile } \frac{H_{ult}}{nD^3} = 23$$

$$\text{Short pile } \frac{H_{ult}}{nD^3} > 50$$

$$H_{ult} = 23 \times 258.7 \times 0.5^3 = 744 \text{ kN}$$

$$\text{For group } H_{ult} = \underline{\underline{2.975 \text{ MN}}}$$

Tunnelling question:

$$s_u = 80 \text{ kPa} \quad \gamma = 18 \text{ kN/m}^3$$

$$D = 7 \text{ m} \quad H = 18 \text{ m} \quad C = H - D/2 = 14.5 \text{ m} \quad P = 1.5 \text{ m}$$

$$(a) \quad P/D = 1.5/7 \approx 2.0$$

$$C/D = 14.5/7 \approx 2.1$$

from chart $N_c \approx 8.0$

$$N = \frac{\gamma H + \sigma_s - \sigma_t}{s_u} = \frac{18 \times 18}{80} \approx 4.1$$

$$FS = \frac{N_c}{N} = \frac{8.0}{4.1} \approx 2.0$$

$$(b) \quad N = \frac{18 \times 18 + \sigma_s}{80} = N_c = 8.0$$

$$\sigma_s = 8 \times 80 - 18 \times 18 = 316 \text{ kPa}$$

This is a rather high value considering that an average building applies 10 kPa/floor

$$(c) \quad N = \frac{18 \times 18}{s_u} = N_c = 8.0$$

$$s_u = \frac{18 \times 18}{8} = 40.5 \text{ kPa}$$

$$(d) \quad F = \frac{80}{40.5} \approx 2.0 \quad (\text{same as on actions})$$

(e) mechanised tunnelling,
either ~~EPB~~ EPB or SS.

Support pressure required at face.
in the EPB shield face support is provided
by the excavated material (+ foam/slurry/
other additives). The spoil is extracted
from the excavation chamber using a

screw conveyor (cochlea) that allows to regulate the pressure at the face of the TBM

(2)

In the slurry shield face support is provided by a suspension of bentonite (slurry) pumped into the excavation chamber and pressurised by air.

The mixture of natural soil and bentonite is pumped from the excavation chamber (hydraulic mucking) to a separation plant that enables the slurry to be recycled.

In both cases segmental lining is mounded inside the shield using an erector. A set of hydraulic jacks push the shield forward reacting on the already installed lining.

(f) to improve the safety factor we can either increase N_c or decrease N .

To minimise $N = \frac{\gamma H + \sigma_s - \sigma_t}{s_u}$ we

can:

- ~~1. increase s_u by consolidation or other treatment??~~
- ~~2. decrease H~~

Parts a to d were generally well done with only a few candidates making mistakes. The discussion in parts e and f were of varying quality, probably due to time constraints as this was many candidates 3rd question

- 1. increase s_u
not so easy, in clay limited options for ground treatment.
- 2. decrease H (shallower tunnel)
- 3. decrease surcharge σ_s
- 4. apply internal support pressure σ_t

By looking at the stability chart, to increase N_c we can ~~increase~~

- 5. increase cover, C
- 6. decrease the unsupported length, P
- 7. decrease diameter, D (e.g. by sequential excavation)

Note that decreasing H (as in point 2) will reduce N_c , but also reduce C/D , that is reduce N_c . Also, in most natural clay deposits s_u increases with depth, so reducing H , will reduce s_u , which is bad news. If undrained shear strength increases with depth, even by a small amount, then the factor of safety is almost always increased by going deeper.

Retaining wall question

(4)

$$(a) \quad \gamma_{d_{hy}} = \gamma_w \frac{G_s}{1+e} = 10 \times \frac{2.75}{1.8} = 15.3 \text{ KN/m}^3$$

$$\gamma_{sat} = \gamma_w \frac{G_s + e}{1+e} = 10 \times \frac{(2.75 + 0.8)}{1.8} = 19.7 \text{ KN/m}^3$$

$$(b) \quad \varphi_k = 36^\circ \quad \delta_k = \varphi_k / 3 = 12^\circ$$

$$K_{a_k} = \frac{1 - \sin \varphi_k}{1 + \sin \varphi_k} = 0.295$$

$$K_{p_k} = \frac{\cos \delta_k}{1 - \sin \varphi_k} \left(\cos \delta_k + \sqrt{\sin^2 \varphi_k - \sin^2 \delta_k} \right) e^{2\theta_k \tan \varphi_k}$$

$$2\theta_k = \sin^{-1} \left(\frac{\sin \delta_k}{\sin \varphi_k} \right) + \delta_k = 31.5^\circ = 0.550$$

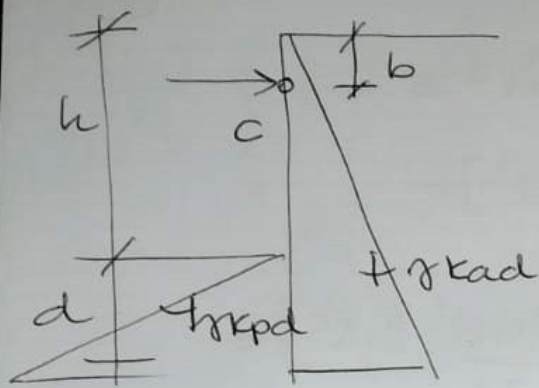
$$K_{p_k} = 4.596$$

$$(c) \quad \varphi_d = \tan^{-1} \frac{3\varphi_k}{1.25} = 27.5^\circ \quad \delta_d = \frac{\varphi_d}{3} = 9.2^\circ$$

$$K_{a_d} = \frac{1 - \sin \varphi_d}{1 + \sin \varphi_d} = 0.369$$

$$2\theta_d = \sin^{-1} \left(\frac{\sin \delta_d}{\sin \varphi_d} \right) + \delta_d = 29.3^\circ = 0.512$$

$$K_{p_d} = 3.394$$



moment equilibrium about c

$$\gamma = \gamma_{dry}$$

(5)

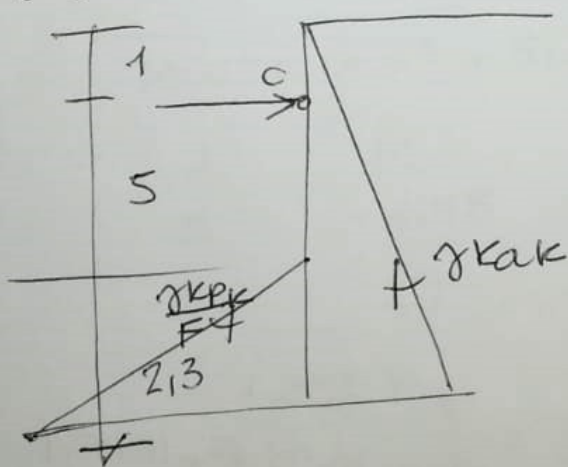
$$M_D = \frac{1}{2} \gamma k_{ad} (h+d)^2 \left[\frac{2}{3} (h+d) - b \right] = 2.82 (6+d)^2 [0.667(6+d) - 1]$$

$$M_R = \frac{1}{2} \gamma k_{pd} d^2 \left(\frac{2}{3} d + h - b \right) = 25.96 d^2 (0.667d + 5)$$

d (m)	M _D (kNm/m)	M _R (kNm/m)	ΔM (kNm/m)
3	1142.79	1635.71	-492.92
2	782.56	657.72	124.84
2.15	951.39	1081.80	-130.41
2.13	881.23	897.32	-16.09 ✓

less than 2% on average between M_D and M_R - no point specifying wall length to cm.

(d)



again moment equilibrium about c

$$M_D = \frac{1}{2} \gamma k_{ak} (h+d)^2 \left[\frac{2}{3} (h+d) - b \right]$$

$$M_R = \frac{1}{2} \gamma \frac{k_{pk}}{F} d^2 \left(\frac{2}{3} d + h - b \right)$$

$$M_D = M_R$$

$$F = \frac{\frac{1}{2} \gamma k_{pk} d^2 \left(\frac{2}{3} d + h - b \right)}{\frac{1}{2} \gamma k_{ak} (h+d)^2 \left[\frac{2}{3} (h+d) - b \right]}$$

$$F = \frac{4.596 \times 2.13^2 \times (0.667 \times 2.13 + 5)}{0.295 \times 8.3^2 \times (0.667 \times 8.3 - 1)} = \frac{158.86}{92.19}$$

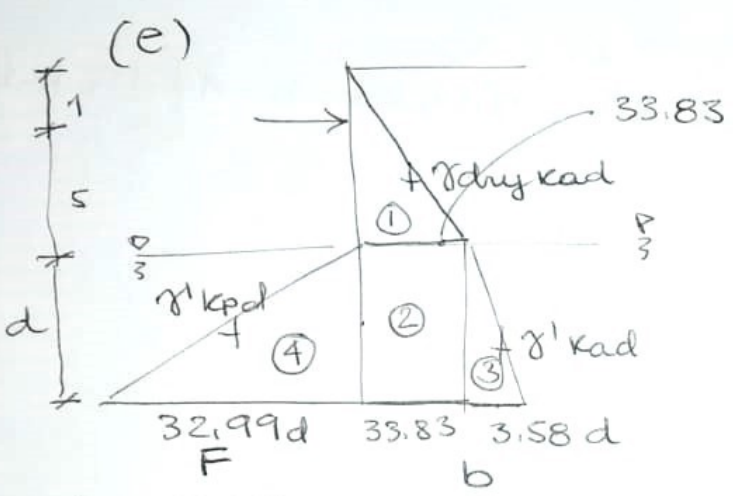
$$F = 1.72$$

$$T = \frac{1}{2} \gamma k_{ak} (h+d)^2 - \frac{1}{2} \gamma \frac{k_{pk}}{F} d^2 =$$

$$= \frac{1}{2} \gamma \left[k_{ak} (h+d)^2 - \frac{k_{pk}}{F} d^2 \right] =$$

$$= \frac{15.3}{2} \left[0.295 (8.3)^2 - \frac{4.596}{1.72} \times 2.3^2 \right] =$$

$$= 47.3 \text{ kN/m}$$



	F	b	M
1	101.49	3	304.47
2	33.83d	0.5d + 5	16.92d ² + 169.15d
3	1.79d ²	0.667d + 5	1.19d ³ + 8.95d ²
4	-16.50d ²	0.667d + 5	-11.01d ³ - 82.50d ²

$$M = 11.01d^3 + 56.63d^2 - 169.15d - 304.47 = 0$$

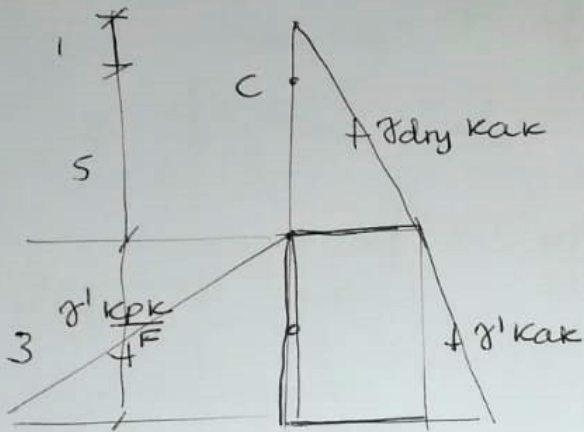
d	M
3	-4.98 ✓
3.1	47

~~$$F = \frac{k_{pk} d^2 (\frac{2}{3}d + h - b)}{k_{ak} (h+d)^2 \left[\frac{2}{3}(h+d) - b \right]}$$

$$= \frac{4.596 \times 9 \times (0.667 \times 3 + 5)}{0.295 \times 8^2 \times (0.667 \times 8 - 1)} = \frac{289.21}{81.84}$$~~

(e)

(7)



$$M_B = \frac{1}{2} \gamma_{dry\ kak} \cdot h^2 \cdot \left(\frac{2}{3}h - b\right) + \gamma_{dry\ kak} h \cdot d \left[\frac{d}{2} + h - b\right] + \frac{1}{2} \gamma'_{kak} \cdot d^2 \cdot \left(\frac{2}{3}d + h - b\right)$$

$$M_R = \frac{1}{2} \gamma'_{KPK} \cdot d^2 \cdot \left(\frac{2}{3}d + h - b\right)$$

$$E_{KPK} \quad F = \frac{9,7 \cdot 4,596 \cdot 3^2 \cdot (2+5)}{\frac{15,3 \cdot 0,295 \cdot 6^2}{2} \left(3 + 15,3 \cdot 0,295 \cdot 6 \cdot 3 [1,5+5]\right) + \frac{9,7}{2} \cdot 0,295 \cdot 3^2 \cdot (2+5)}$$

$$F = \frac{1404,31}{861,95} = 1,63$$

$$T = \frac{1}{2} \gamma_{dry\ kak} h^2 + \gamma_{dry\ kak} h d + \frac{1}{2} \gamma'_{kak} d^2$$

$$- \frac{1}{2} \gamma'_{KPK} d^2 =$$

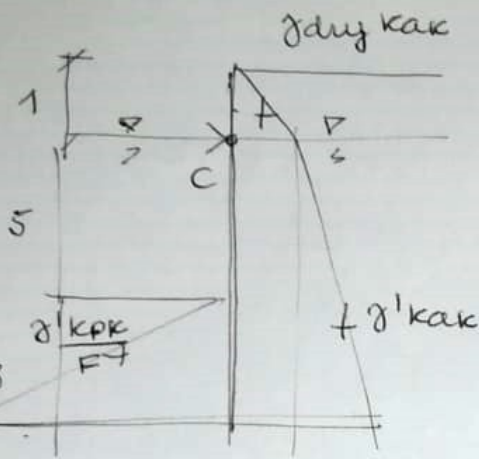
$$0,5 \times 15,3 \times 0,295 \times 36 + 15,3 \times 0,295 \times 6 \times 3 +$$

$$0,5 \times 9,7 \times 0,295 \times 9 - 0,5 \times 9,7 \times \frac{4,596}{1,63} \times 3^2 =$$

$$> 52,29 \text{ kN/m}$$

(f)

⑧



Moment equilibrium
about c

~~there~~

$$\begin{aligned}
 MD &= -\frac{1}{2} \gamma_{dry} kak \cdot b^2 \times \frac{b}{3} = -\frac{15.3}{2} \times 0.295 \times \frac{1}{3} \\
 &+ \gamma_{dry} kak \cdot b \left(\frac{h-b+d}{2} \right)^2 = 15.3 \times 0.295 \times 8^2 \times 0.5 \\
 &+ \frac{1}{2} \gamma' kak \times (h-b+d)^3 \times \frac{2}{3} = 0.5 \times 9.7 \times 0.295 \times 8^3 \times \frac{2}{3}
 \end{aligned}$$

631.29

$$MR = \frac{1}{2} \gamma' \frac{kpk}{F} \cdot 3^2 \left(5 + \frac{2}{3} \cdot 3 \right) =$$

$$\frac{9.7}{2} \cdot 4.596 \times 9 \times 7 \cdot \frac{1}{F} = \frac{1404.3}{F}$$

$$F = \frac{1404.3}{631.3} = 2.22$$

$$T = \frac{1}{2} \gamma_{dry} kak b^2 + \gamma_{dry} kak b (h-b+d) + \frac{1}{2} \gamma' kak (h-b+d)^2$$

$$- \frac{1}{2} \gamma' \frac{kpk}{F} \cdot d^2 =$$

$$= \frac{15.3}{2} \cdot 0.295 + 15.3 \times 0.295 \times 8 + \frac{9.7}{2} \cdot 0.295 \times 8^2 -$$

$$\frac{9.7}{2} \frac{4.596}{2.22} \times 9 = 39.57 \text{ kN/m}$$

(9)

~~the most demanding~~

The most demanding position of the water table is at dredge level, which makes sense as the active pressure is almost the same as for the dry case, while the passive pressure is greatly reduced.

The most challenging case would be high water table and dry excavation

The least popular question on the paper and the least well handled. While standard active and passive pressures were calculated well in part a and b, the reduction of shear strength by 1.25 in part c was not correctly applied by many candidates. In parts d-f many candidates had the right ideas but made mistakes in the calculations and there were signs of running out of time.