Version CNA/3

EGT3
ENGINEERING TRIPOS PART IIB

Wednesday 28 April 2021 9 to 10.40

Module 4D5

FOUNDATION ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet and at the top of each answer sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4D5 Foundation Engineering data sheet (19 pages).
You are allowed access to the electronic version of the Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.

The time taken for scanning/uploading answers is 15 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

1 A rigid circular foundation with diameter D of 2 m is designed to support a vertical advertisement board with surface area A_s of 10 m^2, with its centroid at a height h of 10 m above ground level. The advertisement board, mast and foundation apply a vertical self-weight load V of 200 kN to the foundation. It is founded on clay with undrained shear strength s_u of 100 kPa, and in extreme conditions a wind pressure p of 2 kPa could be applied.

(a) Calculate the factor of safety for the foundation in the following scenarios:

(i) Sliding failure. [20%]

(ii) Combined failure without 'lift-off' using the failure envelope given below, where $f \geq 0$ indicates failure.

$$f = \left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1$$

In this equation, H is the horizontal load, M is the moment and V_{ult}, H_{ult} and M_{ult} are the ultimate uniaxial capacities. [40%]

(b) Describe any assumptions made in these calculations. [20%]

(c) Without performing further calculations, indicate what the most likely mode of failure is for this foundation, for the current design loading scenario. [10%]

(d) What would the effect of long term sustained wind load be on the validity of the calculation for failure without 'lift-off'? How could this condition be checked? [10%]

2 A circular pad foundation, with diameter D of 2 m is installed in clay with undrained shear strength s_u of 100 kPa. The total vertical self-weight load V on the foundation is 200 kN and it is loaded rapidly during construction.

(a) Assuming that the stress-strain response of the soil can be described by a power curve (with half of the undrained shear strength s_u being mobilised at a shear strain $\gamma_{M=2}$), derive an expression for the immediate settlement using the Mobilised Strength Design (MSD) method. [25%]

(b) If the mobilisation strain $\gamma_{M=2}$ is assumed to be 0.02, and the power curve exponent b is taken as the default value of 0.6, calculate the immediate settlement to the nearest millimetre. [10%]

(c) State three reasons why the estimate of part (b) is approximate. [15%]

(d) Justify the value used for the bearing capacity factor N_c in part (b). [10%]

(e) Estimate the immediate and long term settlement of the foundation using elastic methods, taking the shear stiffness G of the soil as $100s_u$ and the Poisson's ratio as 0.3. Assume that the foundation is sufficiently stiff as to be considered fully rigid. Comment on how this estimate compares to that derived via the MSD method. [20%]

(f) A more exhaustive site investigation finds that bedrock lies just 10 metres beneath the foundation. Calculate revised estimates of the immediate and long term settlements. Comment on how the estimates compare with part (e). [20%]

3 The capacity of a steel tubular pile of diameter $D = 1$ m and length $L = 10$ m is being assessed for a pile-supported building. The pile wall thickness is 30 mm. It will be founded on soft normally consolidated clay with undrained shear strength profile $s_u = 2z$ kPa, with z in m, and effective unit weight $\gamma' = 5$ kN m^{-3}.

(a) Describe the stages in the installation and life of a displacement pile in soft clay and their effects on the total stress, pore water pressure and effective stress of the in-situ soil around the pile. [20%]

(b) Calculate the driving force required to install the pile by assuming that the pile becomes plugged in the early phases of the installation process. Assume undrained installation conditions with a pile skin friction factor estimated using the ISO/API method based on Fig. 1. [40%]

(c) Explain why and how a CPT test could be used to assess the pile axial capacity and what the limitations of this technique are. [25%]

(d) The coefficient of horizontal consolidation for this site is $c_h = 16$ m^2 year^{-1}. How soon after the piles are installed can construction of the building start? Use the design chart in Fig. 2. [15%]

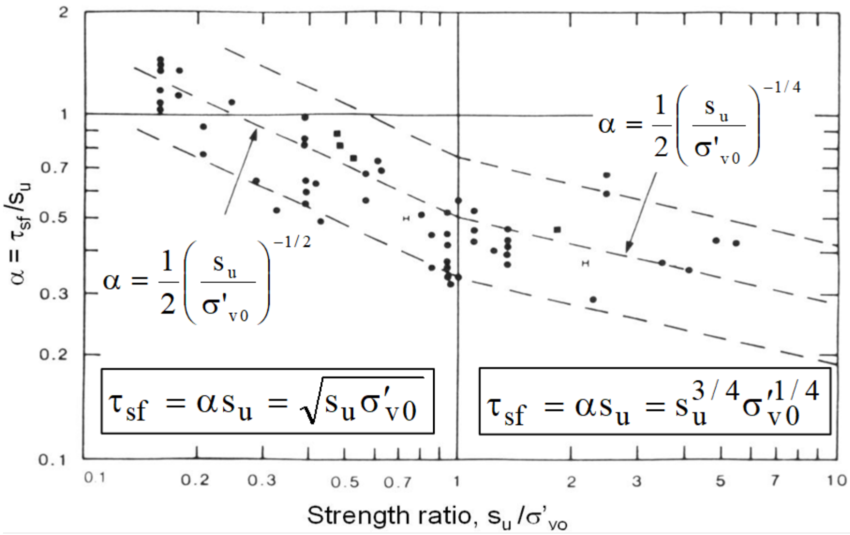
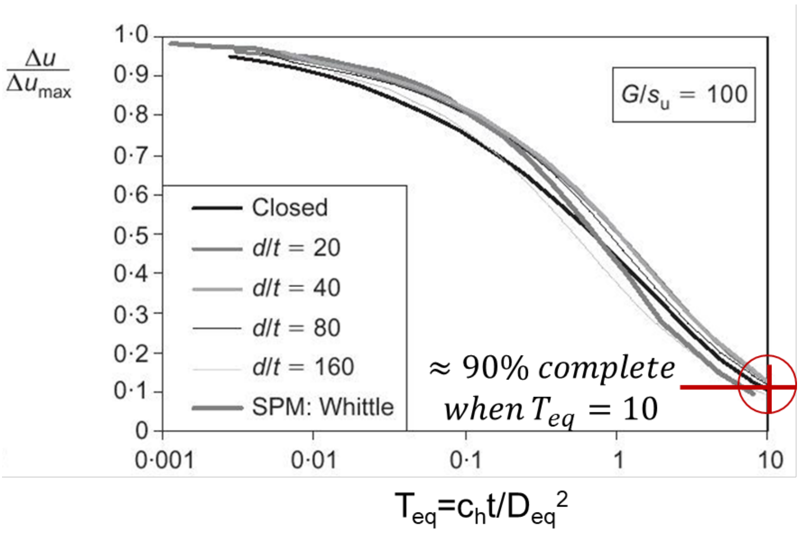


Fig. 1



Note: d/t = diameter / wall thickness; but t on x axis is time

Fig. 2

(TURN OVER

4 A free-headed steel tubular pile of outer diameter D = 0.8 m, length L = 10 m and yield stress σ_y = 350 MPa is being laterally loaded at ground level in a uniform stiff clay of undrained shear strength s_u = 120 kPa. Assume that the lateral soil resistance profile of the clay is constant at all depths, including at shallow depths.

(a) Calculate the ultimate lateral force per unit length p_u assuming a bearing capacity factor of $N_c = 9$ at all depths. [5%]

(b) Assume that the pile fails as a rigid body. Calculate the depth of the point of rotation, with the origin taken at the ground surface, and calculate the ultimate lateral load H_{ult}. [35%]

(c) What is the required pile wall thickness t to prevent structural failure? Note: the plastic moment capacity $M_p = \sigma_y D^2 t$. [35%]

(d) Using a pile wall thickness of 22 mm enables significant cost savings. What is the maximum lateral load that can be applied on this pile before it fails structurally? [25%]

END OF PAPER

Cambridge University Engineering Department

Supplementary Databook

Module 4D5: Foundation Engineering

SAS/CNA January 2021

Section 1: ULS and SLS

Eurocode partial factors for design

Case	Actions			Ground Properties	
	Permanent		Variable	tan ϕ	s_u
	Unfavourable	*Favourable*	*Unfavourable*		
EQU	1.1	0.9	1.5	1.1	1.2
STR (A1)	1.35	1.0	1.5	1.0	1.0
GEO (A2)	1.0	1.0	1.3	1.25	1.4

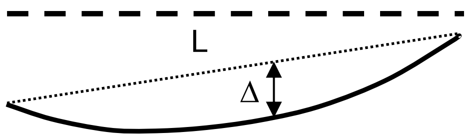
Case EQU : governs overall stability of a structure
Case STR : concerned only with the failure of structural members, including foundations and retaining structures
Case GEO : applies to the design of foundations and earthworks

Damage to buildings from differential settlement

Distortion defined as relative displacement Δ/L:

L is the length of building segment with consistent sagging or hogging

Δ is the maximum settlement of the deformed segment relative to chord L



Distortion linked to local tensile strain ε in elastic beams of various E/G and L/H:

$$\frac{\Delta}{L\,\varepsilon} \approx$$ 1.0 to 1.5 diagonally in end zones due to shear

0.75 to 1.0 longitudinally in middle zone due to sagging

0.25 to 0.5 longitudinally in middle zone due to hogging

Onset of visible (~ 0.1mm) cracks in brick or blockwork walls: $\varepsilon_{max} \approx 0.75\ .10^{-3}$
(after Burland & Wroth, 1974)

Categories of damage associated with masonry walls:

Cat.	Limit	Relative displacement	Description	Action
0	-	$\Delta/L \leq\ 0.5\ 10^{-3}$	negligible	none
1	SLS	$0.5\ 10^{-3} < \Delta/L \leq 0.75\ 10^{-3}$	very slight	redecorate interior
2	SLS	$0.75\ 10^{-3} < \Delta/L \leq\ 1.5\ 10^{-3}$	slight	+ some repointing
3	SLS	$1.5\ 10^{-3} < \Delta/L \leq\ 3.\ 10^{-3}$	moderate	+ significant repointing etc
4	ULS	$3.\ 10^{-3} < \Delta/L \leq\ 10^{-2}$	severe	shore; consider demolition
5	ULS	$10^{-2} < \Delta/L$	very severe	demolish

(after Boscardin & Cording, 1989)

Section 2: Empirical correlations from geotechnical data

2.1 Undrained shear strength of clays (s_u)

Normally consolidated clay $\left(\frac{s_u}{\sigma'_v}\right)_{nc} \approx 0.11 + 0.37\, I_p$ after Skempton (1957)

where I_p is the plasticity index

Overconsolidated clay $\frac{s_u}{\sigma'_v} \approx \left(\frac{s_u}{\sigma'_v}\right)_{nc} n^{\Lambda}$ after Ladd et al (1977)

where n is overconsolidation ratio

exponent $\Lambda \approx 0.8$

Penetrometer correlations $s_u = \frac{(q_{penetrometer} - \sigma_v)}{N_{penetrometer}}$ from q at tip load cell

where $N_{cone} \approx 14 \pm 3$; $N_{T\text{-}bar} \approx 10.5 \pm 1.5$

$s_u \approx 4.5 N_{60}$ kPa from SPT blow-count N_{60}

2.2 Drained shear strength of sands (friction and dilatancy): after Bolton (1986)

Definition of relative dilatancy $I_R = I_D\, I_C - 1$

definition of relative density $I_D = (e_{max} - e)/(e_{max} - e_{min})$

SPT blow-count correlation $I_D \approx [N_{60} / (20 + 0.2\, \sigma'_v \text{ kPa})]^{0.5}$

definition of relative crushability $I_C = \ln(\sigma_c / p')$

aggregate crushing stress $\sigma_c \approx$ 5 000 kPa for shelly sand

20 000 kPa for quartz sand

80 000 kPa for quartz silt

CPT correlation (q_{cone} , σ'_v in kPa) $I_D \approx 0.27(\ln q_{cone} - 0.5 \ln \sigma'_v) - 1.29 \pm 0.15$ (higher if σ_c lower)

Peak friction correlation $(\phi_{max} - \phi_{crit}) \approx 0.8\ \psi_{max} \approx 5° \times I_R$ in plane strain

$(\phi_{max} - \phi_{crit}) \approx 3° \times I_R$ in axisymmetric strain

Peak dilatancy rate $(-\delta\varepsilon_v / \delta\varepsilon_1)_{max} \approx 0.3 \times I_R$ in all conditions

Critical state friction angle $\phi_{crit} \approx 32°$ (uniform, rounded) $\rightarrow$ 40° (well-graded, angular)

2.3 **Stiffness of clays:** after Vardanega & Bolton (2011, 2012, 2013)

Very small strains ($\gamma \sim 10^{-6}$) $\quad G_0 = \frac{B}{(1+e)^3}(p')^{0.5}$ with G_0, p' in kPa

soil fabric factor $\quad B \approx 25\,000$ within factor 2

Small strains ($10^{-6} < \gamma < 10^{-2}$) $\quad \frac{G}{G_0} \approx \frac{1}{1+\left(\frac{\gamma}{\gamma_{ref}}\right)^a}$

hyperbolic curvature parameter $\quad a = 0.74 \pm 10\%$

reference shear strain $\quad \gamma_{ref} = 2.2\ I_p 10^{-3} \pm 50\%$

Moderate mobilizations of strength ($0.2s_u < \tau_{mob} < 0.8s_u$; typical $\gamma > 0.1\%$)

mobilised shear strength $\quad \frac{\tau_{mob}}{s_u} \approx 0.5\left(\frac{\gamma}{\gamma_{M=2}}\right)^b$

mobilization strain $\quad \gamma_{M=2} \approx 0.004\ n^{0.7}$

power curve exponent $\quad b \approx 0.37 + 0.01\ n$ (default value 0.6)

Conventional linearised stiffness modulus $\quad G_{50}$ *or* $G_{M=2} = 0.5\ s_u/\gamma_{M=2}$

2.4 **Stiffness of sands:** after Oztoprak & Bolton (2012)

Very small strains ($\gamma \sim 10^{-6}$) $\quad G_0 = \frac{B}{(1+e)^3}(p')^{0.5}$ with G_0, p' in kPa

soil fabric factor $\quad B \approx 50\,000$ within factor 2

Small strains ($10^{-6} < \gamma < 10^{-2}$) $\quad \frac{G}{G_0} = \frac{1}{1+\left(\frac{\gamma-\gamma_e}{\gamma_{ref}}\right)^a}$

hyperbolic curvature parameter $\quad a = U_c^{-0.075}$

e.g. $a = 0.9$ at uniformity coefficient $U_c = 4$

reference shear strain $\quad \gamma_{ref} = U_c^{-0.3}\ p'\ 10^{-6} + 8\,e\,I_D\,10^{-4}$

limiting elastic strain $\quad \gamma_e = 0.012\ \gamma_{ref} + 2.10^{-6}$

Section 3: Plasticity theory

This section is common to the Soil Mechanics Databook supporting modules 3D1 and 3D2. Undrained shear strength ('cohesion' in a Tresca material) is denoted by s_u rather than c_u.

3.1 Plasticity: Tresca material, $\tau_{max} = s_u$

Limiting stresses

Tresca $\quad |\sigma_1 - \sigma_3| = q_u = 2s_u$

von Mises $\quad (\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3}q_u^2 = 2s_u^2$

q_u= undrained triaxial compression strength; s_u= undrained plane shear strength.

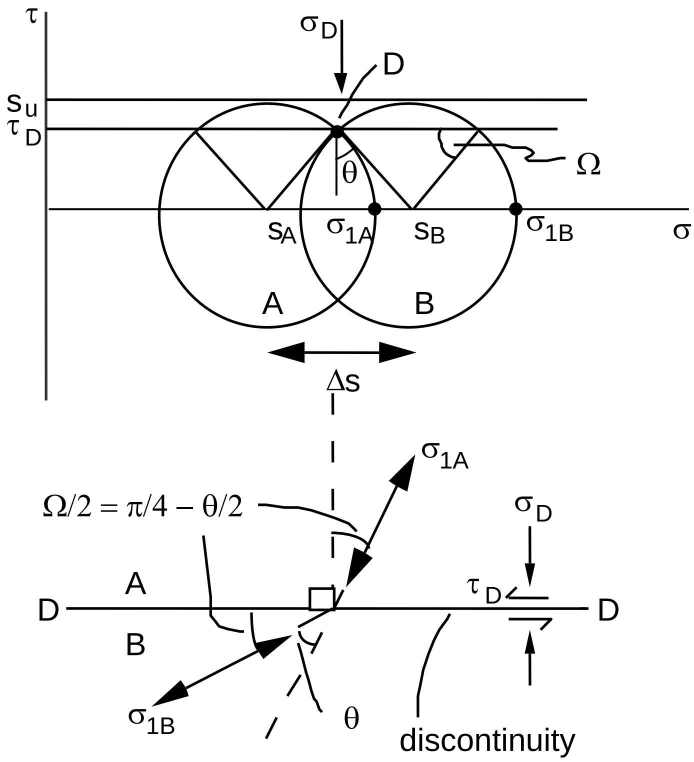
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = s_u \delta\varepsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength s_u , this becomes

$$D = A s_u x$$

3.2 Stress conditions across a discontinuity:



Rotation of major principal stress

$$\theta = \pi/2 - \Omega$$

$s_B - s_A = \Delta s = 2s_u \sin\theta$

$\sigma_{1B} - \sigma_{1A} = 2s_u \sin\theta$

In limit with $\theta \to 0$

$$ds = 2s_u \, d\theta$$

Useful example:

$\theta = 30^o$

$\sigma_{1B} - \sigma_{1A} = s_u$

$\tau_D / s_u = 0.87$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

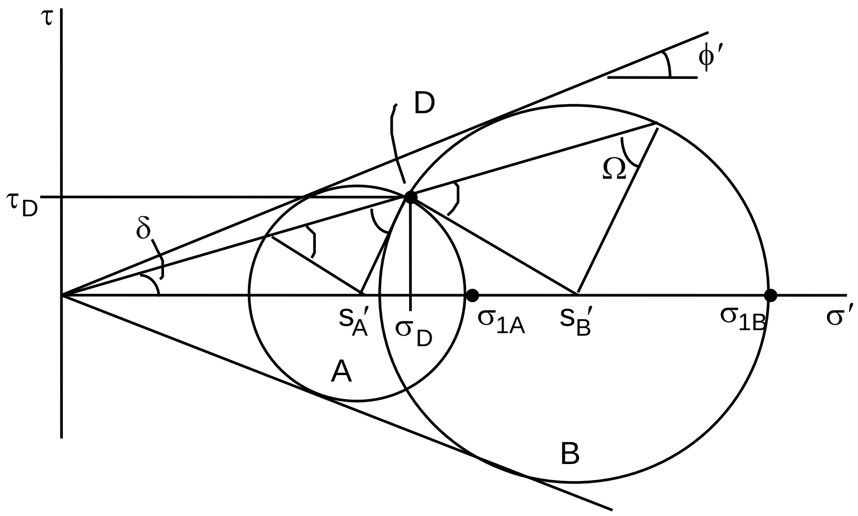
3.3 Plasticity: Coulomb material $(\tau/\sigma')_{max} = \tan\phi$

Limiting stresses

$$\sin\phi = (\sigma'_{1f} - \sigma'_{3f})/(\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f})/(\sigma_{1f} + \sigma_{3f} - 2u)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principal total stresses at failure, and u is the pore pressure.

3.4 Stress conditions across a discontinuity



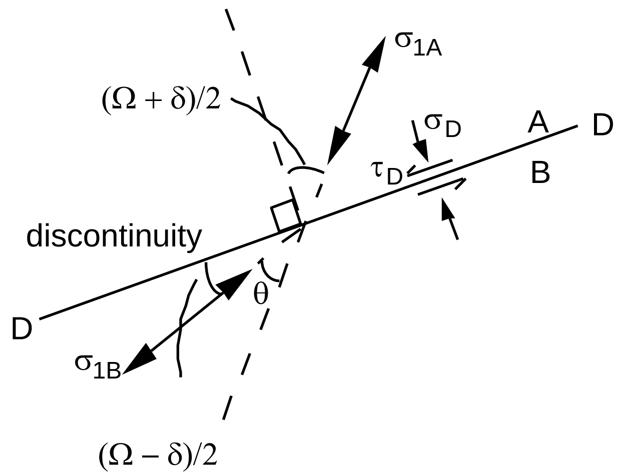
Rotation of major principal stress

$\theta = \pi/2 - \Omega$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

$\tan\delta = \tau_D / \sigma'_D$



$\sin\Omega = \sin\delta / \sin\phi'$

$s'_B/s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$

In limit, $d\theta \rightarrow 0$ and $\delta \rightarrow \phi'$

$ds' = 2s'.\, d\theta \tan\phi'$

$s'_B/s'_A = \exp(2\theta \tan\phi')$

Section 4: Bearing capacity of shallow foundations

4.1 Tresca soil, with undrained strength s_u

4.1.1 Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$N_c = 2 + \pi$ (Prandtl, 1921)

Shape correction factor:
For a rectangular footing of length L and breadth B (Eurocode 7):

$s_c = 1 + 0.2\ B / L$

The exact solution for a rough circular foundation (B/L=1) is q_f= 6.05s_u, hence s_c= 1.18~ 0.2.

Embedment correction factor:
A fit to Skempton's (1951) embedment correction factors, for an embedment of h, is:

$d_c = 1 + 0.33 \tan^{-1} (h/D)$ (or h/B for a strip or rectangular foundation)

4.1.2 Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

If $V/V_{ult} > 0.5$: $$\frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{H}{H_{ult}}} \quad \text{or} \quad \frac{H}{H_{ult}} = 1 - \left(2\frac{V}{V_{ult}} - 1\right)^2$$

If $V/V_{ult} < 0.5$: $H = H_{ult} = Bs_u$

4.1.3 Combined V-H-M loading

With lift-off: combined Green-Meyerhof ($V\rho_{ult}$ = bearing capacity of effective area B-e)

If $V/V\rho_{ult} < 0.5$: $$\frac{H}{H_{ult}} = \left(1 - 2\frac{M}{VB}\right)$$

Without lift-off: $$\left(\frac{V}{V_{ult}}\right)^2 + \left[\frac{M}{M_{ult}}\left(1 - 0.3\frac{H}{H_{ult}}\right)\right]^2 + \left|\left(\frac{H}{H_{ult}}\right)^3\right| - 1 = 0$$ (Taiebat & Carter 2000)

4.2 Frictional (Coulomb) soil, with friction angle ϕ

4.2.1 Vertical loading

The vertical bearing capacity, q_f, of a shallow foundation under drained loading (Coulomb soil) is:

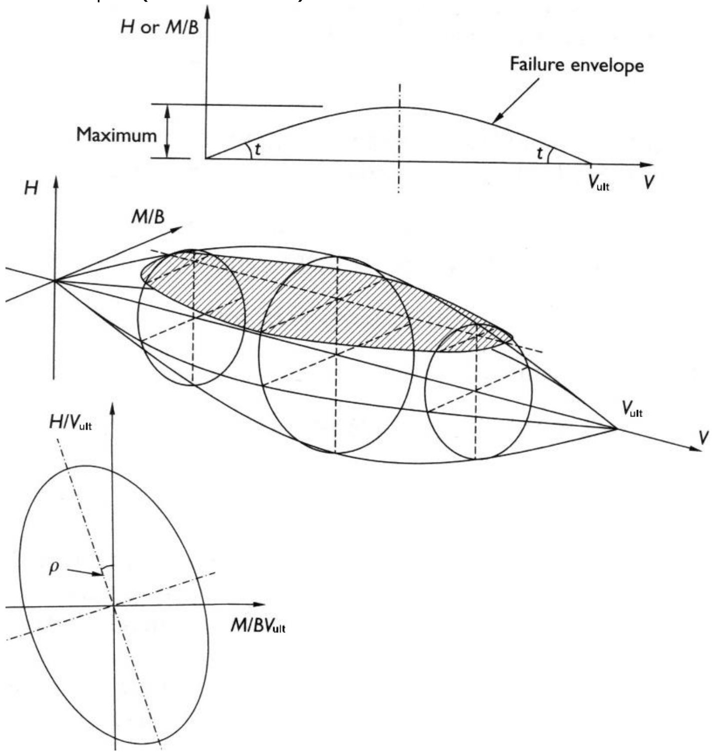
$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$N_q = \tan^2(\pi/4 + \phi/2)\, e^{(\pi \tan \phi)}$ (Prandtl 1921)

An empirical relationship to estimate N_γ from N_q is (Eurocode 7):



$N_\gamma = 2\,(N_q - 1) \tan \phi$

Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

Rough base: $N_\gamma = 0.1054\, e^{9.6\phi}$

Smooth base: $N_\gamma = 0.0663\, e^{9.3\phi}$

Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$s_q = 1 + (B \sin \phi) / L$
$s_\gamma = 1 - 0.3\, B / L$

For circular footings assume L = B.

4.2.2 Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

4.2.3 Combined V-H-M loading
(with lift-off- drained conditions- see failure surface shown above)

$$\left[\frac{H/V_{ult}}{t_h}\right]^2 + \left[\frac{M/BV_{ult}}{t_m}\right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m}\right] = \left[\frac{V}{V_{ult}}\left(1 - \frac{V}{V_{ult}}\right)\right]^2$$

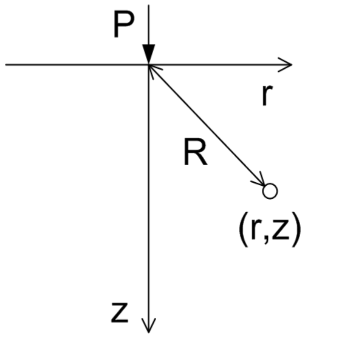
where $C = \tan\left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2 t_h t_m}\right)$ (Butterfield & Gottardi 1994)

Typically, t_h~0.5, t_m~0.4 and ρ~15°. t_h is the friction coefficient, H/V= tan ϕ, during sliding.

Section 5: Settlement of shallow foundations

5.1 Elastic stress distributions below point, strip and circular loads

Point loading (Boussinesq solution)



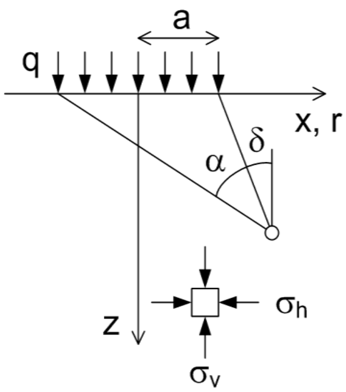
Vertical stress $$\sigma_z = \frac{3Pz^3}{2\pi R^5}$$

Radial stress $$\sigma_r = \frac{P}{2\pi R^2}\left[\frac{3r^2 z}{R^3} - \frac{(1-2\nu)R}{R+z}\right]$$

Tangential stress $$\sigma_\theta = \frac{P(1-2\nu)}{2\pi R^2}\left[\frac{R}{R+z} - \frac{z}{R}\right]$$

Shear stress $$\tau_{rz} = \frac{3Prz^2}{2\pi R^5}$$

Uniformly-loaded strip



Vertical stress $$\sigma_v = \frac{q}{\pi}\left[\alpha + \sin\alpha\cos(\alpha + 2\delta)\right]$$

Horizontal stress $$\sigma_h = \frac{q}{\pi}\left[\alpha - \sin\alpha\cos(\alpha + 2\delta)\right]$$

Shear stress $$\tau_{vh} = \frac{q}{\pi}\sin\alpha\sin(\alpha + 2\delta)$$

Principal stresses

$$\sigma_1 = \frac{q}{\pi}(\alpha + \sin\alpha) \qquad \sigma_3 = \frac{q}{\pi}(\alpha - \sin\alpha)$$

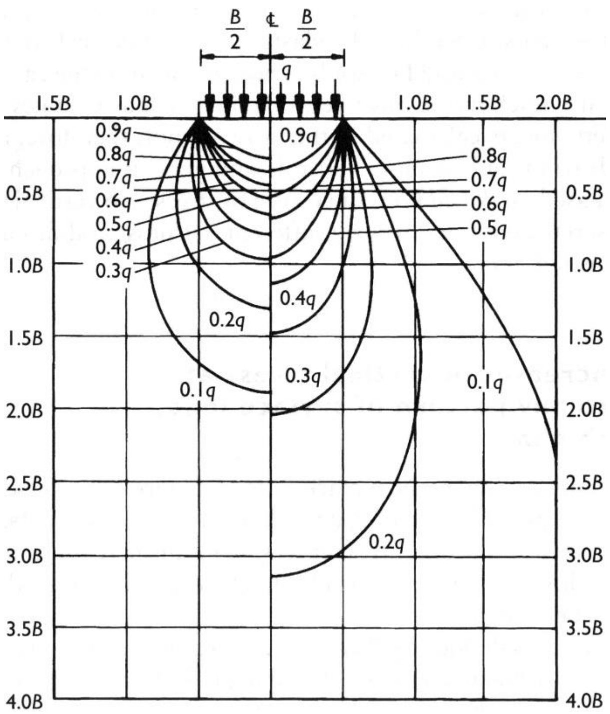
Uniformly-loaded circle
(on centerline, r=0)

Vertical stress

$$\sigma_v = q\left[1 - \left(\frac{1}{1+(a/z)^2}\right)^{\frac{3}{2}}\right]$$

Horizontal stress

$$\sigma_h = \frac{q}{2}\left[(1+2\nu) - \frac{2(1+\nu)z}{(a^2+z^2)^{1/2}} + \frac{z^3}{(a^2+z^2)^{3/2}}\right]$$



Contours of vertical stress below uniformly-loaded circular (left) and strip footings (right)

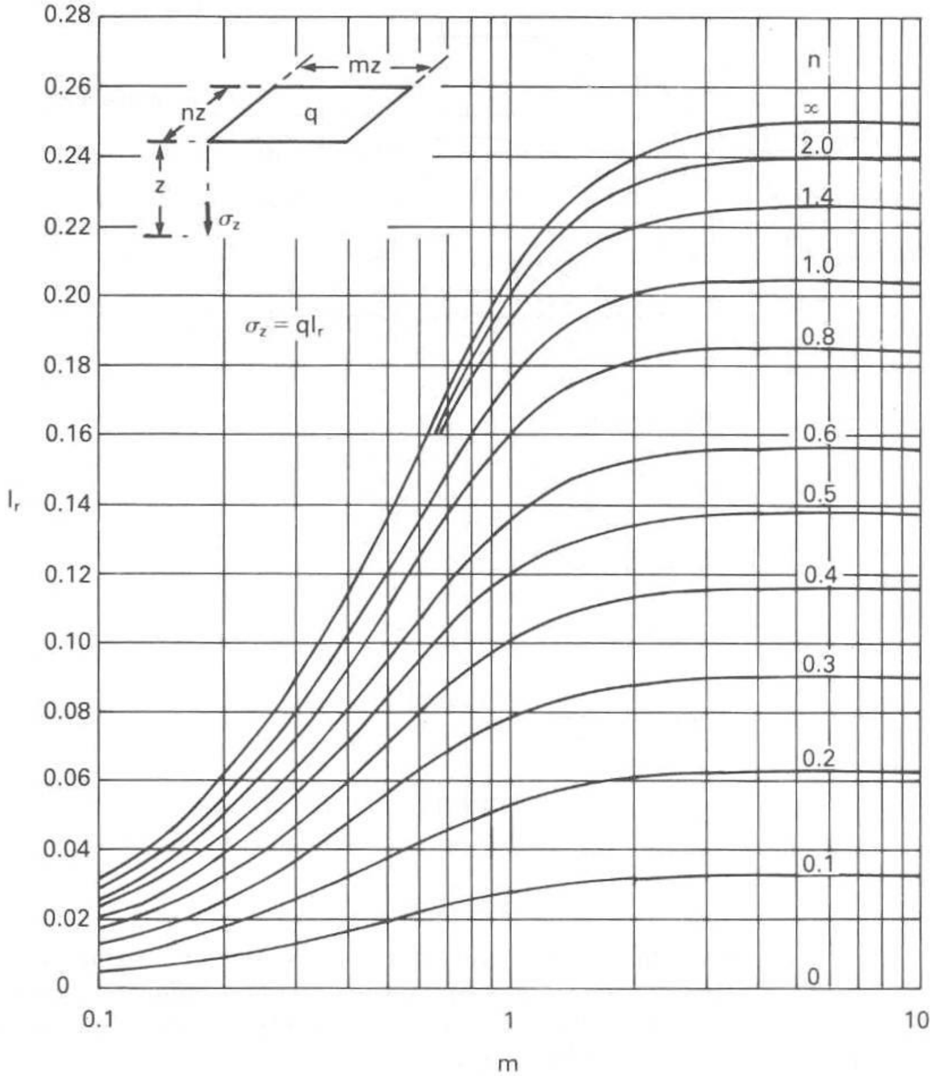
5.2 Elastic stress distribution below rectangular area

The vertical stress, σ_z, below the corner of a uniformly-loaded rectangle ($L \times B$) is:

$$\sigma_z = I_r q$$

I_r is found from m (=L/z) and n (=B/z) using Fadum's chart or the expression below (L and B are interchangeable), which are from integration of Boussinesq's solution.

$$I_r = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right)+\tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right]$$



Influence factor, I_r, for vertical stress under the corner of a uniformly-loaded rectangular area (Fadum's chart)

5.3 Elastic solutions for surface settlement

5.3.1 Isotropic, homogeneous, elastic half-space (semi-infinite)

Point load (Boussinesq solution)

Settlement, w, at distance s: $w(s) = \frac{1}{2\pi}\frac{(1-\nu)}{G}\frac{P}{s}$

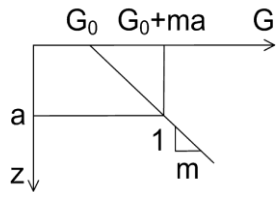
Circular area (radius a), uniform soil

Uniform load: central settlement: $w_o = \frac{(1-\nu)}{G}qa$

edge settlement: $w_e = \frac{2}{\pi}\frac{(1-\nu)}{G}qa$

Rigid punch: ($q_{avg}= V/\pi a^2$) $w_r = \frac{\pi}{4}\frac{(1-\nu)}{G}q_{avg}a$

Circular area, stiffness increasing with depth



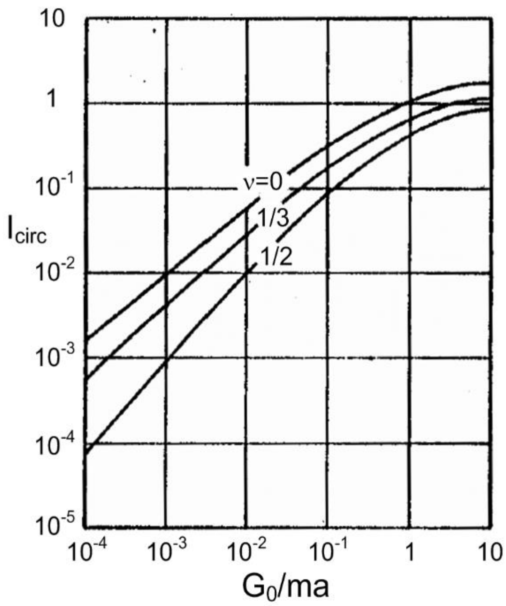
For $G_0= 0$, $\nu= 0.5$:

$w= q/2m$ under loaded area of any shape
$w= 0$ outside loaded area

For $G_0> 0$, central settlement:

$$w_o = \frac{qa}{2G_0}I_{circ}$$

For $\nu= 0.5$, $w_o \approx \frac{qa}{2(G_0 + ma)}$



Rectangular area, uniform soil

Uniform load, corner settlement:

$$w_c = \frac{(1-\nu)}{G}\frac{qB}{2}I_{rect}$$

Where I_{rect} depends on the aspect ratio, L/B:

L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}
1	0.561	1.6	0.698	2.4	0.822	5	1.052
1.1	0.588	1.7	0.716	2.5	0.835	6	1.110
1.2	0.613	1.8	0.734	3	0.892	7	1.159
1.3	0.636	1.9	0.750	3.5	0.940	8	1.201
1.4	0.658	2	0.766	4	0.982	9	1.239
1.5	0.679	2.2	0.795	4.5	1.019	10	1.272

Rigid rectangle: $w_r = \frac{(1-\nu)}{G}\frac{q_{avg}\sqrt{BL}}{2}I_{rgd}$ where I_{rgd} varies from $0.9\rightarrow0.7$ for L/B = 1-10.

Note: $G = \frac{E}{2(1+\nu)}$ where ν= Poisson's ratio, E= Young's modulus.

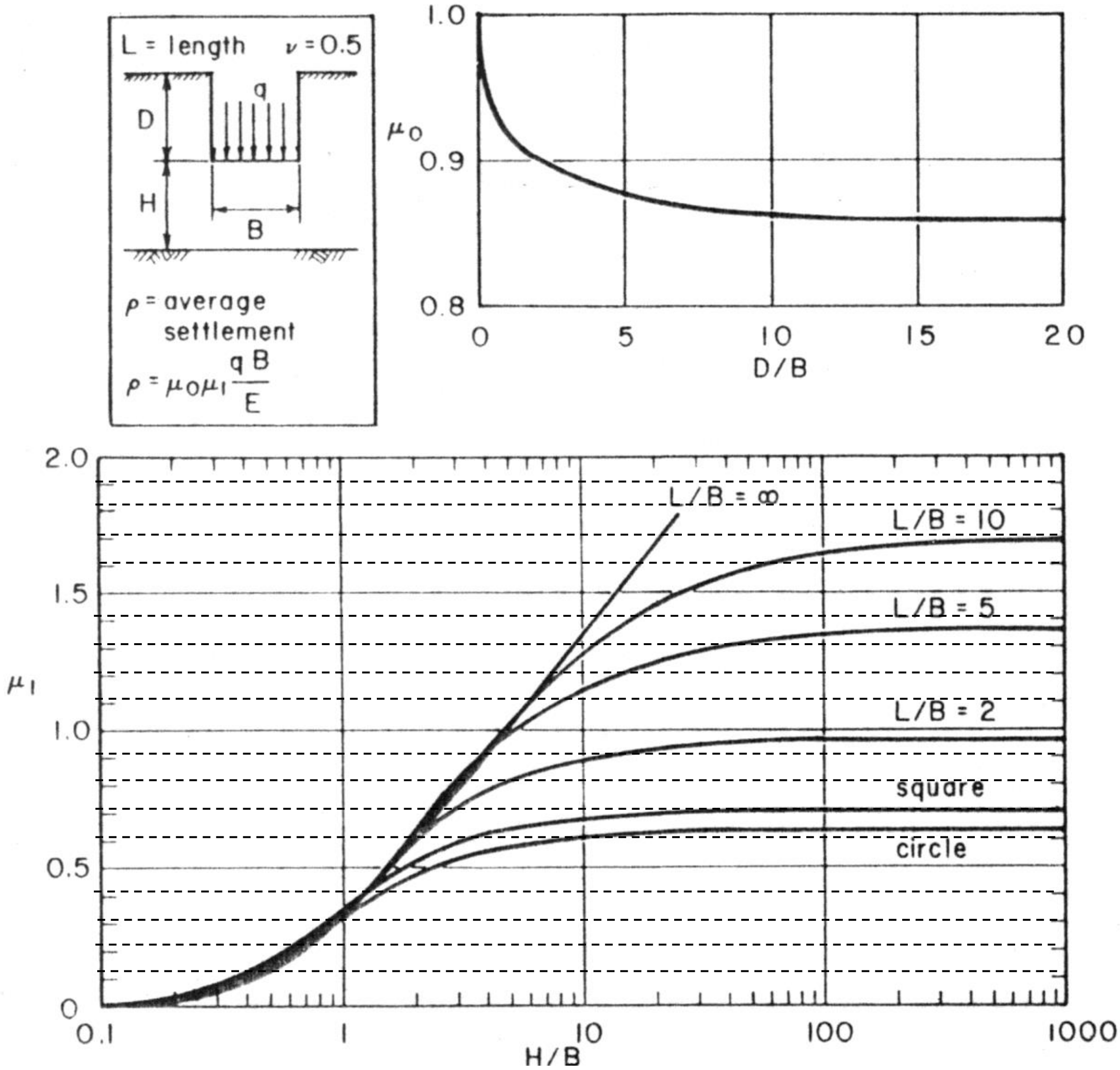
5.3.2 Isotropic, homogeneous, elastic finite space

Elastic layer of finite thickness

The mean settlement of a uniformly loaded foundation embedded in an elastic layer of finite thickness can be found using the charts below, for $\nu \sim 0.5$.

$$w_{avg} = \mu_0 \mu_1 \frac{qB}{E} \qquad\qquad E = 2G(1+\nu)$$

The influence factor μ_1 accounts for the finite layer thickness. The influence factor μ_0 accounts for the embedded depth.



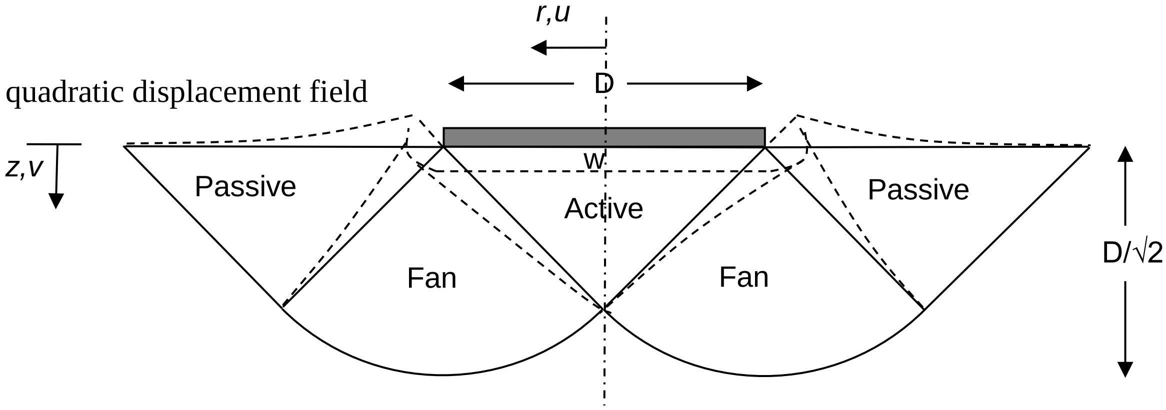
Average immediate settlement of a uniformly loaded finite thickness layer

Christian & Carrier (1978) Canadian Geotechnical Journal (15) 123-128
Janbu, Bjerrum and Kjaernsli's chart reinterpreted

5.4 Mobilizable Strength Design (MSD) solutions

Rigid circular foundation on incompressible half-space: Osman & Bolton (2005)

Vertical deformation mechanism



Average shear strain within deformation mechanism: $\gamma_{mob} = M_c\ w/D \approx 1.3\ w/D$

Average shear stress mobilized within mechanism: $\tau_{mob} = q\ /\ N_c \approx q\ /\ 6$

Representative depth for stress-strain behaviour: $z_{rep} = 0.3D$

V *or* H *or* M loading: Osman et al. (2007) Geotechnique 57 (9) 729-737

Vertical loading	$V = 1.5\pi D^2\,\tau_{mob}$	corresponding to	$\gamma_{mob} = 1.3\,w/D$
Horizontal loading	$H = 0.25\pi D^2\,\tau_{mob}$	corresponding to	$\gamma_{mob} = 8.5\,u/D$
Moment loading	$M = 0.17\pi D^3\,\tau_{mob}$	corresponding to	$\gamma_{mob} = 2\theta$

5.5 Atkinson's Equivalent Stiffness G*: Osman, White, Britto & Bolton (2007)

Rigid smooth circular foundation on a deep homogeneous bed

Vertical loading V for linear soil: settlement $w = \dfrac{\pi(1-\nu)}{4\quad G} q_{avg} a = \dfrac{(1-\nu)}{2}\dfrac{V}{GD}$

for power-law soil: use G* determined at $\gamma = \dfrac{w}{2D}$

Moment loading M for linear soil: rotation $\theta = \dfrac{3(1-\nu)M}{GD^3}$

for power-law soil: use G* determined at $\gamma = 0.68\ \theta$

Rigid rough circular foundation on a deep homogeneous bed

Shear loading H for linear soil: displacement $u = \dfrac{16(1-\nu)}{(7-8\nu)}\dfrac{H}{GD}$

for power-law soil: use G* determined at $\gamma = \dfrac{1.15\,u}{D}$

Rectangular foundations: Use $D = \sqrt{B\,L}$

Section 6: Bearing capacity of deep foundations

6.1 Axial capacity: API (2000) design method for driven piles

6.1.1 Sand

Unit shaft resistance: $\tau_{sf} = \sigma'_{hf} \tan\delta = K\sigma'_{v0} \tan\delta \leq \tau_{s,lim}$

Closed-ended piles: $K = 1$
Open-ended piles: $K = 0.8$

Unit base resistance: $q_b = N_q\ \sigma'_{vo} < q_{b,limit}$

Soil category	*Soil density*	*Soil type*	*Soil-pile friction angle, δ (°)*	*Limiting value $\tau_{s,lim}$ (kPa)*	*Bearing capacity factor, N_q*	*Limiting value, $q_{b,lim}$ (MPa)*
1	Very loose Loose Medium	Sand Sand-silt Silt	15	50	8	1.9
2	Loose Medium Dense	Sand Sand-silt Silt	20	75	12	2.9
3	Medium Dense	Sand Sand-silt	25	85	20	4.8
4	Dense Very dense	Sand Sand-silt	30	100	40	9.6
5	Dense Very dense	Gravel Sand	35	115	50	12

API (2000) recommendations for driven pile capacity in sand

6.1.2 Clay

American Petroleum Institute (API) (2000) guidelines for driven piles in clay.

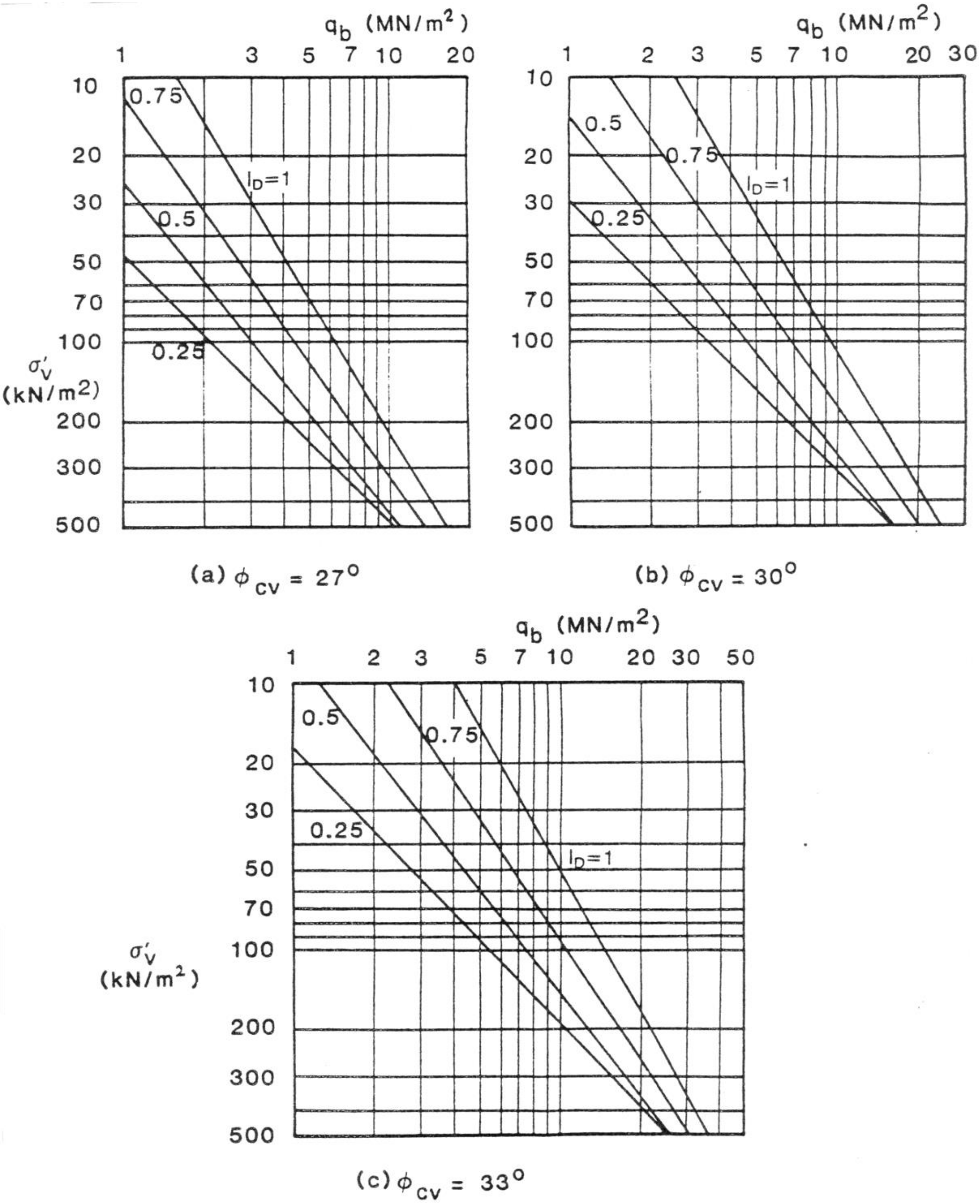
***Unit shaft resistance*:** $$\alpha = \frac{\tau_s}{s_u} = 0.5 \cdot \mathrm{Max}\left[\left(\frac{\sigma'_{vo}}{s_u}\right)^{0.5}, \left(\frac{\sigma'_{vo}}{s_u}\right)^{0.25}\right]$$

It is assumed that equal shaft resistance acts inside and outside open-ended piles.

***Unit base resistance*:** $q_b = N_c\ s_u$ $N_c = 9$.

6.2 Axial capacity: base resistance in sand using Bolton's stress dilatancy

Unit base resistance, q_b, is expressed as a function of relative density, I_D, constant volume (critical state) friction angle, ϕ_{cv}, and in situ vertical effective stress, σ'_v.



Design charts for base resistance in sand
(Randolph 1985, Fleming et al 1992)

6.3 Axial capacity: pile plugging

Resulting stress at the base of the plug:

$$q_{bf-plug} = \gamma' h_p \left(\frac{e^{\lambda} - 1}{\lambda}\right)$$

Where:

$$\lambda = 4\beta \frac{h_p}{D}$$

$$\beta = \frac{sin\phi \sin(\Delta - \delta)}{1 + sin\phi \cos(\Delta - \delta)}$$

$$sin\Delta = \frac{sin\delta}{sin\phi}$$

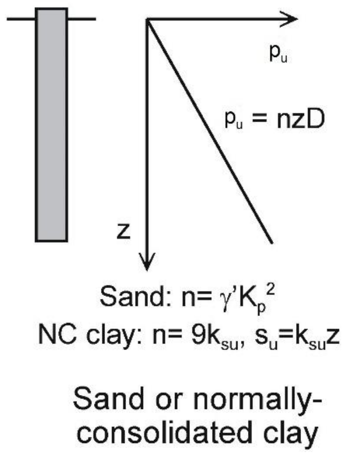
6.4 Lateral capacity: linearly increasing lateral resistance with depth

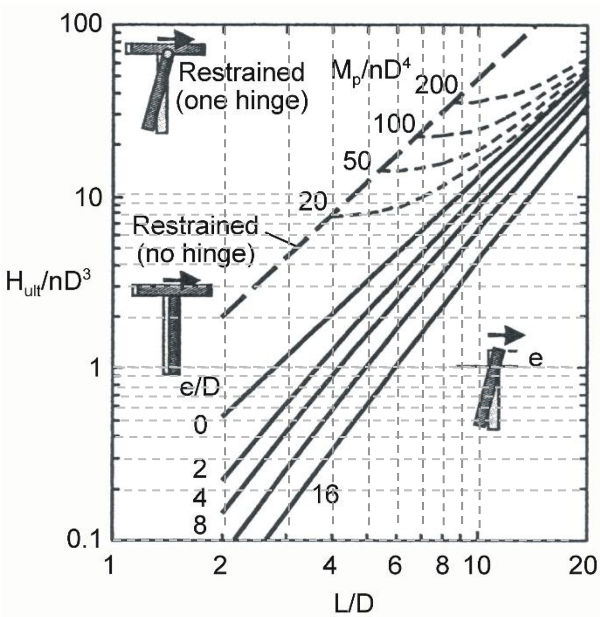
Lateral soil resistance (force per unit length), $p_u = nzD$

In sand, $n = \gamma' K_p^2$
In normally consolidated clay with strength gradient k; $s_u = kz$; $n=9k$

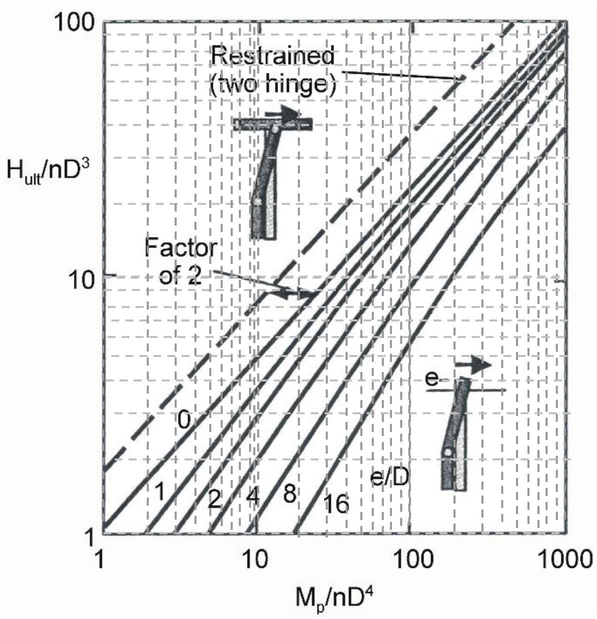
H_{ult} ultimate horizontal load on pile

M_p plastic moment capacity of pile
D pile diameter
L pile length
e load level above pile head
(=M/H for H-M pile head loading)
γ' effective unit weight
K_p passive earth pressure coefficient,
$K_p = (1 + \sin\phi)/(1 - \sin\phi)$





Short pile failure mechanism



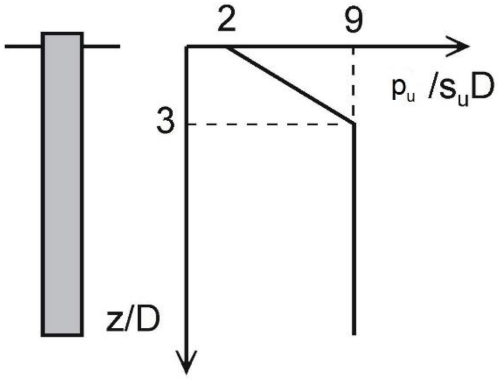
Long pile failure mechanism

Lateral pile capacity
(linearly increasing lateral resistance with depth)

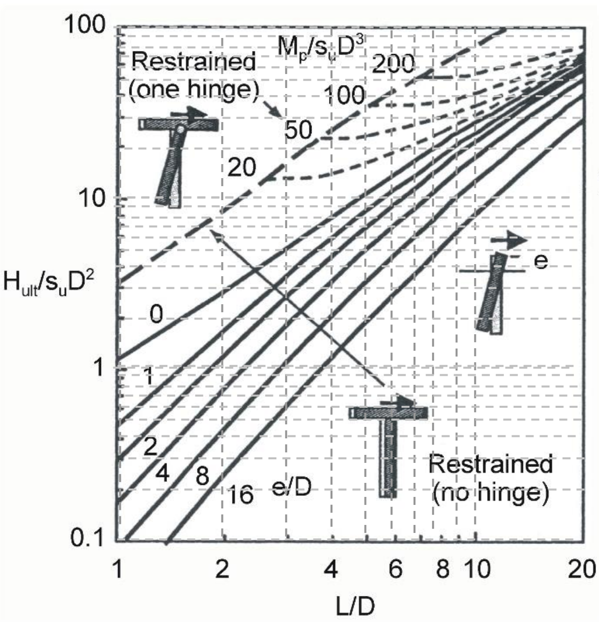
6.5 Lateral capacity: uniform clay

Lateral soil resistance (force per unit length), p_u, increases from $2s_uD$ at surface to $9s_uD$ at 3D depth then remains constant.

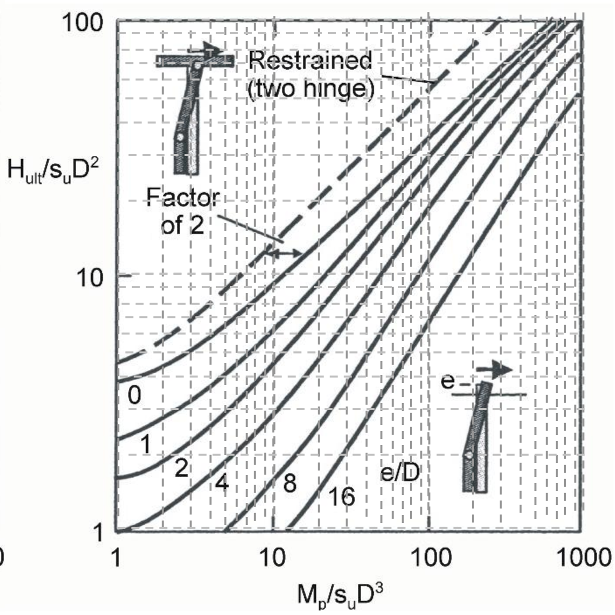
H_{ult}	ultimate horizontal load on pile
M_p	plastic moment capacity of pile
D	pile diameter
L	pile length
e	load level above pile head (=M/H for H-M pile head loading)
s_u	undrained shear strength



Uniform clay



Short pile failure mechanism



Long pile failure mechanism

Lateral pile capacity
(uniform clay lateral resistance profile)

Section 7: Settlement of deep foundations

7.1 Settlement of a rigid pile

Shaft response:

Equilibrium:

$$\tau = \tau_s \frac{R}{r}$$

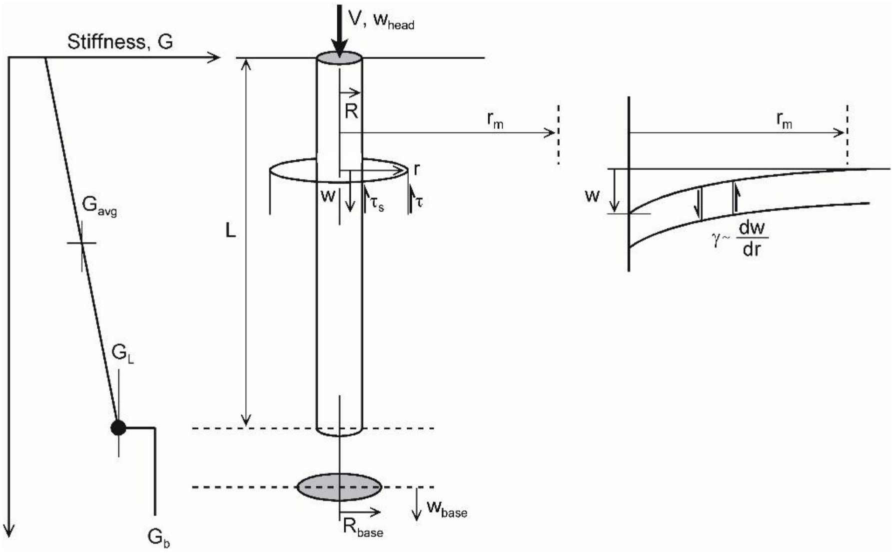
Compatibility:

$$\gamma \approx \frac{dw}{dr}$$

Elasticity:

$$\frac{\tau}{\gamma} = G$$

Integrate to magical radius, r_m, for shaft stiffness, τ_s/w.



Nomenclature for settlement analysis of single piles

Combined response of base (rigid punch) and shaft:

$$\frac{V}{w_{head}} = \frac{Q_b}{w_{base}} + \frac{Q_s}{w} \qquad \frac{V}{w_{head}} = \frac{4R_{base}G_{base}}{1-\nu} + \frac{2\pi L G_{avg}}{\zeta}$$

$$\frac{V}{w_{head}DG_L} = \frac{2}{1-\nu}\frac{G_{base}}{G_L}\frac{D_{base}}{D} + \frac{2\pi}{\zeta}\frac{G_{avg}}{G_L}\frac{L}{D} \qquad \frac{V}{w_{head}DG_L} = \frac{2}{1-\nu}\frac{\eta}{\xi} + \frac{2\pi}{\zeta}\rho\frac{L}{D}$$

These expressions are simplified using dimensionless variables:

Base enlargement ratio, eta	$\eta = R_{base}/R = D_{base}/D$	Slenderness ratio	L/D
Stiffness gradient ratio, rho	$\rho = G_{avg}/G_L$	Base stiffness ratio, xi	$\xi = G_L/G_{base}$

It is often assumed that the dimensionless zone of influence, $\zeta = \ln(r_m/R) = 4$.

More precise relationships, checked against numerical analysis are:

$$\zeta = \ln\left\{\{0.5 + (5\rho(1-\nu) - 0.5)\xi\}\frac{L}{D}\right\} \qquad \text{for } \xi = 1: \qquad \zeta = \ln\left\{5\rho(1-\nu)\frac{L}{D}\right\}$$

7.2 Settlement of a compressible pile

$$\frac{V}{w_{head}DG_L} = \frac{\dfrac{2\eta}{(1-\nu)\xi} + \rho\dfrac{2\pi}{\zeta}\dfrac{\tanh\mu L}{\mu L}\dfrac{L}{D}}{1 + \dfrac{1}{\pi\lambda}\dfrac{8\eta}{(1-\nu)\xi}\dfrac{\tanh\mu L}{\mu L}\dfrac{L}{D}}$$

where $\mu = \dfrac{\sqrt{8/\zeta\lambda}}{D}$ Pile compressibility

$\lambda = E_p/G_L$ Pile-soil stiffness ratio

Pile head stiffness, $\dfrac{V}{w_{head}}$, is maximum when L ≥1.5D $\sqrt{\lambda}$