

EGT3
ENGINEERING TRIPOS PART IIB

Thursday 26 April 2018 2 to 3.40

Module 4D5

FOUNDATION ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed

Attachment: 4D5 Foundation Engineering Data Sheet (18 pages)

Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A building site is underlain by a deep overconsolidated clay deposit. Field and laboratory testing have established the clay has an undrained shear strength of 30 kPa and a saturated unit weight of 17 kN m^{-3} . The water table is at the ground surface.

A typical column carries a vertical load of 225 kN and a moment of 150 kN m parallel to one of the sides of the footing.

- (a) Calculate the ultimate vertical bearing capacity of the clay for a square footing with embedment of 1 m without lift-off. [10%]
- (b) Suggest a size for the footing, using the allowable bearing capacity calculated according to Eurocode 7. [40%]
- (c) Derive the V-M failure surface and calculate the factors of safety for an increasing vertical load alone and for an increasing moment alone. [40%]
- (d) Explain how a skirted foundation may provide benefits against lift-off and the limitations of such a solution. [10%]

2 A 1 m diameter steel tubular pile is driven 20 m into a sand deposit with a dry unit weight of 16 kN m^{-3} and a saturated unit weight of 20 kN m^{-3} . The steel has a yield strength of 250 MPa.

- (a) If the water table is coincident with the ground surface, calculate:
 - (i) the lateral load capacity of the pile for a load applied at the ground surface; [30%]
 - (ii) the wall thickness of the pile required to prevent plastic failure. [20%]
- (b) If the water table falls to a depth 5 m below the ground surface, what is the new lateral capacity of the pile if the load is applied at ground level? [50%]

3 A new building is designed with a basement on a rectangular mat foundation with dimensions of 25 m by 35 m. The base of the foundation is 4 m below the ground surface. The superstructure is exerting loads resulting in a uniform pressure of 200 kPa on the foundation.

The soil profile consists of a 50 m thick layer of overconsolidated clay overlying bedrock. The clay has a saturated unit weight of 20 kN m^{-3} and shear modulus of 10 MPa estimated from unconsolidated undrained triaxial tests. The water table can be taken to be at the ground surface.

- (a) Calculate the net building pressure. [10%]

- (b) Assuming the foundation to be fully flexible, estimate the immediate settlement at the centre of the foundation and the maximum deflection ratio. Comment on this result in relation to the performance of the foundation. [30%]

- (c) Predict the immediate settlement assuming the foundation is fully rigid. Comparing the results in part (b) and part (c), which is likely to be more representative of the actual foundation, and why? [15%]

- (d) Consider the situation in which the bedrock is instead at a depth of 25 m. What is the effect of the shallower bedrock? Estimate the immediate settlement in this case. [30%]

- (e) The actual measured settlement is lower than any of the estimates. Why? [15%]

4 (a) A single reinforced concrete pile, 15 m long and 0.3 m in diameter is bored in a clay stratum with a stiffness at the surface of 15 MPa increasing linearly with depth to a value of 25 MPa at the base of the pile.

(i) Why might the pile base stiffness be ignored when calculating the resultant vertical pile stiffness? [10%]

(ii) Assuming the pile itself to be rigid, calculate the stiffness of the pile to vertical loads ignoring the stiffness of the pile base. [20%]

(b) Nine piles identical to those in part (a) are now installed in a 3×3 grid with a pile centre spacing of 1.5 m and their heads connected by a rigid pile cap.

(i) Why does the stiffness of a group of piles differ from that of the individual piles? [10%]

(ii) Ignoring any bearing capacity of the pile cap, calculate the resultant stiffness of the pile group to vertical loads. What is the efficiency of the pile group relative to the individual pile? [60%]

END OF PAPER

Cambridge University Engineering Department

Supplementary Databook

Module 4D5: Foundation Engineering

GB/SKH January 2018

Section 1: ULS and SLS

Eurocode partial factors for design

Case	Actions			Ground Properties	
	Permanent		Variable	tan ϕ	s _u
	<i>Unfavourable</i>	<i>Favourable</i>	<i>Unfavourable</i>		
EQU	1.1	0.9	1.5	1.1	1.2
STR (A1)	1.35	1.0	1.5	1.0	1.0
GEO (A2)	1.0	1.0	1.3	1.25	1.4

Case EQU : governs overall stability of a structure

Case STR : concerned only with the failure of structural members, including foundations and retaining structures

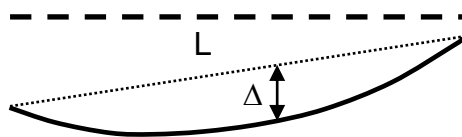
Case GEO : applies to the design of foundations and earthworks

Damage to buildings from differential settlement

Distortion defined as relative displacement Δ/L :

L is the length of building segment with consistent sagging or hogging

Δ is the maximum settlement of the deformed segment relative to chord L



Distortion linked to local tensile strain ε in elastic beams of various E/G and L/H:

$$\frac{\Delta}{L\varepsilon} \approx 1.0 \text{ to } 1.5 \text{ diagonally in end zones due to shear}$$

0.75 to 1.0 longitudinally in middle zone due to sagging

0.25 to 0.5 longitudinally in middle zone due to hogging

Onset of visible (~ 0.1 mm) cracks in brick or blockwork walls: $\varepsilon_{\max} \approx 0.75 \cdot 10^{-3}$
(after Burland & Wroth, 1974)

Categories of damage associated with masonry walls:

Cat.	Limit	Relative displacement	Description	Action
0	-	$\Delta/L \leq 0.5 \cdot 10^{-3}$	negligible	none
1	SLS	$0.5 \cdot 10^{-3} < \Delta/L \leq 0.75 \cdot 10^{-3}$	very slight	redecorate interior
2	SLS	$0.75 \cdot 10^{-3} < \Delta/L \leq 1.5 \cdot 10^{-3}$	slight	+ some repointing
3	SLS	$1.5 \cdot 10^{-3} < \Delta/L \leq 3 \cdot 10^{-3}$	moderate	+ significant repointing etc
4	ULS	$3 \cdot 10^{-3} < \Delta/L \leq 10^{-2}$	severe	shore; consider demolition
5	ULS	$10^{-2} < \Delta/L$	very severe	demolish

(after Boscardin & Cording, 1989)

Section 2: Empirical correlations from geotechnical data

2.1 Undrained shear strength of clays (s_u)

Normally consolidated clay $\left(\frac{s_u}{\sigma_v'}\right)_{nc} \approx 0.11 + 0.37 I_p$ after Skempton (1957)
 where I_p is the plasticity index

Overconsolidated clay $\frac{s_u}{\sigma_v'} \approx \left(\frac{s_u}{\sigma_v'}\right)_{nc} n^\Lambda$ after Ladd et al (1977)
 where n is overconsolidation ratio
 exponent $\Lambda \approx 0.8$

Penetrometer correlations $s_u = \frac{(q_{\text{penetrometer}} - \sigma_v')}{N_{\text{penetrometer}}}$ from q at tip load cell
 where $N_{\text{cone}} \approx 14 \pm 2$; $N_{\text{T-bar}} \approx 12 \pm 2$
 $s_u \approx 4.5 N_{60}$ kPa from SPT blow-count N_{60}

2.2 Drained shear strength of sands (friction and dilatancy): after Bolton (1986)

Definition of relative dilatancy $I_R = I_D I_C - 1$

definition of relative density $I_D = (e_{\text{max}} - e)/(e_{\text{max}} - e_{\text{min}})$

SPT blow-count correlation $I_D \approx [N_{60}/(20 + 0.2 \sigma_v', \text{kPa})]^{0.5}$

definition of relative crushability $I_C = \ln(\sigma_c/p')$

aggregate crushing stress $\sigma_c \approx$ 5 000 kPa for shelly sand
 20 000 kPa for quartz sand
 80 000 kPa for quartz silt

CPT correlation (q_{cone} , σ_v' in kPa) $I_D \approx 0.27(\ln q_{\text{cone}} - 0.5 \ln \sigma_v') - 1.29 \pm 0.15$ (higher if σ_c lower)

Peak friction correlation $(\phi_{\text{max}} - \phi_{\text{crit}}) \approx 0.8 \psi_{\text{max}} \approx 5^\circ \times I_R$ in plane strain

$(\phi_{\text{max}} - \phi_{\text{crit}}) \approx 3^\circ \times I_R$ in axisymmetric strain

Peak dilatancy rate $(-\delta\varepsilon_v / \delta\varepsilon_1)_{\text{max}} \approx 0.3 \times I_R$ in all conditions

Critical state friction angle $\phi_{\text{crit}} \approx 32^\circ$ (uniform, rounded) $\rightarrow 40^\circ$ (well-graded, angular)

2.3 Stiffness of clays: after Vardanega & Bolton (2011, 2012, 2013)

Very small strains ($\gamma \sim 10^{-6}$) $G_0 = \frac{B}{(1+e)^3} (p')^{0.5}$ with G_0, p' in kPa

soil fabric factor $B \approx 25\,000$ within factor 2

Small strains ($10^{-6} < \gamma < 10^{-2}$) $\frac{G}{G_0} \approx \frac{1}{1 + \left(\frac{\gamma}{\gamma_{ref}}\right)^a}$

hyperbolic curvature parameter $a = 0.74 \pm 10\%$

reference shear strain $\gamma_{ref} = 2.2 I_p 10^{-3} \pm 50\%$

Moderate mobilizations of strength ($0.2s_u < \tau_{mob} < 0.8s_u$; typical $\gamma > 0.1\%$)

mobilised shear strength $\frac{\tau_{mob}}{s_u} \approx 0.5 \left(\frac{\gamma}{\gamma_{M=2}}\right)^b$

mobilization strain $\gamma_{M=2} \approx 0.004 n^{0.7}$

power curve exponent $b \approx 0.37 + 0.01 n$ (default value 0.6)

Conventional linearised stiffness modulus G_{50} or $G_{M=2} = 0.5 s_u / \gamma_{M=2}$

2.4 Stiffness of sands: after Oztoprak & Bolton (2012)

Very small strains ($\gamma \sim 10^{-6}$) $G_0 = \frac{B}{(1+e)^3} (p')^{0.5}$ with G_0, p' in kPa

soil fabric factor $B \approx 50\,000$ within factor 2

Small strains ($10^{-6} < \gamma < 10^{-2}$) $\frac{G}{G_0} = \frac{1}{1 + \left(\frac{\gamma - \gamma_e}{\gamma_{ref}}\right)^a}$

hyperbolic curvature parameter $a = U_c^{-0.075}$

e.g. $a = 0.9$ at uniformity coefficient $U_c = 4$

reference shear strain $\gamma_{ref} = U_c^{-0.3} p' 10^{-6} + 8e I_D 10^{-4}$

limiting elastic strain $\gamma_e = 0.012 \gamma_{ref} + 2 \cdot 10^{-6}$

Section 3: Plasticity theory

This section is common to the Soil Mechanics Databook supporting modules 3D1 and 3D2. Undrained shear strength ('cohesion' in a Tresca material) is denoted by s_u rather than c_u .

3.1 Plasticity: Tresca material, $\tau_{max} = s_u$

Limiting stresses

Tresca $|\sigma_1 - \sigma_3| = q_u = 2s_u$

von Mises $(\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2s_u^2$

q_u = undrained triaxial compression strength; s_u = undrained plane shear strength.

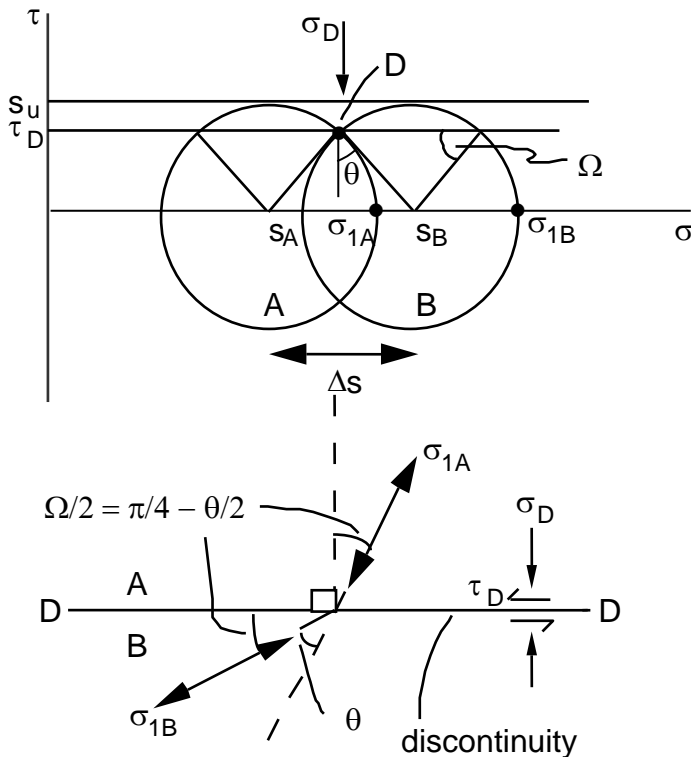
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = s_u \delta \epsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength s_u , this becomes

$$D = A s_u x$$

3.2 Stress conditions across a discontinuity:



Rotation of major principal stress

$$\theta = \pi/2 - \Omega$$

$$s_B - s_A = \Delta s = 2s_u \sin \theta$$

$$\sigma_{1B} - \sigma_{1A} = 2s_u \sin \theta$$

In limit with $\theta \rightarrow 0$

$$ds = 2s_u d\theta$$

Useful example:

$$\theta = 30^\circ$$

$$\sigma_{1B} - \sigma_{1A} = s_u$$

$$\tau_D / s_u = 0.87$$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

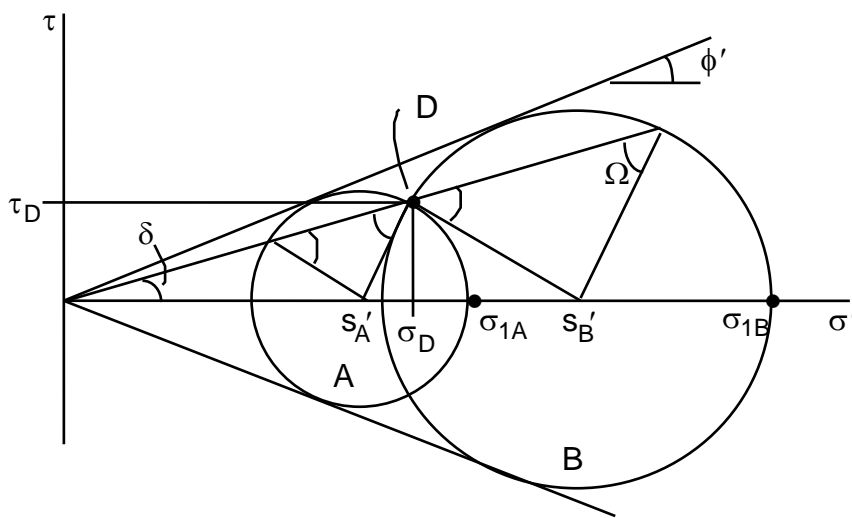
3.3 Plasticity: Coulomb material $(\tau/\sigma')_{\max} = \tan \phi$

Limiting stresses

$$\sin \phi = (\sigma'_{1f} - \sigma'_{3f}) / (\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f}) / (\sigma_{1f} + \sigma_{3f} - 2u)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principal total stresses at failure, and u is the pore pressure.

3.4 Stress conditions across a discontinuity



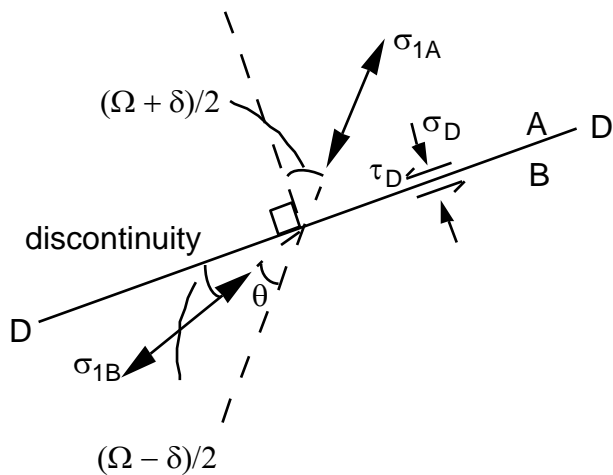
Rotation of major principal stress

$$\theta = \pi/2 - \Omega$$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

$$\tan \delta = \tau_D / \sigma'_D$$



$$\sin \Omega = \sin \delta / \sin \phi'$$

$$s'_B/s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$$

In limit, $d\theta \rightarrow 0$ and $\delta \rightarrow \phi'$

$$ds' = 2s' \cdot d\theta \tan \phi'$$

$$s'_B/s'_A = \exp(2\theta \tan \phi')$$

Section 4: Bearing capacity of shallow foundations

4.1 Tresca soil, with undrained strength s_u

4.1.1 Vertical loading

The vertical bearing capacity, q_f , of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$$N_c = 2 + \pi \quad (\text{Prandtl, 1921})$$

Shape correction factor:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_c = 1 + 0.2 B / L$$

The exact solution for a rough circular foundation ($B/L=1$) is $q_f = 6.05s_u$, hence $s_c = 1.18 \sim 0.2$.

Embedment correction factor:

A fit to Skempton's (1951) embedment correction factors, for an embedment of h , is:

$$d_c = 1 + 0.33 \tan^{-1} (h/D) \quad (\text{or } h/B \text{ for a strip or rectangular foundation})$$

4.1.2 Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

$$\text{If } V/V_{ult} > 0.5: \quad \frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{H}{H_{ult}}} \quad \text{or} \quad \frac{H}{H_{ult}} = 1 - \left(2 \frac{V}{V_{ult}} - 1 \right)^2$$

$$\text{If } V/V_{ult} < 0.5: \quad H = H_{ult} = B s_u$$

4.1.3 Combined V-H-M loading

With lift-off: combined Green-Meyerhof ($V_{\rho_{ult}}$ = bearing capacity of effective area $B-e$)

$$\text{If } V/V_{\rho_{ult}} < 0.5: \quad \frac{H}{H_{ult}} = \left(1 - 2 \frac{M}{VB} \right)$$

$$\text{Without lift-off: } \left(\frac{V}{V_{ult}} \right)^2 + \left[\frac{M}{M_{ult}} \left(1 - 0.3 \frac{H}{H_{ult}} \right) \right]^2 + \left| \left(\frac{H}{H_{ult}} \right)^3 \right| - 1 = 0 \quad (\text{Taiebat \& Carter 2000})$$

4.2 Frictional (Coulomb) soil, with friction angle ϕ

4.2.1 Vertical loading

The vertical bearing capacity, q_f , of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$$N_q = \tan^2(\pi/4 + \phi/2) e^{(\pi \tan \phi)} \quad (\text{Prandtl 1921})$$

An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$$N_\gamma = 2 (N_q - 1) \tan \phi$$

Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

$$\text{Rough base: } N_\gamma = 0.1054 e^{9.6\phi}$$

$$\text{Smooth base: } N_\gamma = 0.0663 e^{9.3\phi}$$

Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_q = 1 + (B \sin \phi) / L$$

$$s_\gamma = 1 - 0.3 B / L$$

For circular footings assume $L = B$.

4.2.2 Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

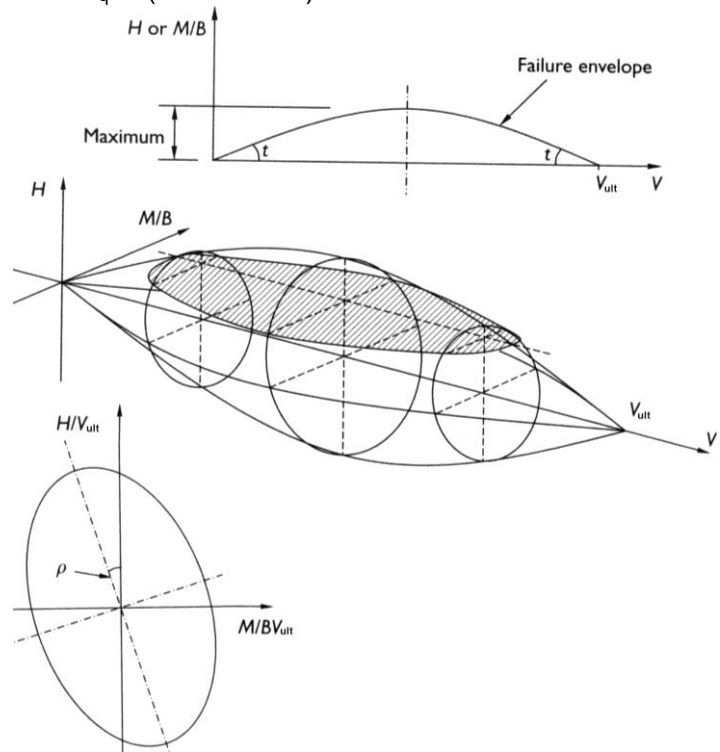
4.2.3 Combined V-H-M loading

(with lift-off- drained conditions- see failure surface shown above)

$$\left[\frac{H/V_{ult}}{t_h} \right]^2 + \left[\frac{M/BV_{ult}}{t_m} \right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m} \right] = \left[\frac{V}{V_{ult}} \left(1 - \frac{V}{V_{ult}} \right) \right]^2$$

$$\text{where } C = \tan \left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2t_h t_m} \right) \quad (\text{Butterfield \& Gottardi 1994})$$

Typically, $t_h \sim 0.5$, $t_m \sim 0.4$ and $\rho \sim 15^\circ$. t_h is the friction coefficient, $H/V = \tan \phi$, during sliding.



Section 5: Settlement of shallow foundations

5.1 Elastic stress distributions below point, strip and circular loads

Point loading (Boussinesq solution)

Vertical stress

$$\sigma_z = \frac{3Pz^3}{2\pi R^5}$$

Radial stress

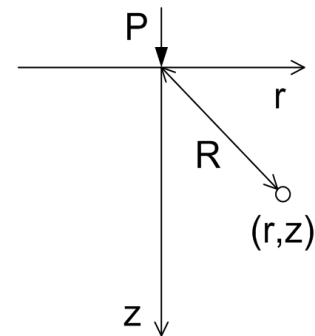
$$\sigma_r = \frac{P}{2\pi R^2} \left[\frac{3r^2z}{R^3} - \frac{(1-2\nu)R}{R+z} \right]$$

Tangential stress

$$\sigma_\theta = \frac{P(1-2\nu)}{2\pi R^2} \left[\frac{R}{R+z} - \frac{z}{R} \right]$$

Shear stress

$$\tau_{rz} = \frac{3Prz^2}{2\pi R^5}$$



Uniformly-loaded strip

Vertical stress

$$\sigma_v = \frac{q}{\pi} [\alpha + \sin\alpha \cos(\alpha + 2\delta)]$$

Horizontal stress

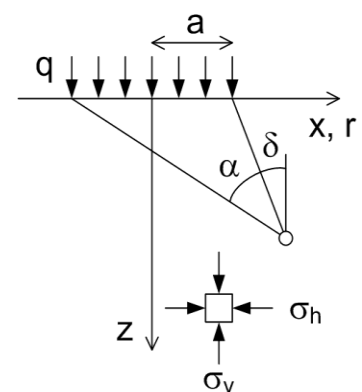
$$\sigma_h = \frac{q}{\pi} [\alpha - \sin\alpha \cos(\alpha + 2\delta)]$$

Shear stress

$$\tau_{vh} = \frac{q}{\pi} \sin\alpha \sin(\alpha + 2\delta)$$

Principal stresses

$$\sigma_1 = \frac{q}{\pi} (\alpha + \sin\alpha) \quad \sigma_3 = \frac{q}{\pi} (\alpha - \sin\alpha)$$



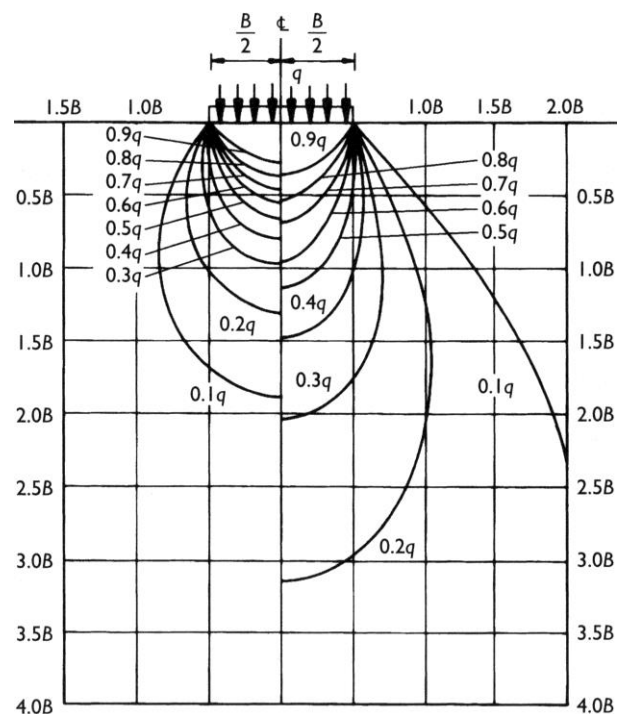
Uniformly-loaded circle (on centerline, r=0)

Vertical stress

$$\sigma_v = q \left[1 - \left(\frac{1}{1 + (a/z)^2} \right)^{\frac{3}{2}} \right]$$

Horizontal stress

$$\sigma_h = \frac{q}{2} \left[(1 + 2\nu) - \frac{2(1 + \nu)z}{(a^2 + z^2)^{1/2}} + \frac{z^3}{(a^2 + z^2)^{3/2}} \right]$$



Contours of vertical stress below uniformly-loaded circular (left) and strip footings (right)

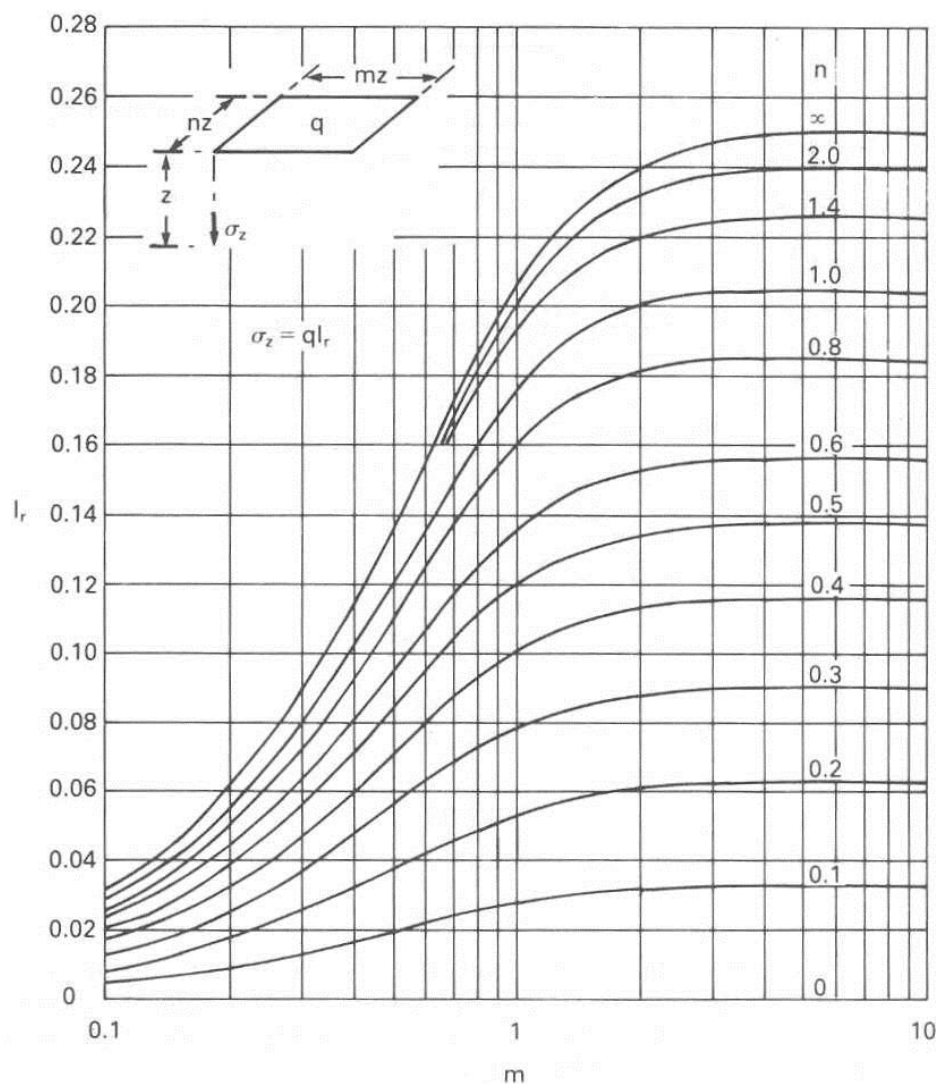
5.2 Elastic stress distribution below rectangular area

The vertical stress, σ_z , below the corner of a uniformly-loaded rectangle ($L \times B$) is:

$$\sigma_z = I_r q$$

I_r is found from m ($=L/z$) and n ($=B/z$) using Fadum's chart or the expression below (L and B are interchangeable), which are from integration of Boussinesq's solution.

$$I_r = \frac{1}{4\pi} \left[\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + m^2n^2 + 1} \left(\frac{m^2 + n^2 + 2}{m^2 + n^2 + 1} \right) + \tan^{-1} \left(\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 - m^2n^2 + 1} \right) \right]$$



Influence factor, I_r , for vertical stress under the corner of a uniformly-loaded rectangular area (Fadum's chart)

5.3 Elastic solutions for surface settlement

5.3.1 Isotropic, homogeneous, elastic half-space (semi-infinite)

Point load (Boussinesq solution)

Settlement, w , at distance s :
$$w(s) = \frac{1}{2\pi} \frac{(1-\nu) P}{G s}$$

Circular area (radius a), uniform soil

Uniform load: central settlement:
$$w_o = \frac{(1-\nu)}{G} qa$$

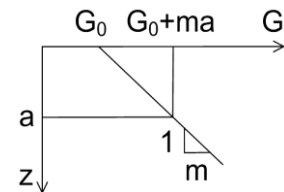
edge settlement:
$$w_e = \frac{2(1-\nu)}{\pi G} qa$$

Rigid punch: ($q_{avg} = V/\pi a^2$)
$$w_r = \frac{\pi(1-\nu)}{4 G} q_{avg} a$$

Circular area, stiffness increasing with depth

For $G_0 = 0$, $\nu = 0.5$:

$w = q/2m$ under loaded area of any shape
 $w = 0$ outside loaded area



For $G_0 > 0$, central settlement:

$$w_o = \frac{qa}{2G_0} I_{circ}$$

For $\nu = 0.5$, $w_o \approx \frac{qa}{2(G_0 + ma)}$

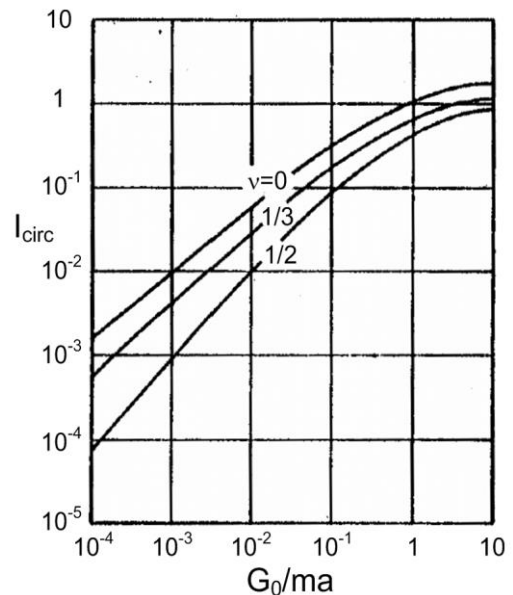
Rectangular area, uniform soil

Uniform load, corner settlement:

$$w_c = \frac{(1-\nu) qB}{G} \frac{1}{2} I_{rect}$$

Where I_{rect} depends on the aspect ratio, L/B :

L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}	L/B	I_{rect}
1	0.561	1.6	0.698	2.4	0.822	5	1.052
1.1	0.588	1.7	0.716	2.5	0.835	6	1.110
1.2	0.613	1.8	0.734	3	0.892	7	1.159
1.3	0.636	1.9	0.750	3.5	0.940	8	1.201
1.4	0.658	2	0.766	4	0.982	9	1.239
1.5	0.679	2.2	0.795	4.5	1.019	10	1.272



Rigid rectangle: $w_r = \frac{(1-\nu) q_{avg} \sqrt{BL}}{G} I_{rgd}$ where I_{rgd} varies from 0.9 \rightarrow 0.7 for $L/B = 1-10$.

Note: $G = \frac{E}{2(1+\nu)}$ where ν = Poisson's ratio, E = Young's modulus.

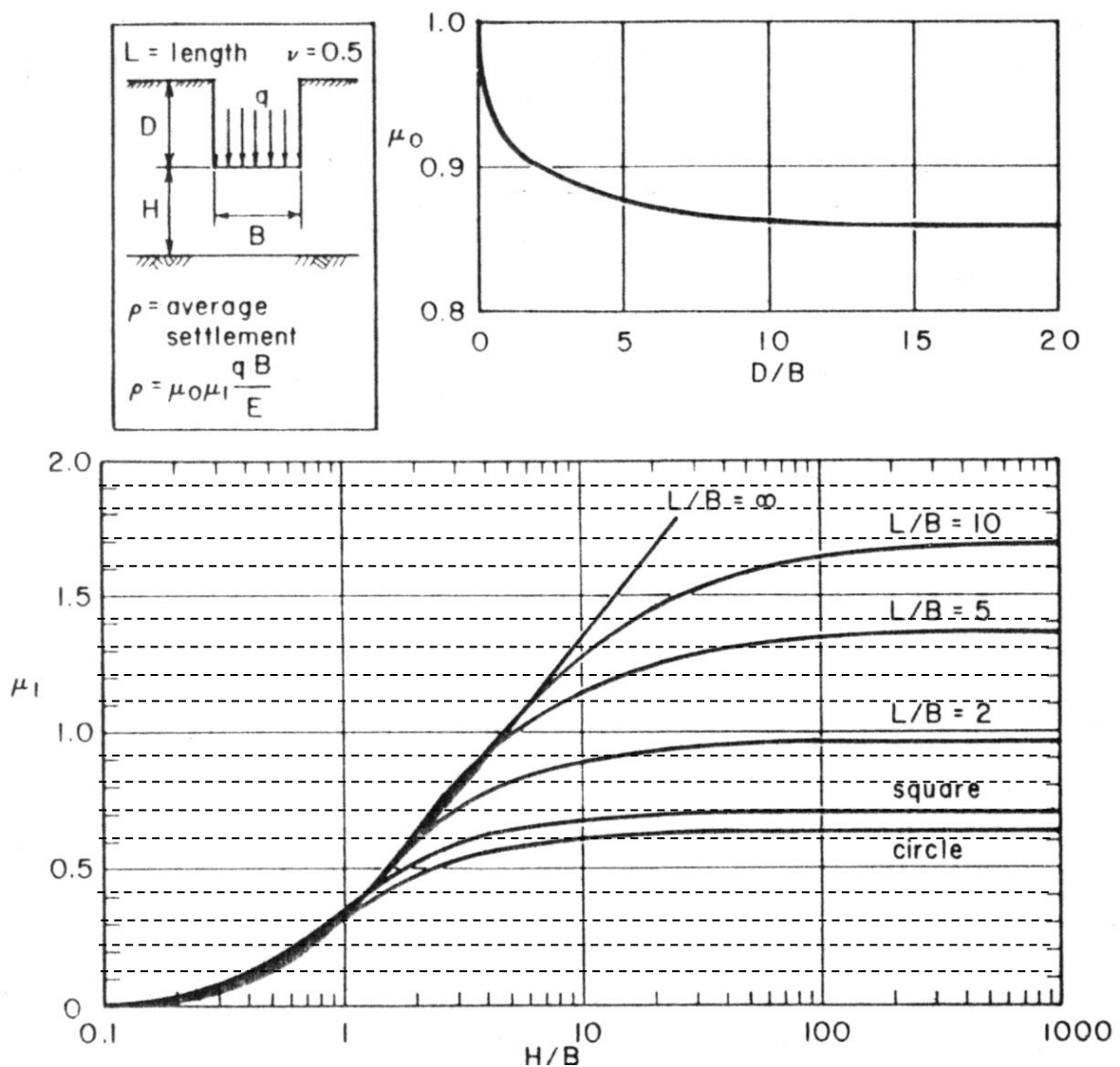
5.3.2 Isotropic, homogeneous, elastic finite space

Elastic layer of finite thickness

The mean settlement of a uniformly loaded foundation embedded in an elastic layer of finite thickness can be found using the charts below, for $\nu \sim 0.5$.

$$W_{avg} = \mu_0 \mu_1 \frac{qB}{E} \quad E = 2G(1 + \nu)$$

The influence factor μ_1 accounts for the finite layer thickness. The influence factor μ_0 accounts for the embedded depth.



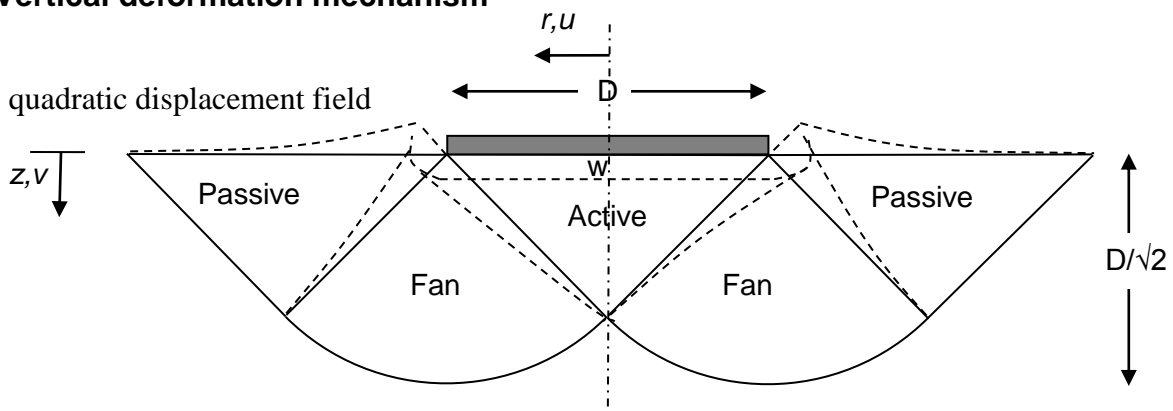
Average immediate settlement of a uniformly loaded finite thickness layer

Christian & Carrier (1978) Canadian Geotechnical Journal (15) 123-128
Janbu, Bjerrum and Kjaernsli's chart reinterpreted

5.4 Mobilizable Strength Design (MSD) solutions

Rigid circular foundation on incompressible half-space: Osman & Bolton (2005)

Vertical deformation mechanism



Average shear strain within deformation mechanism: $\gamma_{\text{mob}} = M_c w/D \approx 1.3 w/D$

Average shear stress mobilized within mechanism: $\tau_{\text{mob}} = q / N_c \approx q / 6$

Representative depth for stress-strain behaviour: $z_{\text{rep}} = 0.3D$

V or H or M loading: Osman et al. (2007) Geotechnique 57 (9) 729-737

Vertical loading $V = 1.5 \pi D^2 \tau_{\text{mob}}$ corresponding to $\gamma_{\text{mob}} = 1.3 w/D$

Horizontal loading $H = 0.25 \pi D^2 \tau_{\text{mob}}$ corresponding to $\gamma_{\text{mob}} = 8.5 u/D$

Moment loading $M = 0.17 \pi D^3 \tau_{\text{mob}}$ corresponding to $\gamma_{\text{mob}} = 2\theta$

5.5 Atkinson's Equivalent Stiffness G^* : Osman, White, Britto & Bolton (2007)

Rigid smooth circular foundation on a deep homogeneous bed

Vertical loading V for linear soil: settlement $w = \frac{\pi(1-\nu)}{4G} q_{\text{avg}} a = \frac{(1-\nu)}{2} \frac{V}{GD}$

for power-law soil: use G^* determined at $\gamma = \frac{w}{2D}$

Moment loading M for linear soil: rotation $\theta = \frac{3(1-\nu)M}{GD^3}$

for power-law soil: use G^* determined at $\gamma = 0.68 \theta$

Rigid rough circular foundation on a deep homogeneous bed

Shear loading H for linear soil: displacement $u = \frac{16(1-\nu)}{(7-8\nu)} \frac{H}{GD}$

for power-law soil: use G^* determined at $\gamma = \frac{1.15u}{D}$

Rectangular foundations: Use $D = \sqrt{BL}$

Section 6: Bearing capacity of deep foundations

6.1 Axial capacity: API (2000) design method for driven piles

6.1.1 Sand

Unit shaft resistance: $\tau_{sf} = \sigma'_{hf} \tan \delta = K \sigma'_{v0} \tan \delta \leq \tau_{s,lim}$

Closed-ended piles: $K = 1$

Open-ended piles: $K = 0.8$

Unit base resistance: $q_b = N_q \sigma'_{v0} < q_{b,limit}$

Soil category	Soil density	Soil type	Soil-pile friction angle, δ ($^\circ$)	Limiting value $\tau_{s,lim}$ (kPa)	Bearing capacity factor, N_q	Limiting value, $q_{b,lim}$ (MPa)
1	Very loose Loose Medium	Sand Sand-silt Silt	15	50	8	1.9
2	Loose Medium Dense	Sand Sand-silt Silt	20	75	12	2.9
3	Medium Dense	Sand Sand-silt	25	85	20	4.8
4	Dense Very dense	Sand Sand-silt	30	100	40	9.6
5	Dense Very dense	Gravel Sand	35	115	50	12

API (2000) recommendations for driven pile capacity in sand

6.1.2 Clay

American Petroleum Institute (API) (2000) guidelines for driven piles in clay.

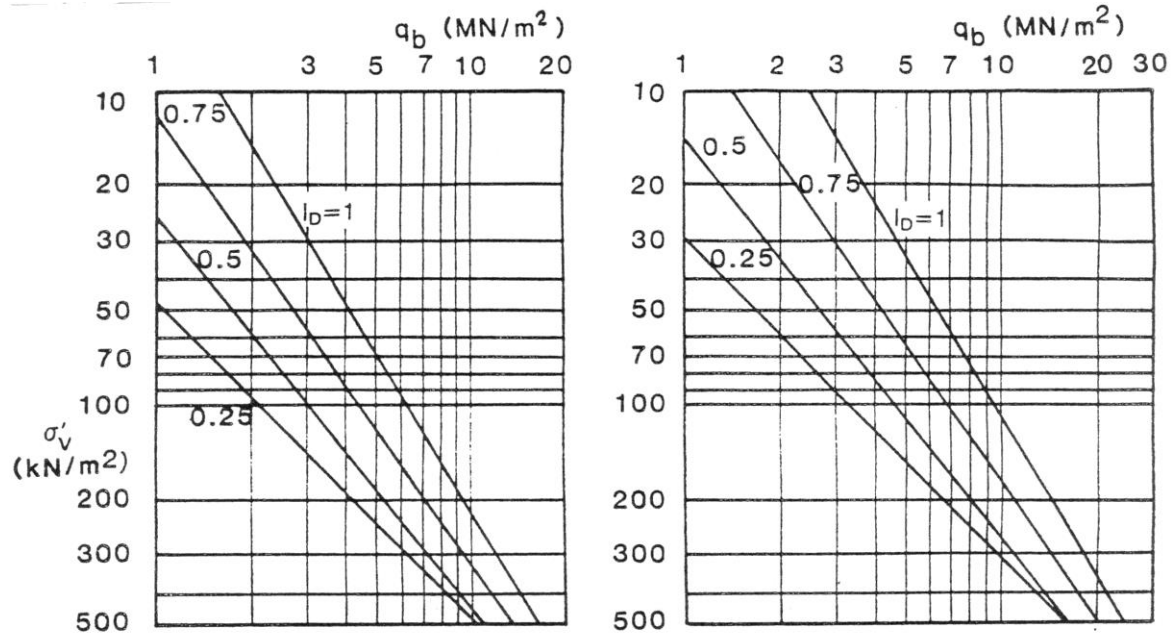
Unit shaft resistance: $\alpha = \frac{\tau_s}{s_u} = 0.5 \cdot \text{Max} \left[\left(\frac{\sigma'_{v0}}{s_u} \right)^{0.5}, \left(\frac{\sigma'_{v0}}{s_u} \right)^{0.25} \right]$

It is assumed that equal shaft resistance acts inside and outside open-ended piles.

Unit base resistance: $q_b = N_c s_u$ $N_c = 9.$

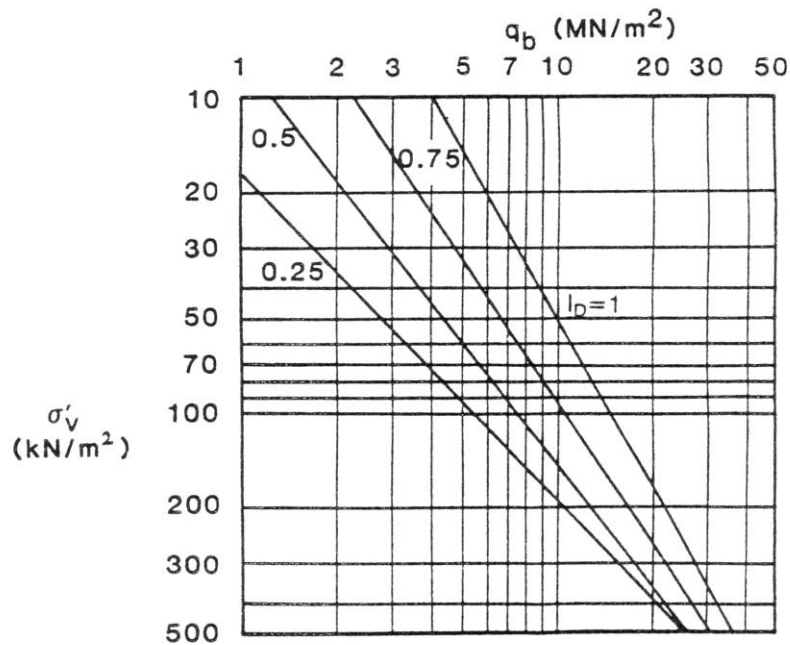
6.2 Axial capacity: base resistance in sand using Bolton's stress dilatancy

Unit base resistance, q_b , is expressed as a function of relative density, I_D , constant volume (critical state) friction angle, ϕ_{cv} , and in situ vertical effective stress, σ'_v .



(a) $\phi_{cv} = 27^\circ$

(b) $\phi_{cv} = 30^\circ$



(c) $\phi_{cv} = 33^\circ$

Design charts for base resistance in sand
(Randolph 1985, Fleming et al 1992)

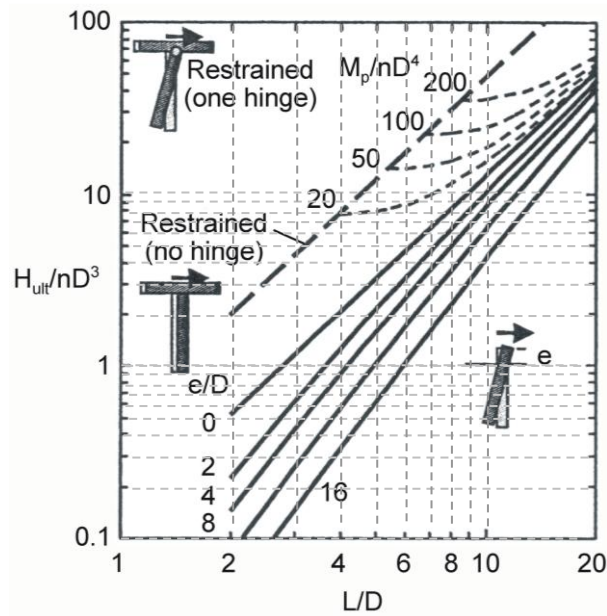
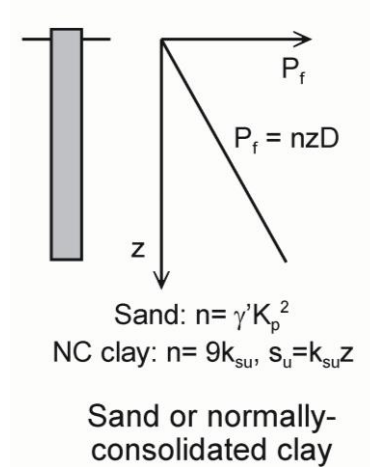
6.3 Lateral capacity: linearly increasing lateral resistance with depth

Lateral soil resistance (force per unit length), $P_u = nzD$

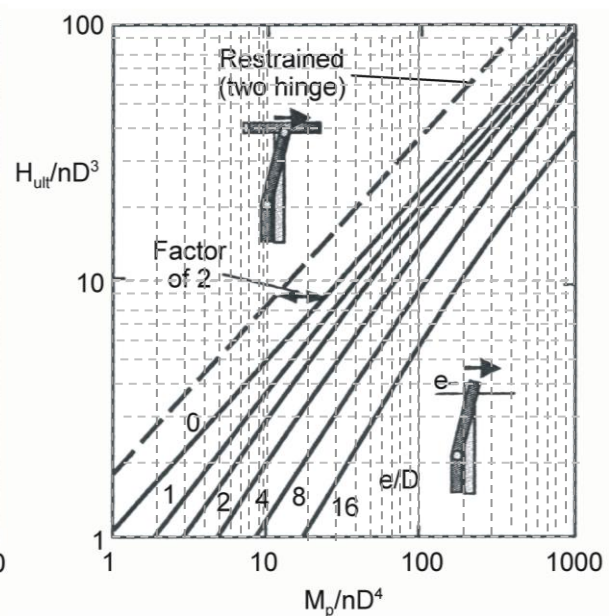
In sand, $n = \gamma'K_p^2$

In normally consolidated clay with strength gradient k ; $s_u = kz$; $n=9k$

- H_{ult} ultimate horizontal load on pile
- M_p plastic moment capacity of pile
- D pile diameter
- L pile length
- e load level above pile head
(=M/H for H-M pile head loading)
- γ' effective unit weight
- K_p passive earth pressure coefficient,
 $K_p = (1 + \sin \phi) / (1 - \sin \phi)$



Short pile failure mechanism



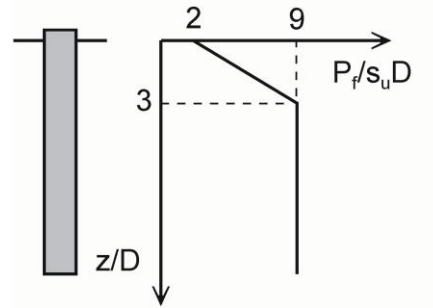
Long pile failure mechanism

Lateral pile capacity
(linearly increasing lateral resistance with depth)

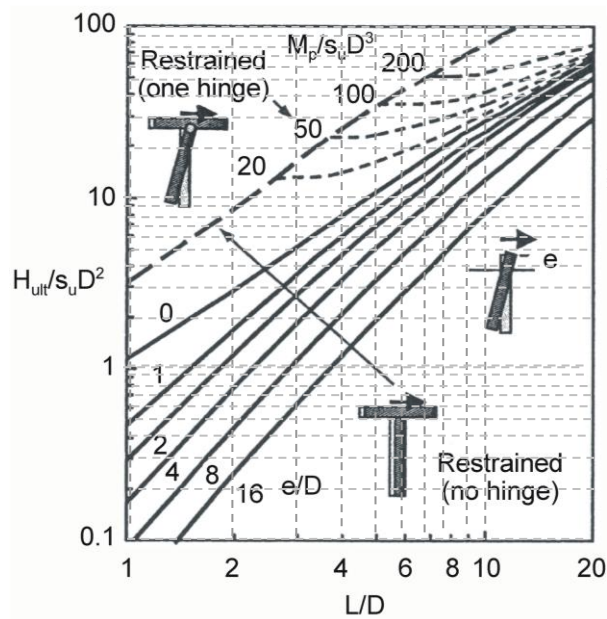
6.4 Lateral capacity: uniform clay

Lateral soil resistance (force per unit length), P_u , increases from $2s_uD$ at surface to $9s_uD$ at $3D$ depth then remains constant.

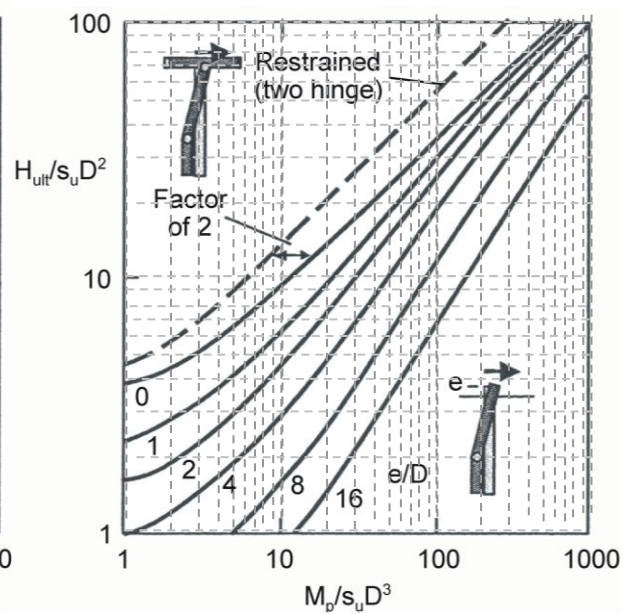
H_{ult} ultimate horizontal load on pile
 M_p plastic moment capacity of pile
 D pile diameter
 L pile length
 e load level above pile head
 (= M/H for H-M pile head loading)
 s_u undrained shear strength



Uniform clay



Short pile failure mechanism



Long pile failure mechanism

Lateral pile capacity
 (uniform clay lateral resistance profile)

Section 7: Settlement of deep foundations

7.1 Settlement of a rigid pile

Shaft response:

Equilibrium:

$$\tau = \tau_s \frac{R}{r}$$

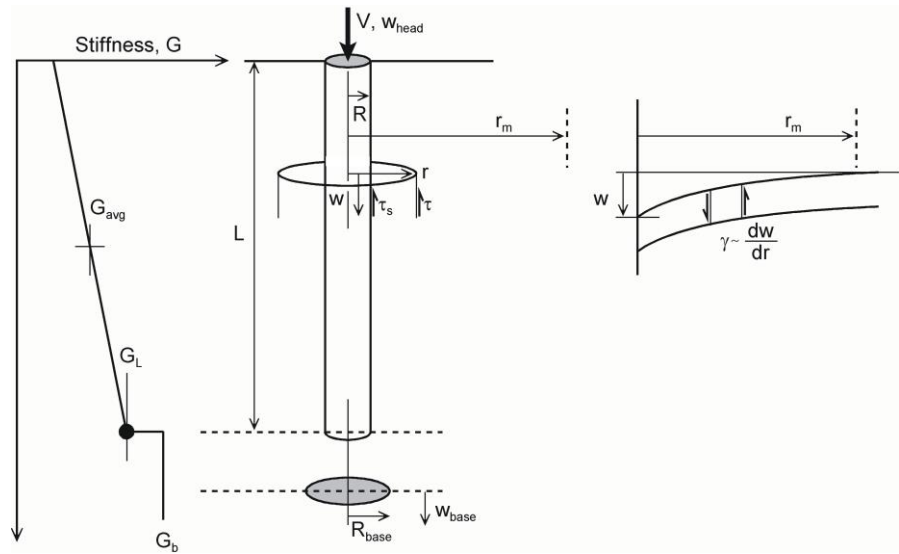
Compatibility:

$$\gamma \approx \frac{dw}{dr}$$

Elasticity:

$$\frac{\tau}{\gamma} = G$$

Integrate to magical radius, r_m , for shaft stiffness, τ_s/w .



Nomenclature for settlement analysis of single piles

Combined response of base (rigid punch) and shaft:

$$\frac{V}{w_{head}} = \frac{Q_b}{w_{base}} + \frac{Q_s}{w}$$

$$\frac{V}{w_{head}} = \frac{4R_{base}G_{base}}{1-\nu} + \frac{2\pi LG_{avg}}{\zeta}$$

$$\frac{V}{w_{head}DG_L} = \frac{2}{1-\nu} \frac{G_{base}}{G_L} \frac{D_{base}}{D} + \frac{2\pi}{\zeta} \frac{G_{avg}}{G_L} \frac{L}{D}$$

$$\frac{V}{w_{head}DG_L} = \frac{2}{1-\nu} \frac{\eta}{\xi} + \frac{2\pi}{\zeta} \rho \frac{L}{D}$$

These expressions are simplified using dimensionless variables:

Base enlargement ratio, eta $\eta = R_{base}/R = D_{base}/D$ Slenderness ratio L/D

Stiffness gradient ratio, rho $\rho = G_{avg}/G_L$ Base stiffness ratio, xi $\xi = G_L/G_{base}$

It is often assumed that the dimensionless zone of influence, $\zeta = \ln(r_m/R) = 4$.

More precise relationships, checked against numerical analysis are:

$$\zeta = \ln \left\{ \left[0.5 + (5\rho(1-\nu) - 0.5)\xi \right] \frac{L}{D} \right\} \quad \text{for } \xi=1: \quad \zeta = \ln \left\{ 5\rho(1-\nu) \frac{L}{D} \right\}$$

7.2 Settlement of a compressible pile

$$\frac{V}{w_{head}DG_L} = \frac{\frac{2\eta}{(1-\nu)\xi} + \rho \frac{2\pi \tanh \mu L}{\zeta} \frac{L}{\mu L} \frac{L}{D}}{1 + \frac{1}{\pi\lambda} \frac{8\eta}{(1-\nu)\xi} \frac{\tanh \mu L}{\mu L} \frac{L}{D}}$$

$$\text{where } \mu = \frac{\sqrt{8/\zeta\lambda}}{D}$$

Pile compressibility

$$\lambda = E_p/G_L$$

Pile-soil stiffness ratio

Pile head stiffness, $\frac{V}{w_{head}}$, is maximum when $L \geq 1.5D\sqrt{\lambda}$