Q 2) 1) Ground floor

$$k_1 = 12\,EI\left[\frac{1}{h_1^3}+\frac{1}{h_2^3}\right]$$

$m_1 = 20000$ kg

$$= 12\times10^6 \times 6 \times \left[\frac{1}{6^3}+\frac{1}{3^3}\right] = 3\times10^6 \text{ N/m}$$

First floor

$$k_2 = 12\,EI \times \frac{2}{h_3^3} = 12\times60\times10^6\times\frac{2}{5^3} = 1.15\times10^6 \text{ N/m}$$ $m_2 = 20000$ kg

$\therefore$ $KE = \frac{1}{2} m \sum \dot{u}^2$ Mode shape given $[0.34 \quad 1]$.

$$\therefore \tfrac{1}{2}M_{eq} = 20000\times(1^2+0.34^2) = 22312 \text{ kg.}$$

$$PE = \frac{1}{2}k_1 u_1^2 + \frac{1}{2}k_2(u_2-u_1)^2$$

$$\therefore \frac{1}{2}K_{eq} = 3\times10^6\times0.34^2 + 1.15\times10^6(1-0.34)^2$$

$$= 847740 \text{ N/m}$$

$\therefore$ Natural frequency $\omega_n = \sqrt{\frac{K_{eq}}{M_{eq}}}$

$$\omega_n = \sqrt{\frac{847740}{22312}} = 6.16 \text{ rad/s}$$

Natural frequency $f_n = \frac{\omega_n}{2\pi} = \underline{\underline{0.98 \text{ Hz}}}$

$$T_n = \frac{1}{0.98} \simeq 1.02 \text{ sec}$$

Use the tripartite graph with the above natural period $\simeq 1$ sec

This natural frequency will be either equal to or greater than the true natural frequency depending on the accuracy of the assumed mode shape

[20%]

b) Modal participation factor $\Gamma = \frac{\Sigma m \bar{u}}{\Sigma m \bar{u}^2} = \frac{0.34+1}{0.34^2+1^2} = 1.2012$.

Fraction of critical damping, ξ = 0, 2, 5, 10, 20%

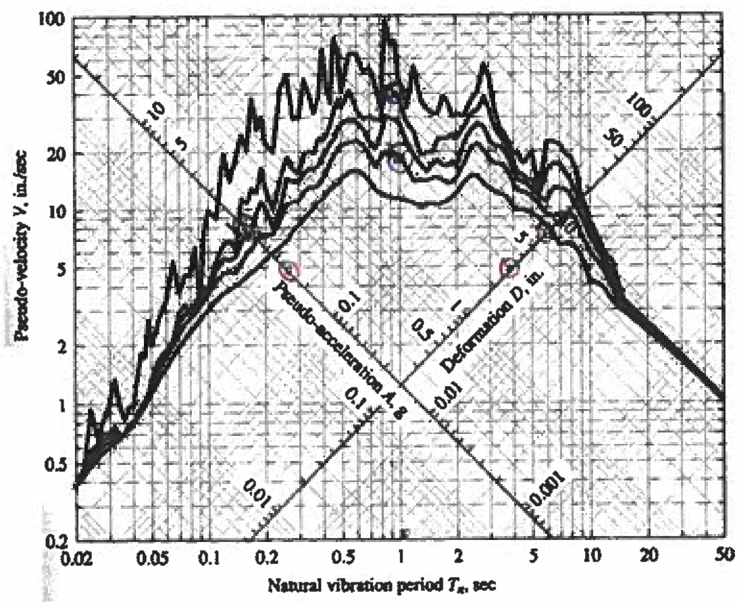


Fig. 3

$\xi = 2\%$ Damping

$S_a = 0.8g$

$S_d = 7.5'' = 7.5 \times 25.4$ mm

$U_{max} = \Gamma \, S_d$

$= 1.2012 \times$

$= 228.82$ mm.

$U_{1max} = 0.34 \times 228.82$

$= 77.80$ mm

$U_{2max} = 1 \times 228.82$ mm

$\ddot{y}_{max} = \Gamma \, S_a$

$= 1.2012 \times 0.8g$

$= 9.427 \, m/s^2$

$\ddot{y}_{1max} = 0.34 \times 9.427$

$= 3.2051 \, m/s^2$

$\ddot{y}_{2max} = 1 \times \ddot{y}_{max} = 9.427 \, m/s^2$

$\xi = 10\%$ Damping

$S_a = 0.3g$

$S_d = 3'' = 3 \times 25.4$ mm

$U_{max} = \Gamma \, S_d$

$= 1.2012 \times 76.2$

$= 91.53144$

$U_{1max} = 0.34 \, U_{max}$

$= 31.12$ mm.

$U_{2max} = 91.53144$ mm

$\ddot{y}_{max} = \Gamma S_a = 1.2012 \times 0.3g$

$= 3.5351 \, m/s^2$

$\ddot{y}_{1max} = 0.34 \, \ddot{y}_{max}$

$= 1.2019 \, m/s^2$

$\ddot{y}_{2max} = 3.5351 \, m/s^2$

c)

For moment equilibrium, the shear force

$\xi = 2\%$

$$SF = 12EI\delta\left[\frac{1}{h_1^3} + \frac{1}{h_2^3}\right]$$

$$= 12 \times 6\times10^6 \times 77.80\times10^{-3}\left[\frac{1}{6^3} + \frac{1}{3^3}\right]$$

$$= 233400\ N$$

or $SF = \Sigma Ma = 20000[3.2051 + 9.427]$

$\simeq 252642\ N$.

$\xi = 10\%$

$$SF = 12 \times 6\times10^6 \times \frac{31.12}{1000}\left[\frac{1}{6^3} + \frac{1}{3^3}\right]$$

$$= 93360\ N$$

or $SF = \Sigma Ma = 20000[1.2019 + 3.1351]$

$= 94740\ N$.

As both methods are giving quite close answers, the assumed mode shape is good.

Increasing the damping in the structure from 2% to 10%, the shear force at the base reduces by a factor of 2.5!

It is therefore important to estimate the damping in the structure carefully.

[30%]

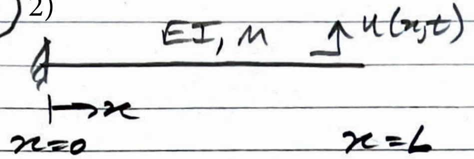
d) In an actual earthquake the response spectra may be different. This is why codes of practice suggest uses of 'averaged' design spectra rather than one earthquake spectrum.

Also the fixity of ~~cto~~ columns to foundations may not be truly 'fixed'. This is why soil-structure interaction is important to consider.

[10%]

Part IIB 4D6: Dynamics in Civil Engineering

1 2)



Assume $\bar{u}(x) = a\left(\frac{x}{L}\right)^3 + b\left(\frac{x}{L}\right)^2 + c\left(\frac{x}{L}\right) + d$

BCs: $\bar{u}(0) = 0$ ① $\bar{u}'(0) = 0$ (no slope) ②

a). $\bar{u}''(L) = 0$ (no tip moment) ③ [2]

① $\Rightarrow d = 0$

$\bar{u}' = \frac{3ax^2}{L^3} + \frac{2bx}{L^2} + c$ $\therefore$ ② $\Rightarrow c = 0$

$\bar{u}'' = \frac{6ax}{L^3} + \frac{2b}{L^2}$ $\therefore$ ③ $\Rightarrow \frac{6a}{L^2} + \frac{2b}{L^2} = 0,\ b = -3a$

$\therefore\ \bar{u}'' = \frac{6ax}{L^3} - \frac{6a}{L^2}$ Setting (arbitrarily) $a = 1$ gives:

$\bar{u}(x) = \left(\frac{x}{L}\right)^3 - 3\left(\frac{x}{L}\right)^2$, $\bar{u}'(x) = \frac{3x^2}{L^3} - 6\frac{x}{L^2}$, $\bar{u}''(x) = \frac{6x}{L^3} - \frac{6}{L^2}$ [2]

$$M_{eq} = \int_0^L m\bar{u}^2\,dx = m\int_0^L \left(\frac{x}{L}\right)^6 - 6\left(\frac{x}{L}\right)^5 + 9\left(\frac{x}{L}\right)^4 dx = m\left[\frac{x^7}{7L^6} - \frac{6x^6}{6L^5} + \frac{9x^5}{5L^4}\right]_0^L$$

$$= \frac{33mL}{35} = \frac{33}{35} \times 500 \times 20 = 9{,}429\ \text{kg}$$ [2]

$$K_{eq} = \int_0^L EI(\bar{u}'')^2\,dx = EI\int_0^L \frac{36x^2}{L^6} - \frac{72x}{L^5} + \frac{36}{L^4}\,dx = EI\left[\frac{36x^3}{3L^6} - \frac{72x^2}{2L^5} + \frac{36x}{L^4}\right]_0^L$$

$$= \frac{12EI}{L^3} = \frac{12 \times 1 \times 10^{10}}{20^3} = 15 \times 10^6\ \text{N/m}$$ [2]

$$\therefore\ \omega_1 = \sqrt{\frac{K_{eq}}{M_{eq}}} = \sqrt{\frac{15 \times 10^6}{9429}} = 40\ \text{rad/s} \equiv 6.4\ \text{Hz}$$ [1]

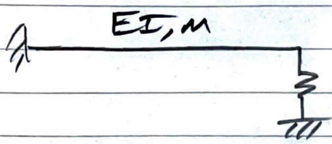
b). Now, $M_{eq} = \frac{33mL}{35} + M\left(\bar{u}(x=L)\right)^2 = \frac{33mL}{35} + M(1-3)^2 = \frac{33mL}{35} + 4M$

$= 9429 + 4000 = 13{,}429$ kg. [2]

$\Rightarrow M_{eq} \times \frac{13429}{9429} \Rightarrow \omega_1 \times \sqrt{\frac{9429}{13429}}$ ie. $\omega_1 = 34$ rad/s $\equiv 5.3$ Hz. [1]

c)

The cable introduces a spring at the tip of the cantilever:



Now, $K_{eq} = \frac{12EI}{L^3} + k(\bar{u}(x=L))^2 = \frac{12EI}{L^3} + \frac{4AE}{l}$.

$= 15\times10^6 + \frac{4\times10^{-4}\times210\times10^9}{5} = 31.8\times10^6$ N/m [3]

$\Rightarrow K_{eq} \times \frac{31.8}{15} \Rightarrow \omega_1 \times \sqrt{\frac{31.8}{15}}$ i.e. $\omega_1 = 58$ rad/s $= 9.3$ Hz. [1]

d). The addition of mass may be straightforward (eg. filling a water tank) but may introduce unacceptable stresses

A tensioned cable may be difficult to anchor, depending on what is beneath the bridge. Determining the tension may be difficult in advance.
A much better solution would be to add a tuned-mass damper, tuned to the problem resonance frequency. [4]

4D6

Q3 a) Loose sands contract when subjected to shearing from earthquake loading, while dense sands dilate. In case of saturated soil, the contractile tendency of the loose sand is manifested as an increase in ~~exc~~ pore water pressure. As the pore water pressure increases the effective stress in the soil decreases following the Terazaghi's effective stress principle. $\downarrow \sigma' = \sigma - u \uparrow$

This can lead to full liquefaction of the soil when u matches σ and $\sigma' \simeq 0$.

In dense sand, the dilative tendency causes a lowering of pore pressure and a temporary increase in effective stress, thereby reducing any risk of liquefaction.

[10 %]

b) In order to carry out FE analyses of two phase materials like saturated soil, the solid phase and the fluid phase are discretised separately. The solid mesh is then superposed over fluid mesh in space. This is called u - p formulation (u - solid node displacement, p - fluid node pressure).

The equation of motion for solid phase has a cross coupling term Q;

$$[M]\ddot{u} + [C]\dot{u} + [K]u + [Q]p = [f_s]$$

Similarly for fluid phase, we can have

$$[T]\ddot{p} + (R)\dot{p} + [B]p + [Q]u = [f_f].$$

These two equations can be solved iteratively at every time step until consistent pore pressure and solid node displacements are obtained according to the constitutive model.

Usually the fluid acceleration $\ddot{p}$ is ignored in the u-p formulations

[15%]

c) Soil layers extend to large distances laterally. This is normally called the semi-infinite extent of a soil layer or soil half space. We consider soil layers go from $-\infty$ to $+\infty$ relative to the size of the structure.

However, this causes a difficulty in FE analyses as we cannot discretise the soil domain from $-\infty$ to $+\infty$. FE mesh must have a reasonable size to keep the computational effort in check. So the semi-infinite extent is modelled in FE analyses using special mathematical formulations like

a) Smith - Cundall boundary
b) Lysmer - Kuhlemeyer Boundary
c) Compound parabolic collector - CPC boundary.

CPC boundary is particularly simple as we shape the boundary region into compound parabolic collector shape into which the seismic wave can enter and get multiply reflected. The analyses time is organised such that it finishes before any reflections can reenter the soil domain. This works well for short duration such as earthquake loading. [15%]

d) $\gamma_{sat} = 19.6\ kN/m^3$, $G_s = 2.65$ $\gamma_w = 10\ kN/m^3$.

$$\frac{(G_s + e S_r)\gamma_w}{1+e} = \gamma_{sat}$$

$$\therefore \underset{(S_r=1)}{\gamma_{sat}} = \frac{(G_s + e)}{1+e}\gamma_w = 19.6.$$

$$\Rightarrow e = 0.72.$$

Using Hardin & Drnevich (1972) equation from Data book.

$$G_{max} = 100\ \frac{[3-e]^2}{1+e}(p')^{0.5}$$

$$p' = \left(\frac{1+2K_0}{3}\right)\sigma'_v \qquad K_0 = \frac{\nu}{1-\nu} = \frac{0.3}{0.7} = 0.428$$

$$= \left(\frac{1+2\times 0.428}{3}\right)\sigma'_v = 0.619\ \sigma'_v.$$

$$\sigma_v = \gamma_{sat}\, z = 19.6 \times 3 = 58.8\ kPa.$$

$$u_{hyd} = \gamma_w\, z = 10\times 3 = 30\ kPa$$

$$\therefore \sigma'_v = \sigma_v - u = 58.8 - 30 = 28.8\ kPa, \quad p' = 0.619\ \sigma'_v = 17.82\ kPa$$

$$\therefore G_{max} = 100\ \frac{[3-0.72]^2}{1.72}\times\left[\frac{17.82}{1000}\right]^{1/2} = \underline{\underline{40.35}}\ MPa$$

$$v_s = \sqrt{\frac{G_{max}}{\rho}} \qquad \gamma_{dry} = \frac{G_s\gamma_w}{1+e} = \frac{2.65\times 10}{1.72} = 15.4059\ kN/m^3 \simeq 1540.59\ kg/m^3$$

$$\therefore v_s = \sqrt{\frac{40.35\times 10^6}{1540.59}} = \underline{\underline{161.84}}\ m/s$$

[20%]

3 e) $\sigma_v = 58.8$ kPa

$u_{hy} = 30$ kPa $\quad u_{ex} = 20$ kPa

$\therefore u_{tot} = 50$ kPa.

$\therefore \sigma'_v = 58.8 - 50 = 8.8$ kPa.

$p' = 0.619 \times \sigma'_v = 5.4472$ kPa

$$G_{max} = 100 \frac{[3-e]^2}{1+e} \sqrt{p'} = 100 \frac{[3-0.72]^2}{1.72} \times \left[\frac{5.4472}{1000}\right]^{1/2}$$

$$= 22.305 \text{ MPa}.$$

Due to cyclic shear strains $G = \frac{G_{max}}{10}$

$\therefore G = 2.2305$ MPa.

Using Wolf's formulae from Data book.

$$K_z = \frac{Gb}{2-\nu}\left[3.1\left(\ell/b\right)^{0.75} + 1.6\right]\left[1 + \left\{0.25 + 0.25\frac{b}{\ell}\right\}\left[e/b\right]^{0.8}\right]$$

$2\ell = 2$ m $\quad \ell/b = 2$

$2b = 1$ m

$e = 0.5$ m $\quad e/b = 1.$

$$\therefore K_z = G\frac{0.5}{2-0.3}\left\{3.1 \times 2^{0.75} + 1.6\right]\left[1 + (0.25 + 0.25 \times 0.5) \times 1^{0.8}\right]$$

$$K_z = 2.755\,G.$$

$$K_{ry} = \frac{G\,b^3}{1-\nu}\left[3.73\left(\ell/b\right)^{2.4} + 0.27\right]\left[1 + \cancel{0.25} + e/b + \frac{1.6}{0.35 + (\ell/b)^4}\left(e/b\right)^2\right]$$

$$= \frac{G \times 0.5^3}{0.7}\left\{3.73 \times 2^{2.4} + 0.27\right]\left\{1 + 1 + \frac{1.6}{0.35 + 2^4} \times 1^2\right]$$

$$K_{ry} = 7.47626\,G$$

$\therefore$ Max vertical settlement $= \frac{1500 \times 10^3}{2.755 \times 2.2305 \times 10^6} = 0.244$ m or $\underline{244 \text{ mm}}$!

Max rotation $= \frac{300 \times 10^3}{7.47626 \times 2.2305 \times 10^6} = 0.0173$ rad

or $\approx \underline{1.03^\circ}$

Smaller rotation compared to settlement. Hardin & Drnevich (1972) relation over estimates shear modulus at small effective stresses. Therefore in reality the settlement & rotation may be larger during a real earthquake event. [40%]

a) $B = 31\,m$ $U = 30\,m/s$ $q_p = 3{,}5$

$\xi_s = 0.002$ $C_D = 0.4$ $I_u = 0.1$

$\omega_p = 0{,}2 \cdot 2\pi$ $\rho = 1{,}2\,kg/m^3$ $m = 22740\,kg/m$

$x_m = 0.185\,m$; $B^2 = 13{,}70$ $q_p = 3{,}5$

-> PEAK RESPONSE

$$x_p = x_m + q_p \cdot \sigma_x$$

$$\sigma_x^2 = 4 \cdot x_m^2 \cdot \frac{\sigma_u^2}{U^2}\left[B^2 + R^2\right]$$

$$I_u = \frac{\sigma_u}{U} = 0.1$$

$$R^2 = \chi(\omega_p) \cdot \frac{S_{uu}(\omega_p)}{\sigma_u^2} \cdot \frac{\omega_p}{8 \cdot \xi}$$ -> RESONANT FACTOR

$$\xi = \xi_s + \xi_a; \quad \xi_a = \frac{\rho \cdot U \cdot C_D \cdot B}{2 \cdot m \cdot \omega_p} = \frac{1{,}2 \cdot 30 \cdot 0{,}4 \cdot 31}{2 \cdot 22740 \cdot 0{,}2 \cdot 2\pi}$$

$$\xi_a = 0.0078$$ -> AERODYNAMIC DAMPING

$$\xi = 0.002 + 0.0078 = 0.0098$$ -> TOTAL DAMPING

$\omega_p = 0.2 \cdot 3{,}14 \cdot 2 = 1{,}26\ rad/s$ => $\chi(\omega_p) = 1$, $\frac{S_{uu}(\omega_p)}{\sigma_u^2} = 10$ } FROM FIGURES 2 and 3 ON EXAM SHEET

$$R^2 = 1 \cdot 10 \cdot \frac{0{,}2 \cdot 2 \cdot \pi}{8 \cdot 0.0098} = 160{,}20$$

$$\sigma_x^2 = 4 \cdot 0{,}185^2 \cdot 0{,}1^2 \cdot \left[13{,}70 + 160{,}20\right] = 0{,}238\,m^2$$

$\sigma_x = 0{,}488\,m$ => PEAK RESPONSE $x_p = 0.185 + 3{,}5 \cdot 0{,}49 = \boxed{1{,}9\,m}$

2

b) TYPES OF AEROELASTIC INSTABILITIES

→ GALLOPING $H_1^* > 0$

→ TORSIONAL FLUTTER $A_2^* > 0$ → $\boxed{\text{PRESENT CASE}}$

→ COUPLED FLUTTER

$\boxed{A_2^* = 0 \text{ at } V_{cr} = 6.1} \rightarrow V_r = \dfrac{U}{\frac{\omega_\alpha B}{2\pi}} \Rightarrow U_{cr} = V_{r,cr} \cdot B \cdot \dfrac{\omega_\alpha}{2\pi}$

↑ FROM Figure 4 IN EXAM

$$U_{cr} = 6{,}1 \cdot 31 \cdot 0{,}23 \cdot \frac{2\pi}{2\pi}$$

$$U_{cr} = 43{,}49 \text{ m/s}$$

---

c)

RANK	WIND SPEED [m/s]	P(u)	y
6	18.1	0.1935	-0,4961
30	28.6	0.9672	3,4176

$$P(u) = \frac{n}{N+1}$$

$$P(u) = 1 - \frac{1}{T}$$

- GUMBEL DISTRIBUTION $P(u) = \exp(-\exp(-y))$

reduced variate $y = a(m - u)$

$$U = m - \frac{1}{a} \cdot y$$

METHOD I (ANALYTICAL)

SLOPE $-\dfrac{1}{a} = \dfrac{28.6 - 18.1}{3{,}41 + 0{,}4961} = 2{,}68$

$m = U + \dfrac{1}{a} \cdot y$ → At $U = 28{,}6$ m/s AND $y = 3{,}42$

$m = 28.6 - 2{,}68 \cdot 3{,}41$

$$\boxed{m = 19{,}43}$$

$U_{cr} = 43,49$ m/s (Flutter velocity)

$$y = -\frac{1}{2,68}(19,43 - 43,49)$$

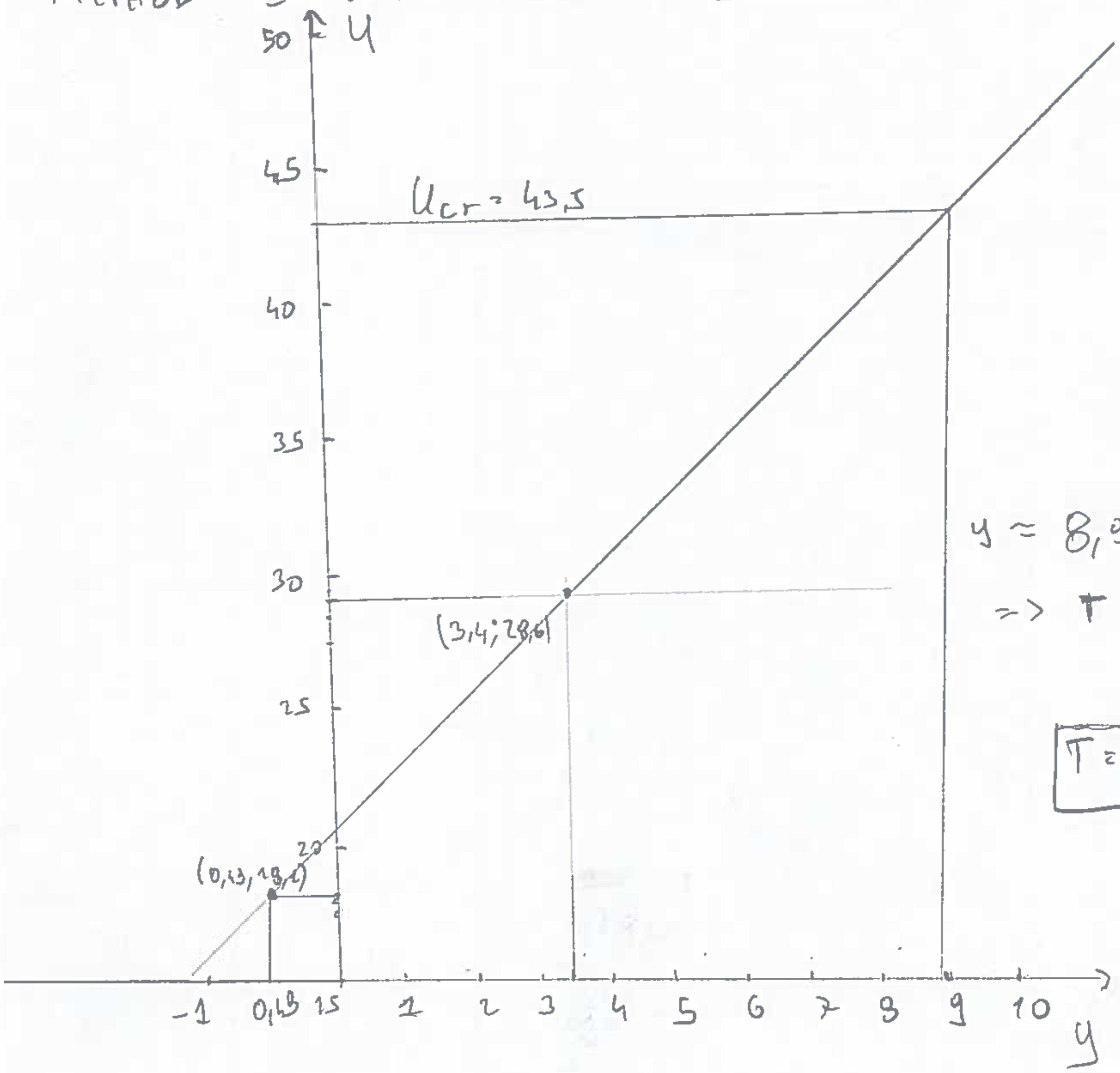
$$y = 8,97$$

$$1 - \frac{1}{T} = \exp(-\exp(-y))$$

$$\Rightarrow T = \frac{1}{1-\exp(-\exp(-y))} = \boxed{7854 \text{ years}}$$

$\boxed{\text{if } U_{cr} = 45 \text{ m/s} \rightarrow T = 13\,773 \text{ years}}$ → if U_{cr} is not determined from question b)

Method II (GRAPHICAL) → ALTERNATIVE



$y \simeq 8,9$

$$\Rightarrow T = \frac{1}{1-\exp(-\exp(-y))}$$

$\boxed{T = 7333 \text{ years}}$