Version FAM/2

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 3 May 2022 14.00 to 15.40

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachment: 4D6 Data Sheet (6 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 Figure 1(a) shows the dynamic model used to assess the blast response of a glazing panel for a new building. This comprises a uniform beam of total length $2L$, flexural rigidity EI and mass per unit length m, which is fully restrained at both ends. A glazing bar is modelled as a point mass $M = mL/7$ at the mid-span location.

(a) Using Rayleigh's Principle and assuming a fourth-order polynomial for the fundamental mode shape, estimate the fundamental period of the panel. Membrane action may be neglected. [40%]

(b) The blast load is equivalent to a uniformly distributed force per unit length, which varies in time according to the triangular force pulse shown in Fig. 1(b). Estimate the maximum deflection of the panel due to the blast load when $2L$ = 1.4 m, EI = 8 kN m^2 and m = 26 kg m^{-1}. [40%]

(c) In practice, a resilient seal is used between the glazing and its supporting frame. Explain how you think this would affect your estimate of the deflection. [20%]

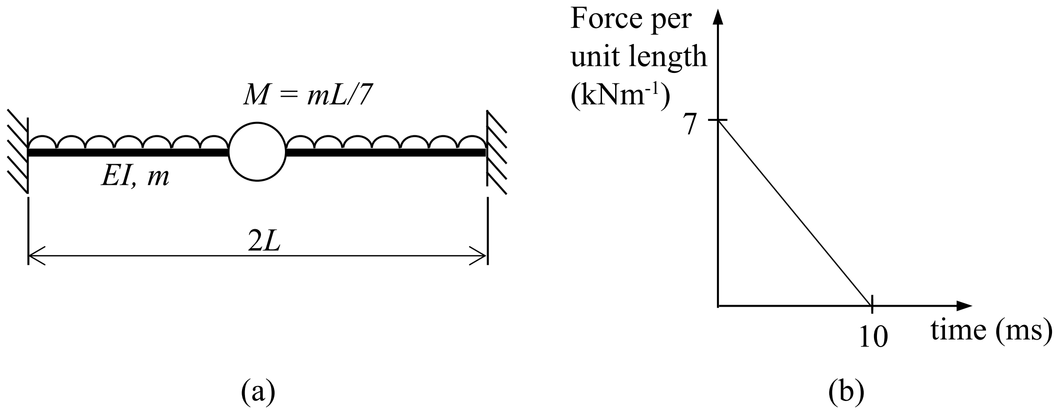


Fig. 1

2 (a) Explain the term 'dynamic soil-structure interaction' by considering a structure founded on a deep soil layer overlying bed rock. Consider vertically propagating horizontal shear S_h waves. [10%]

(b) Explain the terms *full liquefaction* and *partial liquefaction* and briefly state what dangers each of these can pose to structures. [10%]

(c) At a site in Dubai, a loose sand layer is present that extends from the surface to a depth of 12 m below which bedrock is present. The unit weight of the sand is 14.3 kN m^{-3}. The friction angle and the Poisson's ratio of the sand are 33° and 0.3 respectively. Determine the small-strain shear modulus of the sand at depths of 3 m, 6 m and 12 m. Sketch the variation of shear wave velocity with depth, i.e. from the surface to the bedrock, marking the values at 3 m, 6 m and 12 m. [30%]

(d) A water tank is supported on a concrete foundation with dimensions of 3 m × 3 m × 1 m on the sand in part (c). The foundation is embedded to a depth of 1 m below the surface. The tower and its contents may be treated as a cantilever carrying a point mass of 15,000 kg at a height 10 m above the sand surface. The tower was found to have a natural frequency of 2.3 Hz. Calculate the flexural stiffness of the tower. Give your answer in the units of MN m^2. State any assumptions you make. [20%]

(e) A strong earthquake was experienced at the site. Due to the large cyclic strains, the shear modulus of the sand has reduced to 50% of the values you calculated in part (c). The mass moment of inertia of the foundation and the soil participating in the rotational mode is estimated to be 5 times that of the water tank. Calculate the rocking frequency of the foundation by considering a simple discrete model with one degree of freedom. The reference plane for this structure can be taken at a depth of 3 m below the surface. Discuss whether you would expect significant damage to the water tank due to this earthquake. [30%]

3 (a) Figure 2 shows a three-storey sway frame. The masses of the floors are $[3M, 2M, M]$ as shown, where M = 11 tonnes. Each column is of height L = 3.1 m and has flexural rigidity EI = 9000 kN m^2. The column feet are pinned at ground level, and all other connections are fixed.

The structural damping is 5% of critical. The structure experiences an earthquake with the response spectrum shown in Fig. 3.

(i) Write down expressions for stiffness and mass matrices relevant for in-plane sway oscillations. [20%]

(ii) Show that a set of lateral floor deflections in the proportion $[0.848, 0.961, 1]$ is a vibration mode, and determine the corresponding vibration frequency. [20%]

(iii) Determine the maximum ground acceleration and displacement. [10%]

(iv) Considering only the mode given, determine the maximum column shear force. [20%]

(b) The structure described in part (a) is being designed using the inelastic design spectra in Fig. 4 for an earthquake with peak ground acceleration of 0.3g. Considering only the mode given above, determine the required ductility factor μ if each column has a shear capacity of 40 kN. [30%]

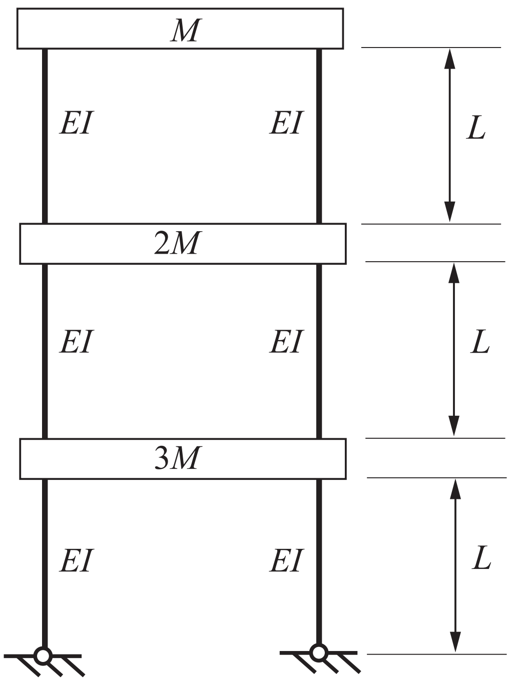


Fig. 2

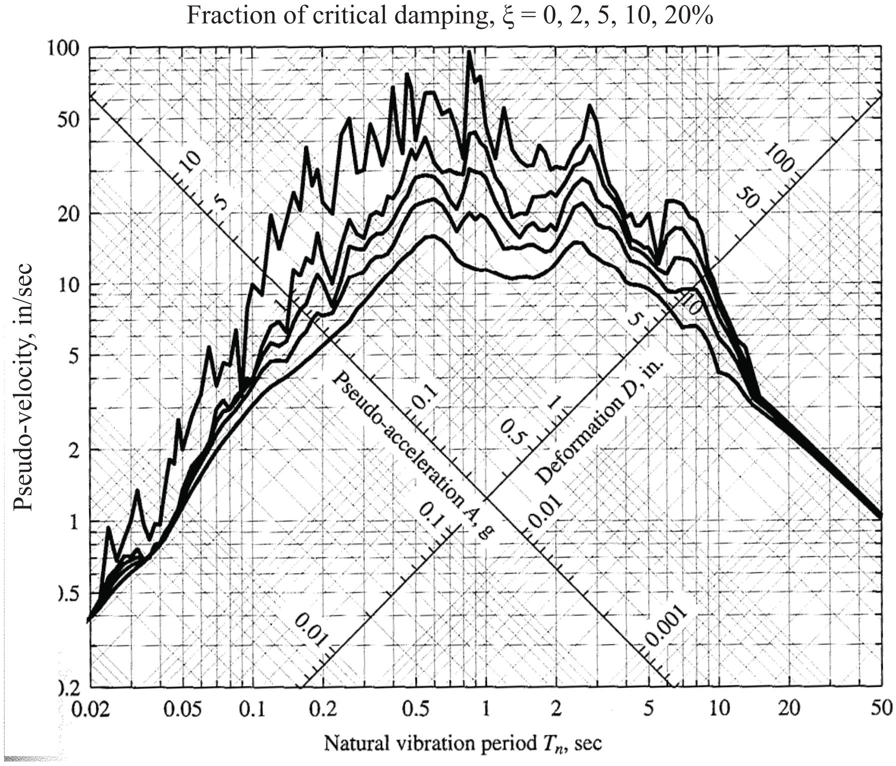


Fig. 3

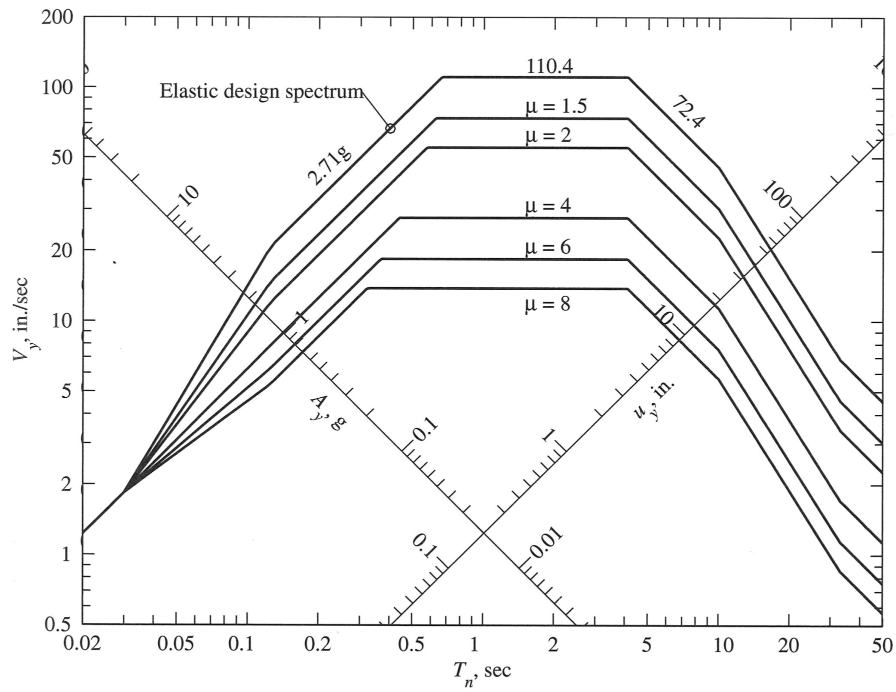


Fig. 4

 (TURN OVER

4 (a) The characteristics of the shock front produced by the free air blast of a high explosive charge are often described by a set of curves on a log-log plot. Two of these curves may be approximated by the following simple formulae:

$$\log_{10} P = -1.5 \log_{10} Z + 6.05$$
$$\log_{10} i^* = -1.45 \log_{10} Z + 3.65$$

Here, P is the peak of the positive pressure pulse above atmospheric (in Pa), i^* is the associated scaled impulse (in Pa.s.kg$^{-1/3}$) and $Z = R/W^{1/3}$ is the scaled distance (in m.kg$^{-1/3}$) where R is the standoff distance (in m) and W is the mass of the explosive (in kg of TNT equivalent).

Stating your assumptions:

(i) derive an approximate formula for the duration of the scaled positive pressure pulse T_s^* as a function of scaled distance Z ; [20%]

(ii) estimate the peak pressure, impulse and pulse duration at a distance of 100 m from a fertiliser truck bomb of 4 tonnes TNT equivalent; [20%]

(iii) briefly describe two effects which may make the peak pressure experienced by a structure significantly greater than that given by the formula above. [10%]

(b) Explain why a simple linear eigenvalue analysis of a small-displacement elastic finite element model of a suspension bridge may lead to a significant underestimate of the stiffness against horizontal vibrations. [10%]

(c) The deck of a long-span bridge has a closed box section which is 30 m wide and 4 m deep, with a mass of 24 tonnes per metre length, and the mass radius of gyration of any cross-section of the deck is 10 m. The first vertical bending and first torsional modes have natural periods of 11 s and 3 s respectively. For each mode, the damping is a fraction 0.5 percent of critical.

(i) Assuming a Strouhal number of 0.15 (based on deck depth), estimate the critical wind speed for vortex-induced vertical vibrations and the corresponding amplitude of vertical oscillations. [20%]

(ii) Estimate the critical wind speed for classical flutter. [20%]

END OF PAPER

Module 4D6: Dynamics in Civil Engineering

Data Sheets

Equivalent SDOF Systems

For an *n*-DOF system, with mass matrix $\underline{\underline{M}}$ and stiffness matrix $\underline{\underline{K}}$, responding in mode shape $\underline{\overline{u}}$ to an applied force $\underline{F}$, the parameters of the equivalent SDOF system are:

$$M_{eq} = \underline{\overline{u}}^T \underline{\underline{M}}\, \underline{\overline{u}} \qquad K_{eq} = \underline{\overline{u}}^T \underline{\underline{K}}\, \underline{\overline{u}} \qquad F_{eq} = \underline{F}^T \underline{\overline{u}}$$

For a continuous beam, of length *L*, mass per unit length *m* and bending stiffness *EI*, responding in mode shape $\overline{u}(x)$ to an applied force $f(x)$, the parameters of the equivalent SDOF system are:

$$M_{eq} = \int_0^L m\overline{u}^2 dx \qquad K_{eq} = \int_0^L EI \left(\frac{d^2\overline{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\overline{u}\, dx$$

The corresponding natural frequency is $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}}$

Modal Analysis of a Simply-Supported Beam

$$\overline{u}_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\,eq} = \frac{mL}{2} \qquad K_{i\,eq} = \frac{(i\pi)^4 EI}{2L^3}$$

Ground Motion Participation Factor

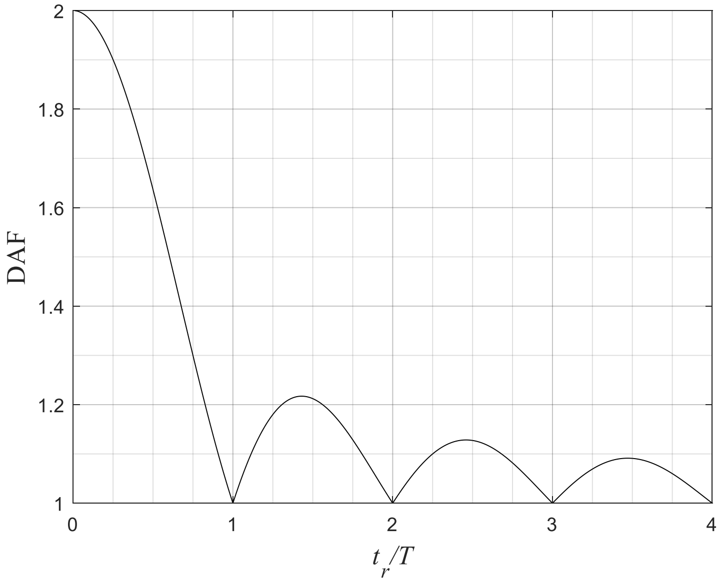
For an *n*-DOF system, with mass matrix $\underline{\underline{M}}$ and stiffness matrix $\underline{\underline{K}}$, responding to ground acceleration $\ddot{u}_g$, the parameters of the equivalent SDOF system are:

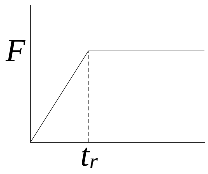
$$M_{eq} = \underline{\overline{u}}^T \underline{\underline{M}}\, \underline{\overline{u}} \qquad K_{eq} = \underline{\overline{u}}^T \underline{\underline{K}}\, \underline{\overline{u}} \qquad F_{eq} = -\Gamma M_{eq}\ddot{u}_g$$

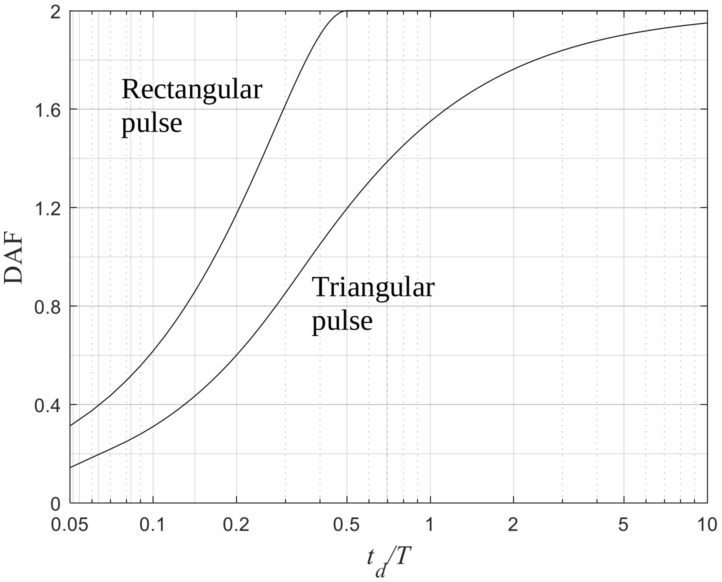
where $\underline{\overline{u}}$ is defined relative to ground and Γ is the modal participation factor

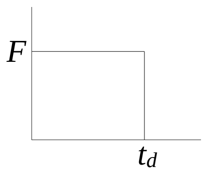
$$\Gamma = \frac{M_1\overline{u}_1 + M_2\overline{u}_2 + \cdots + M_n\overline{u}_n}{M_{eq}}$$

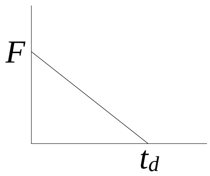
Dynamic Amplification Factors

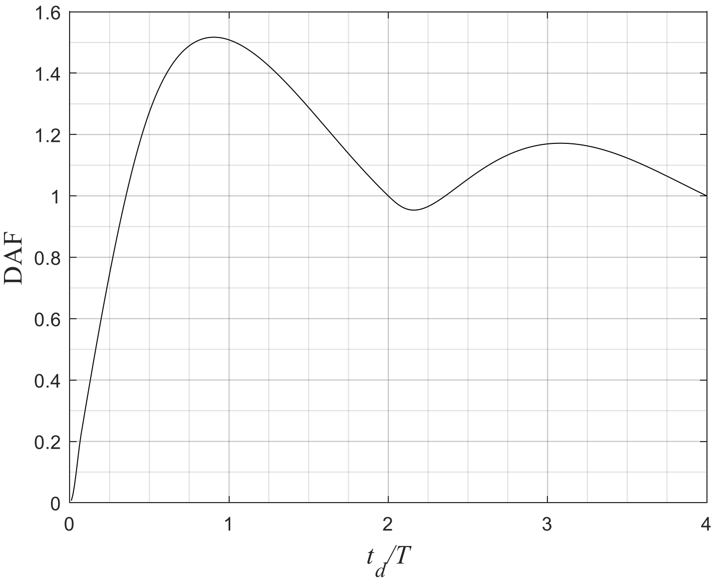


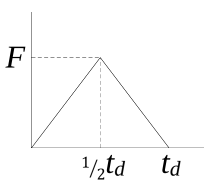






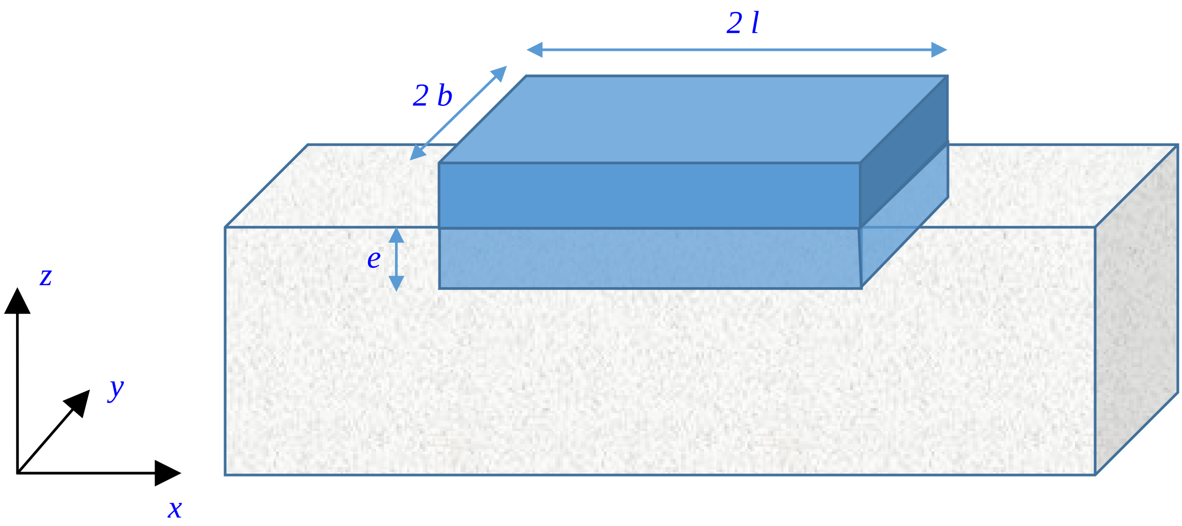






Soil Stiffness for Embedded Footings

Approximate relations for evaluating the soil stiffness for an embedded, prismatic footing of dimensions *2l* and *2b*, embedded to a depth *e*, assuming horizontal shaking in the direction parallel to the *x* axis (i.e. 2*l* of the prismatic structure) are:



$$K_{hx} = \frac{G\,b}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left\{0.33 + \frac{1.34}{1+\frac{l}{b}}\right\}\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{G\,b}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\,\frac{l}{b} + 1.6\right]\left[1 + \left\{0.33 + \frac{1.34}{1+\frac{l}{b}}\right\}\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{v} = \frac{G\,b}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left\{0.25 + \frac{0.25\,b}{l}\right\}\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{G\,b^3}{1-\nu}\left[3.2\,\frac{l}{b} + 0.8\right]\left[1 + \frac{e}{b} + \left(\frac{1.6}{0.35+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{2}\right]$$

$$K_{ry} = \frac{G\,b^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[1 + \frac{e}{b} + \left(\frac{1.6}{0.35+\left(\frac{l}{b}\right)^{4}}\right)\left(\frac{e}{b}\right)^{2}\right]$$

$$K_{tor} = G\,b^3\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[1 + \left(1.3 + 1.32\,\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right]$$

Properties of Soil

Unit weight of soil:

$$\gamma = \frac{(G_s + S_r e)}{1+e}\gamma_w$$

where e is the void ratio, S_r is the degree of saturation and G_s is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_d = \frac{G_s}{1+e}\gamma_w$$

Effective mean confining stress:

$$p' = \sigma_v' \frac{(1+2K_o)}{3}$$

where σ_v' is the effective vertical stress and K_o is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$K_o = \frac{\nu}{1-\nu}$$

Effective stress Principle:

$$p' = p - u$$

The shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{max} = 100\frac{(3-e)^2}{(1+e)}(p')^{0.5}$$

where p' is the effective mean confining pressure (in MPa), e is the void ratio and G_{max} is the small-strain shear modulus (in MPa).

Shear modulus correction for strain may be carried out using the following expressions:

$$\frac{G}{G_{max}} = \frac{1}{1 + \gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r} \left[1 + a\, e^{-b\left(\frac{\gamma}{\gamma_r}\right)}\right]$$

and *a* and *b* are constants depending on soil type. For sandy soil deposits:

$$a = -0.2 \ \ln N$$

$$b = 0.16$$

where *N* is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is the reference shear strain given by

$$\gamma_r = \frac{\tau_{max}}{G_{max}}$$

where

$$\tau_{max} = \sqrt{\left[\left(\frac{1 + K_o}{2} \ \sigma_v' \ sin\varphi'\right)^2 - \left(\frac{1 - K_o}{2} \ \sigma_v' \ \right)^2\right]}$$

Shear modulus is also related to the shear wave velocity ***v_s*** as

$$v_s = \sqrt{\frac{G}{\rho}}$$

where *G* is the shear modulus and ρ is the mass density of the soil.

Wind Engineering

Vortex-induced vibrations

Strouhal Number for flow past a circular cylinder: $S_t = \frac{n_u D}{U}$

where n_u is the vortex-shedding frequency (in Hz), D is the cylinder diameter (in m) and U is the flow velocity (in m/s).

For circular cylinders, $S_t \approx 0.2$

Scruton Number: $S_c = \frac{2\delta_s m}{\rho D^2}$

where m is the actual mass per unit length of the structure, δ_s is the logarithmic decrement of structural damping (= 2π × fraction of critical damping), D is the diameter of the cylinder and ρ is the density of air ($\approx 1.25 \text{kg/m}^3$).

A rough estimate of the amplitude y_{max} of vortex-induced vibrations at resonance can be obtained from

$$\frac{y_{max}}{D} = \frac{1.5}{S_c}$$

Classical flutter

The critical wind velocity v_f for classical flutter of a bridge may be estimated from

$$\frac{v_f}{f_T b} = 1.8 \left[1 - 1.1 \left(\frac{f_B}{f_T}\right)^2\right]^{1/2} \left(\frac{mr}{\rho b^3}\right)^{1/2}$$

where

f_B, f_T are the natural frequencies of the first vertical bending and torsion modes respectively;

b is the deck width;

m is the mass per unit length;

r is the mass radius of gyration of the cross-section;

ρ is the density of air.

FAM, JPT, SPGM

January, 2022