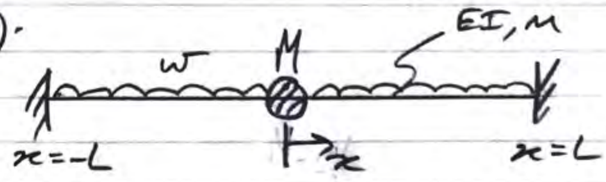
Part IIB 4D6: Dynamics in Civil Engineering

1.



$M = \frac{mL}{7}$

a).

Symmetry $\Rightarrow \bar{u}(x) = a + bx^2 + cx^4$, $\bar{u}'(x) = 2bx + 4cx^3$

$\bar{u}(0) = 1 \Rightarrow a = 1$

$\bar{u}(\pm L) = 0 \Rightarrow 1 + bL^2 + cL^4 = 0$

$\bar{u}'(\pm L) = 0 \Rightarrow \pm 2bL \pm 4cL^3 = 0 \Rightarrow b = -2L^2c$

$\Rightarrow 1 - 2L^4c + L^4c = 0$

$\therefore c = \frac{1}{L^4}$, $b = -\frac{2}{L^2}$

$\therefore \bar{u}(x) = 1 - 2\left(\frac{x}{L}\right)^2 + \left(\frac{x}{L}\right)^4$

$\bar{u}'(x) = -\frac{4}{L^2}x + \frac{4}{L^4}x^3$; $\bar{u}''(x) = -\frac{4}{L^2} + \frac{12}{L^4}x^2$

$$M_{eq} = \int_{-L}^{L} m\bar{u}^2\,dx + M\bar{u}(0)^2$$

$$= 2m\int_0^L 1 - 4\left(\frac{x}{L}\right)^2 + 6\left(\frac{x}{L}\right)^4 - 4\left(\frac{x}{L}\right)^6 + \left(\frac{x}{L}\right)^8\,dx + \frac{mL}{2}.1$$

$$= 2m\left[x - \frac{4}{3}\left(\frac{x}{L}\right)^3 + \frac{6}{5}\left(\frac{x}{L}\right)^5 - \frac{4}{7}\left(\frac{x}{L}\right)^7 + \frac{1}{9}\left(\frac{x}{L}\right)^9\right]_0^L + \frac{mL}{2}$$

$$= 2mL\left(1 - \frac{4}{3} + \frac{6}{5} - \frac{4}{7} + \frac{1}{9}\right) + \frac{mL}{7}$$

$$= \frac{43}{45}mL$$

$$K_{eq} = \int_{-L}^{L} EI(\bar{u}'')^2\,dx = 2EI\int_0^L \frac{16}{L^4} - \frac{96}{L^6}x^2 + \frac{144}{L^8}x^4\,dx = 2EI\left[\frac{16}{L^4}x - \frac{32}{L^6}x^3 + \frac{144}{5L^8}x^5\right]_0^L$$

$$= \frac{2EI}{L^3}\left(16 - 32 + \frac{144}{5}\right)$$

$$= \frac{128EI}{5L^3}$$

$$\therefore \omega = \sqrt{\frac{128EI}{5L^3}\cdot\frac{45}{43mL}} = \sqrt{\frac{1152EI}{43mL^4}} \quad ; \quad f = \frac{\omega}{2\pi} = 0.824\sqrt{\frac{EI}{mL^4}}$$

$$T = 1.214\sqrt{\frac{mL^4}{EI}}$$

① cont.

b). $2L = 1.4\,m \therefore L = 0.7\,m$; $EI = 8000\,Nm^2$; $m = 26\,kg/m$.

$$\Rightarrow T = 1.214\sqrt{\frac{26 \times 0.7^4}{8000}} = 0.034\,s$$

$$t_d = 10\,ms \quad \therefore \frac{t_d}{T} = \frac{0.01}{0.034} = 0.30 \quad \Rightarrow DAF \simeq 0.85$$

$$F_{eq} = \int_{-L}^{L} w.\bar{u}\,dx = 2w\int_0^L 1 - 2\left(\frac{x}{L}\right)^2 + \left(\frac{x}{L}\right)^4 dx = 2w\left[x - \frac{2}{3L^2}x^3 + \frac{1}{5L^4}x^5\right]_0^L$$

$$= 2wL\left(1 - \frac{2}{3} + \frac{1}{5}\right) = \frac{16wL}{15} = \frac{16}{15} \times 7000 \times 0.7$$

$$= 5.2\,kN$$

$$K_{eq} = \frac{128 \times 8000}{5 \times 0.7^3} = 597.1\,kN/m$$

$$\therefore u_{max} \simeq 0.85\frac{F_{eq}}{K_{eq}} = 0.85 \times \frac{5.2}{597.1} = 7.4 \times 10^{-3} = 7.4\,mm$$

c). Resilient seal would reduce K_{eq}, and the increased mobility of the panel would increase its modal mass M_{eq}. The natural period would therefore increase, leading to a reduction in the DAF. Since the static deflection would increase, only a revised model can indicate whether the dynamic deflection would increase or decrease.

Q1 Rayleigh's Principle on single degree of freedom beam model
A popular and straightforward question, well-answered by most candidates. There was a fair bit of algebra in the integrations of the first part, with inevitable slips by a few.

4D6 Lent 2022

Q 3) a) When a structure founded on a deep soil strata is subjected to earthquake loading, it will be subjected to lateral and rocking oscillations. These oscillations can lead to changes in confining pressure, which in turn changes the shear stiffness of the soil. The dynamic interaction between the soil and the structure and each influencing the response of the other is termed as 'dynamic soil-structure interaction'.

[10%]

b) The stiffness of a soil is dependent on the effective stress given by

$$p' = p - u$$

where p' is the effective mean confining stress, p is the total stress and 'u' is the pore water pressure. When earthquake loading creates shear stresses in the soil, its volume can change. This tendency to suffer volumetric contraction in loose, saturated sands is manifested as excess pore pressure giving

$$p' = p - [u_{hydrostatic} + u_{excess}]$$

If u_{excess} is quite large during an earthquake, p' can reduce to near zero value and this is termed as full liquefaction. This can cause structures to settle and/or rotate severely. If u_{excess} is moderate then this can cause a reduction in ~~p~~ p', causing a degradation in soil stiffness. This is called 'partial liquefaction'. This can alter the natural frequency of the structure, and initially stiff response can degrade moving this frequency towards the earthquake driving frequency. This can cause damage to structure due to resonant vibrations.

[10%]

c) Unit weight of sand $= 14.3\ kN/m^3$. Take $G_s = 2.65$

Void ratio 'e' $= \dfrac{G_s \gamma_w}{1+e} = \gamma_d \Rightarrow e = \dfrac{2.65 \times 9.81}{14.3} - 1 = 0.818$

$$G_0 = 100 \frac{[3-e]^2}{1+e} [p']^{0.5} = 261.88 \sqrt{p'}$$

$$p' = \left(\frac{1+2K_0}{3}\right) \sigma'_v \qquad K_0 = \frac{\nu}{1-\nu} = \frac{0.3}{0.7} = 0.429 \Rightarrow p' = 0.619\, \sigma'_v$$

$$\sigma'_v = \gamma_d z = 14.3 z$$

$$\therefore G_0 = 261.88 \sqrt{\frac{14.3 z \times 0.619}{1000}} \cancel{0.613 \sqrt{z} \times \sqrt{0.619} = 0.482 \sqrt{z}}$$

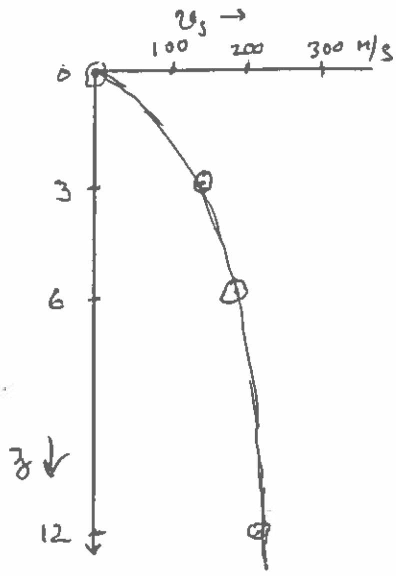
$$= 24.638 \sqrt{z}$$

So at $z = 0$ $\quad G_0 = 0$

$z = 3m$ $\quad G_0 = 42.67$ MPa

$z = 6m$ $\quad G_0 = 60.35$ MPa

$z = 12m$ $\quad G_0 = 85.35$ MPa



v_s Profile

[30%]

Shear wave velocity $= v_s = \sqrt{\frac{G}{\rho}}$.

$\rho_d \times g = \gamma_d$ $\qquad \rho_d = 1457.69$ kg/m³.

$\therefore$ at $z = 0$ $\quad v_s = 0$

$z = 3m$ $\quad v_{s_{30}} = 171.09$ m/s

$z = 6m$ $\quad v_{s_{60}} = 203.47$ m/s

$z = 12m$ $\quad v_{s_{120}} = 241.97$ m

---

3 d) $\quad \omega_n = \sqrt{\frac{K}{m}}$. $\quad f_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$.

$2.3 = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$. $\Rightarrow \sqrt{\frac{k}{m}} = 14.45$

$\therefore k = 3.1326 \times 10^6$ N/m.

Stiffness $k = \frac{3EI}{L^3}$.

$\therefore EI = \frac{kL^3}{3} = \frac{3.1326 \times 10^3 \times 10^6}{3}$.

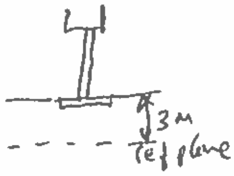
$= 1.044 \times 10^9$ Nm².

$= \underline{\underline{1044.2 \text{ MNm}^2}}$.

Main assumption is that the tower oscillates at its natural frequency due to small oscillations caused by wind.

Second assumption is that the tower is fully fixed at the foundation level.

3e) Due to strong earthquake the shear modulus at the reference plane is reduced to 50% of G_0.



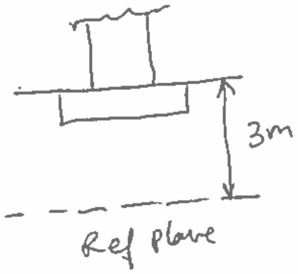
$\therefore G = \frac{50}{100} \times 42.67 = \underline{\underline{21.335}}$ MPa.

1 DoF Discrete model :-

$$I_{tank} = 15000 \times [10+3]^2 = 2535000 \text{ kg-m}^2$$

$$I_{soil} = 5 \times I_{tank} = 12675000 \text{ kg-m}^2$$

$$K_{soil} = K_{rot} = K_{ry}.$$



$2l = 3$ m

$2b = 3$ m

$l/b = 1 \quad e/b = \frac{1}{1.5}$

$$K_{ry} = \frac{G b^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[1 + \frac{e}{b} + \frac{1.6\,(e/b)^2}{0.35 + \left(\frac{l}{b}\right)^4}\right]$$

$$= G \frac{\times 1.5^3}{(1-0.3)}\,[4]\left[1 + \frac{1}{1.5} + \frac{1.6}{1.35} \times \left[\frac{1}{1.5}\right]^2\right]$$

$$= G\ 4.8214 \times 4 \times 2.1934$$

$$= 42.301\ G$$

$$\therefore K_{ry} = 42.301 \times 21.335 \times 10^6 \text{ Nm/rad}$$

$$= 902.499 \times 10^6 \text{ Nm/rad}$$

$\therefore$ Using 1DOF $\quad \omega_n = \sqrt{\frac{K_{ry}}{I_{soil}}} = \sqrt{\frac{902.499 \times 10^6}{12675000}} = 8.438$ rad/s

$$\therefore f_n = \underline{\underline{1.3436}} \text{ Hz}$$

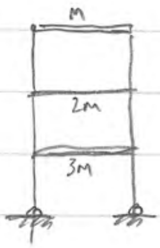
Yes. The rocking frequency and the sway mode frequency are both in the range of 1~5 Hz. Therefore the water tank structure can suffer significant damage. [30%]

Q2 Liquefaction and soil-structure interaction

Another popular question. Students demonstrated a good knowledge of the subject matter covering the effects of earthquakes on soil stiffness.

2022

4D6 Q3



$k_3 = 2(12EI/L^3)$

$k_2 = 2(12EI/L^3)$

$k_1 = 2(3EI/L^3)$

i)

$$K = \begin{bmatrix} 6+24 & -24 & \\ -24 & 24+24 & -24 \\ & -24 & 24 \end{bmatrix} EI/L^3 = \frac{6EI}{L^3}\begin{bmatrix} 5 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 4 \end{bmatrix} = \frac{24EI}{L^3}\begin{bmatrix} 1.25 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} M_0.$$

ii) $\phi = [0.848, 0.961, 1.0]'$

$$\therefore K\phi \propto \begin{bmatrix} 1.25 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0.848 \\ 0.961 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 0.099 \\ 0.074 \\ 0.039 \end{bmatrix}$$

$$M\phi \propto \begin{bmatrix} 3 & & \\ & 2 & \\ & & 1 \end{bmatrix}\begin{bmatrix} 0.848 \\ 0.961 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 2.54 \\ 1.90 \\ 1.00 \end{bmatrix}$$

$$M\phi./K\phi = \begin{bmatrix} 25.7 \\ 25.7 \\ 25.6 \end{bmatrix}$$

$\approx$ constant

$\therefore \quad K\phi = \underset{\uparrow \text{constant}}{\omega^2} M\phi \qquad \therefore \phi$ is an eigenvector.

Specifically

$$K\phi = \begin{bmatrix} 0.099 \\ 0.074 \\ 0.039 \end{bmatrix}\frac{EI}{L^3} = \omega^2 M\phi = \omega^2 \begin{bmatrix} 2.54 \\ 1.90 \\ 1.00 \end{bmatrix} M_0$$

so $$\omega^2 = \frac{24EI}{ML^3}\left[\frac{0.099}{2.54}\right] = \frac{24EI}{25.7ML^3} = 24\frac{(9000\times10^3)\,\text{N m}^2}{(11\times10^3)(3.1)^3(25.7)} = 25.65$$

$\therefore \quad \omega = \sqrt{25.65} = 5.06$ rads/s $= 2\pi f$

$\therefore \quad f = \frac{5.06}{2\pi} = \underline{\underline{0.806\text{Hz}}} \qquad \therefore \quad T = \frac{1}{f} = \underline{\underline{1.24\text{s}}}$

4D6, 2022, Q3 cont'd.

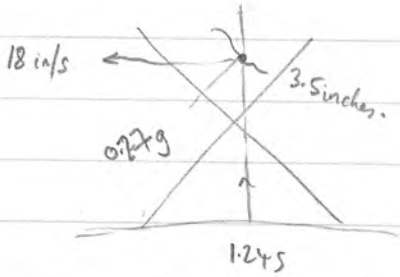
iii) From graph, Fig3,

at RHS, Deformation $\simeq$ 8 inches $= 8 \times 25.4 = 203\,mm$

at LHS Accel $\simeq 0.3g = 2.94\ m/s^2$.

iv) One mode only: $f = 0.806\ Hz$ $\omega = 5.06\ rads/s$

$T = 1.24\ sec.$



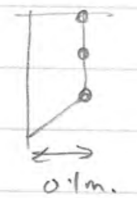
$S_a = 0.25g = 2.45\ m/s^2$

$S_d = 3.5$ inches $= 89\,mm = 0.089\ m$.

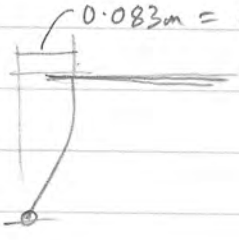
Check $S_d \simeq S_a/\omega^2 = \frac{2.45}{(5.06)^2} = \underline{0.095\,m}$ ✓ $\approx 0.089\,m$. $\Big\}$ $\underline{0.09\,m}$

Modal participation factor $\Gamma_1 = \frac{\sum Mu}{\sum Mu^2} = \frac{3(0.848) + 2(0.961) + 1}{3(0.848)^2 + 2(0.961)^2 + 1}$

$= \frac{5.466}{5.004} = \underline{1.092}$

$u_1 = \Gamma_1 S_{d_1} \phi_1 = (1.092)(0.09\ m) \begin{bmatrix} 0.848 \\ 0.961 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 0.083 \\ 0.094 \\ 0.098 \end{bmatrix} m \approx$ 

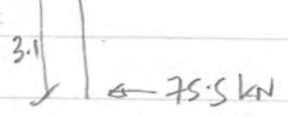
Very flexible lower storey.

Lower floor, 

Shear force $= \frac{3EI\delta}{L^3}$

$= \frac{3(9000 \times 10^3\ Nm^2)(0.083\ m)}{(3.1)^3}$

$= \underline{75.5\ kN}$ Shear

Moment = 

$3.1 \times 75.5\ kN = \underline{234\ kNm}$

2022

4D6 Q3 cont'd.

b) Column shear capacity = 40 kN

but part a) asks for 75.5 kN if fully elastic throughout, with a deflection of 83mm

so we'd like an elastic deflection of $83\text{mm} \times \frac{40}{75.5} = \underline{44\text{ mm}}$

$$\therefore \delta = \Gamma S_d \Phi_{1st\ floor} = (1.092) \frac{S_a}{\omega^2} \Phi_{1st\ floor} = 0.044\text{m}$$

$$\therefore S_a = \frac{\omega^2 / (0.044)}{1.092\,(0.848)} = \frac{(5.06)^2 / (0.044)}{1.092\,(0.848)} = 1.22\text{ m/s}^2$$

$$= \underline{\underline{0.124g}}$$

PGA = 0.3g so require S_a in plot $= \frac{0.124g}{0.3} = 0.41g$

T = 1.24 seconds $S_a = 0.41g \rightarrow \mu = 3.5$

required ductility factor.

Q3 Elastic and Inelastic Response Spectrum Analysis

A popular question with a high average mark, and a significant number scoring full marks. The solutions were helped by having the true mode shape given. This had the further effect that the various ways of approaching part (a)(iv) all led to the same answer for the column shear.

4D6 2022.

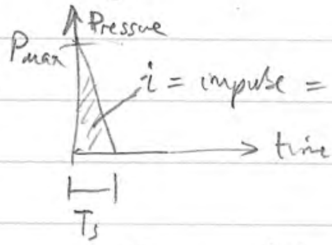
Q4

i) $\log_{10} P_{max} = -1.5 \log_{10} Z + 6.05$ $\quad P = 10^{6.05} Z^{-1.5}$

$\log_{10} i^* = -1.45 \log_{10} Z + 3.65$ $\quad i^* = 10^{3.65} Z^{-1.45}$

Z = Scaled distance $= R/W^{1/3}$

Assume triangular pulse



i = impulse = area $\simeq \frac{1}{2} P_{max} T_s$

$$i = \frac{1}{2} P_{max} T_s \qquad T_s = \frac{2i}{P_{max}}$$

$$T_s^* = \frac{2i^*}{P_{max}} \qquad \text{(divide each side by } W^{1/3}\text{)}$$

$$\therefore T_s^* = \frac{2(10^{3.65})}{10^{6.05}} \frac{Z^{-1.45}}{Z^{-1.5}} = 2(10^{-2.4}) Z^{0.05}$$

$$\therefore \log_{10} T_s^* = \log_{10}(2 \times 10^{-2.4}) + 0.05 \log_{10} Z$$

$$= 0.05 \log_{10} Z - 2.4 + \log_{10} 2$$

$$\underline{\log_{10} T_s^* = 0.05 \log_{10} Z - 2.1}$$

$$\left(\begin{array}{c} T_s^* = T/W^{1/3} \\ \log T_s^* = \log_{10} T - 1/3 \log_{10} W. \end{array} \right)$$

ii) $W = 4$ tonne TNT equiv, $R = 100\,m$

$$Z = \frac{100}{(4000)^{1/3}} = 6.3 \ m.kg^{-1/3}$$

$$\therefore \log_{10} T_s^* = 0.05 \log_{10} 6.3 - 2.1 = -2.06$$

$$T_s^* = 10^{-2.06} = 0.0087 = \frac{T_s}{W^{1/3}}$$

$$T_s = (0.0087)(4000)^{1/3} = 0.14 \text{ seconds.}$$

4D6 2022 Q4 cont'd.

ii) cont'd. $\log_{10} P_{max} = -1.5 \log_{10} 6.3 + 6.05 = 4.85$

$$P_{max} = 10^{4.85} = \underline{71 \text{ kPa}}.$$

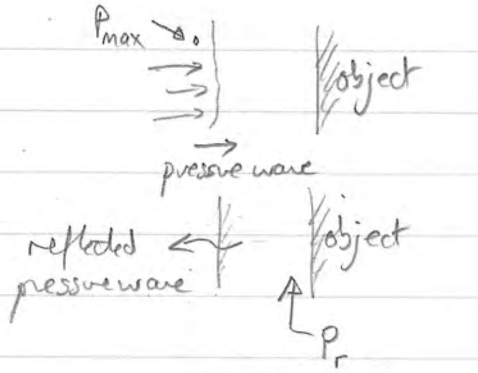
Impulse $\log_{10} i^* = -1.45 \log_{10} 6.3 + 3.65 = 2.49$

$$\therefore i^* = 10^{2.49} = 310$$

$$\therefore i = (310)(4000^{1/3}) = \underline{4916 \text{ Pa s}}.$$

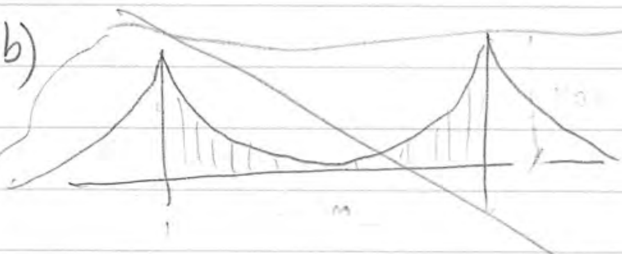
iii) two effects : i) ground reflection

ii) reflection from surface itself
→ Rankine-Hugoniot
→ pressure increase

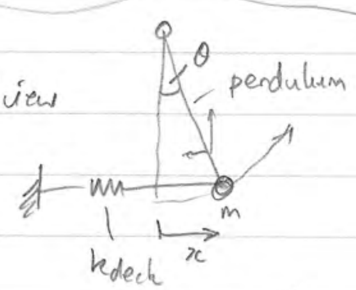


$P_r > P_{max}$ of free air blast wave.

b)



End view



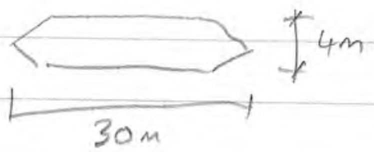
deck → kx
pendulum $mg\theta$ $= mg\frac{x}{H}$

$\therefore$ Stiffness $= \left(k + \frac{mg}{H}\right)$

4D6 2022 Q4 cont'd

b) Answer: a simple linear elastic small displacement finite element analysis is likely to ignore geometric stiffness, and suspension bridges can gain a significant proportion of their stiffness via geometric effects, particularly for lateral sway modes. Can think of this (equivalently) as "pendulum effects", with the deck "hanging" from the tower tops, - or as "tension stiffening" due to the dead load tensions in the suspension cables.

c). i)



24 tonne/m $\quad \zeta = 0.005$ of crit.

$$n_{nat} = \frac{1}{T_{nat}} = \frac{1}{11} = \underline{0.0909 \text{ Hz}}$$

$$St = 0.15 = \frac{n_u D}{U} = \frac{(0.0909)(4m)}{U} = 0.15$$

$$U = \frac{(0.0909 \text{ Hz})(4m)}{0.15} = \underline{\underline{2.42 \text{ m/s}}}$$

$$Sc = \frac{2\delta_s m}{\rho D^2} \qquad \delta_s = 2\pi\zeta$$

$$= \frac{2(2\pi(0.005))(24\times10^3) \text{ kg/m}}{(1.25 \text{ kg/m}^3)(4)^2} = 75.4$$

$$y_{max} \approx \frac{1.5 D}{Sc} \approx \frac{1.5(4)}{75.4} = 0.08 \text{ m} = \underline{8 \text{ cm}}$$

4D6 2022 Q4 (cont'd.)

c) ii)

Flutter.

British Rules:

$$\frac{V_f}{n_t b} = 1.8\left(1 - 1.1\left(\frac{n_b}{n_t}\right)^2\right)^{1/2}\left(\frac{mr}{\rho b^3}\right)^{1/2}$$

$$V_f = \frac{1.8}{T_t} b\left(1 - 1.1\left(\frac{T_T}{T_B}\right)^2\right)^{1/2}\left(\frac{mr}{\rho b^3}\right)^{1/2}$$

$$= \frac{(1.8)(30)}{3}\left(1 - 1.1\left(\frac{3}{11}\right)^2\right)^{1/2}\left(\frac{24\times10^3 . 10}{(1.25)(30)^3}\right)^{1/2}$$

(kg/m) m

(kg/m³) m³

$$= (18)(0.92)(2.67)$$

= 44.1m/s

Q4 Blast response and wind engineering

As is often the case for this final part of the course material, this last question was attempted by only a few candidates, in this case 5. Nevertheless, the standard of submissions was high. For the vortex-induced vibration of the suspension bridge in part (c)(i), the question specified that the given Strouhal number was based on deck depth d, but at least one candidate gave intelligent reasons as to why other dimensions (such as L or sqrt(Ld)) would be more physically reasonable.