Part IIB 4D6: Dynamics in Civil Engineering

①

a). $\bar{u} = 1 - \cos\frac{\pi x}{4L}$

$$M_{eq} = \int_0^{2L} m(x)\,\bar{u}^2\,dx = 2m\int_0^L \left(1-\cos\frac{\pi x}{4L}\right)^2 dx + m\int_L^{2L}\left(1-\cos\frac{\pi x}{4L}\right)^2 dx$$
$$+ M\left(\bar{u}(x=2L)\right)^2$$

$$= 2m\int_0^L 1 - 2\cos\frac{\pi x}{4L} + \frac{1}{2}\left(1+\cos\frac{\pi x}{2L}\right) dx$$
$$+ m\int_L^{2L} 1 - 2\cos\frac{\pi x}{4L} + \frac{1}{2}\left(1+\cos\frac{\pi x}{2L}\right) dx + mL$$

$$= 2m\left[x - \frac{8L}{\pi}\sin\frac{\pi x}{4L} + \frac{x}{2} + \frac{L}{\pi}\sin\frac{\pi x}{2L}\right]_0^L$$
$$+ m\left[x - \frac{8L}{\pi}\sin\frac{\pi x}{4L} + \frac{x}{2} + \frac{L}{\pi}\sin\frac{\pi x}{2L}\right]_L^{2L} + mL$$

$$= 2m\left(\frac{3L}{2} - \frac{8L}{\pi\sqrt{2}} + \frac{L}{\pi}\right) + m\left(3L - \frac{8L}{\pi} - \frac{3L}{2} + \frac{8L}{\pi\sqrt{2}} - \frac{L}{\pi}\right) + mL$$

$$= m\left(3L - \frac{16L}{\pi\sqrt{2}} + \frac{2L}{\pi} + \frac{3L}{2} - \frac{9L}{\pi} + \frac{8L}{\pi\sqrt{2}} + L\right)$$

$$= \left(\frac{11}{2} - \frac{7}{\pi} - \frac{8}{\pi\sqrt{2}}\right)mL = 1.471\,mL$$

$$K_{eq} = \int_0^{2L} EI(x)\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx = 2EI\int_0^L\left(\left(\frac{\pi}{4L}\right)^2\cos\frac{\pi x}{4L}\right)^2 dx + EI\int_L^{2L}\left(\left(\frac{\pi}{4L}\right)^2\cos\frac{\pi x}{4L}\right)^2 dx$$

$$= \cancel{2}EI\cdot\frac{\pi^4}{256L^4}\int_0^L \cancel{\frac{1}{2}}\left(1+\cos\frac{\pi x}{2L}\right)dx + EI\cdot\frac{\pi^4}{256L^4}\int_L^{2L}\frac{1}{2}\left(1+\cos\frac{\pi x}{2L}\right)dx$$

$$= \frac{\pi^4 EI}{256L^4}\left(\left[x + \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right]_0^L + \frac{1}{2}\left[x + \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right]_L^{2L}\right)$$

$$= \frac{\pi^4 EI}{256L^4}\left(L + \frac{2L}{\pi} + L - \frac{L}{2} - \frac{L}{\pi}\right) = \frac{\pi^4 EI}{256L^3}\left(\frac{3}{2} + \frac{1}{\pi}\right) = 0.692\frac{EI}{L^3}$$

$$\therefore \omega_1 = \sqrt{\frac{K_{eq}}{M_{eq}}} = \sqrt{\frac{0.692}{1.471}\frac{EI}{L^3\,mL}} = 0.686\sqrt{\frac{EI}{mL^4}}$$

(1) Cont.

b). $L = 30\,m$, $EI = 2\times10^{11}\ Nm^2$, $m = 3000\ kg/m$

$$\therefore \omega_1 = 0.686\sqrt{\frac{2\times10^{11}}{3000\times30^4}} = 6.224\ rad/s$$

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{6.224} = 1.01\ s$$

$$t_d = 3s \quad \therefore \frac{t_d}{T} = \frac{3}{1.01} \simeq 3 \Rightarrow DAF \simeq 1.17$$

$$F_{eq} = F\bar{u}(L) = F\left(1-\cos\frac{\pi}{4}\right) = 5\times10^6\left(1-\frac{1}{\sqrt{2}}\right) = 1.464\times10^6\ N.$$

$$K_{eq} = 0.692\times\frac{2\times10^{11}}{30^3} = 5.126\times10^6\ N/m$$

$$\therefore U_{max} \simeq 1.17\times\frac{1.464}{5.126} = 0.33\,m$$

$$U_{x=L} = U_{max}\left(1-\cos\frac{\pi}{4}\right) = 0.33\left(1-\frac{1}{\sqrt{2}}\right) = 0.10\,m$$

$$U_{x=2L} = U_{max} = 0.33\,m$$

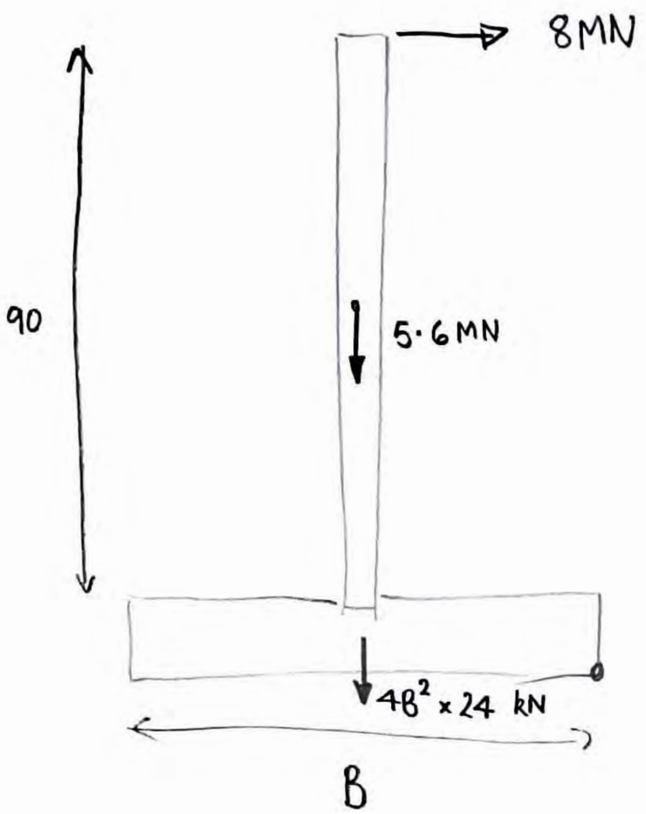
c). ~~For a point-loaded cantilever, static shape is cubic, ie. 3rd order polynomial.~~

See below

~~The '$\frac{1}{4}$ cosine' shape does not satisfy the zero-shear boundary condition at the tip ($x=2L$). This may be satisfied with an appropriate cubic.~~

~~A further improvement to the model would be to consider the distributed nature of the wave loading, rather than assuming a point force. May also wish to consider the contributions from higher modes.~~

The quarter cosine does not satisfy the shear condition at the tip. There is shear due to the lumped mass of the nacelle. A cubic polynomial could represent this (such as the deflected shape of a tip-loaded cantilever). However, the tower mass acts similar to a uniformly-distributed load, so a fourth-order polynomial would be an improvement to better model that.



$$94 \times 8 = 5.6\,\frac{B}{2} + \frac{96B^3}{2000}$$

$$B = \cancel{23.9\,m}\ \ 24.25\,m$$

$$V_s = \sqrt{\frac{G}{\rho}} = 150 \qquad G = 150^2\rho = 45\ MPa$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73\left(\frac{L}{b}\right)^{2.4} + 0.27\right]\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\left(\frac{L}{b}\right)^4}\left(\frac{e}{b}\right)^2\right)$$

$$\frac{L}{b} = 1 \qquad \frac{e}{b} = 0.32 \qquad \nu = 0.3 \qquad b = 12.5$$

$$K_{ry} = \frac{45 \times 12.5^3}{0.7}\,[\,4\,]\left(1.32 + \frac{1.6}{1.35} \times 0.32^2\right)$$

$$= 723.9\ GNm$$

$M_b = 6000$ tonnes

$$I = \underbrace{2\times10^5 \times 94^2}_{\text{nacelle}} + \underbrace{360000 \times \frac{90^2}{12} + 360000 \times 49^2}_{\text{tower}}$$

$$+\ 6\times10^6 \times \frac{4^2}{12} + 6\times10^6 \times 2^2$$

$$= 2.9 \times 10^9\ kg\,m^2$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{\cancel{M}I}} = \frac{1}{2\pi}\sqrt{\frac{723.9\times10^9}{2.9\times10^9}} =$$

$$= \underline{\underline{2.5\ Hz}}$$

c) 1) Soil participating in vibration will increase I hence reduces f_n

2) flexibility of tower will reduce n.f.

3) Shaking can lead to generation of e.p.p leading to reduction in soil stiffness due to partial liquefaction. reduces K so reduces f_n

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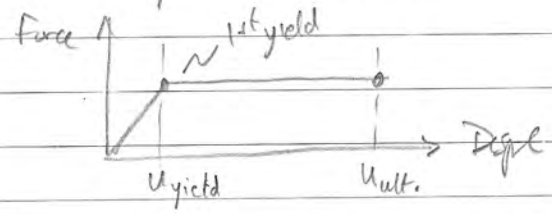
3 (a) Marks will be awarded for descriptions of how the modal participation factor arises as a result

$$\Gamma = \frac{\sum m_i \phi_i}{\sum m_i \phi_i^2}$$

of mode-generalising the D'Alembert forces that arise when writing the equations of motion in a frame moving with the ground displacements.

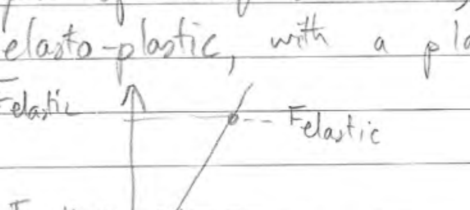
It is a measure of the relative motion of the modal centre of mass relative to the (fixed) ground.

b) Ductility factor is ratio of ultimate displacement (or strain) to displacement (or strain) at first yield (assumes elasto-plastic behaviour:



$$\mu = \frac{u_{ultimate}}{u_{first\,yield}}$$ (~ 5 in the sketch))

Inelastic spectra are obtained by performing ensembles of earthquake simulations on a pair of sdof oscillators; one is purely elastic and the other is elasto-plastic, with a plastic plateau at a height $F_{yield} = \bar{f}\,F_{elastic}$



$$\bar{f} = \frac{F_{yield}}{F_{elastic}}$$ (~1/3 in sketch)

This is repeated for different natural frequencies (of elastic region) and the values of $\bar{f}$ and μ recorded.

The $\bar{f}$ then tell you how much you can reduce the strength by if you provide ductility μ. This can lead to substantial reductions in the strength required.

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Q3.(c) cont'd.

i) $PGA = 0.54g$

$T = 0.7$ seconds

$\phi = [0.2, 0.4, 0.6, 0.8, 1.0]$ $m = 120 \times 10^3$ kg per floor.

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.7} = 8.97 = 9$ rads/sec.

Modal participation factor $\Gamma = \frac{\sum m\phi}{\sum m\phi^2} = \frac{0.2 + 0.4 + 0.6 + 0.8 + 1}{0.04 + 0.16 + 0.36 + 0.64 + 1} = \frac{3}{2.2} = \underline{\underline{1.364}}$

2.20

∴ Elastic for $T = 0.4$ $S_a = \left[\frac{1.5}{0.7}\right][0.54g] = \underline{\underline{1.157g}} = 11.35\ m/s^2$

$S_d = \frac{S_a}{\omega^2} = \frac{11.35\ m/s^2}{9^2\ /s^2} = \underline{\underline{0.14m}}$

∴ $u_{max} = \Gamma S_d = 1.364 \times 0.14 = \underline{\underline{0.19m}}$ - Peak Deflection.

ii) Shear force at base =

Acceleration: $\ddot{u}_{max, top} = \Gamma S_a = 1.364(11.35) = \underline{15.5\ m/s^2}$

∴ $\ddot{u} = [3.1, 6.2, 9.3, 12.4, 15.5]\ m/s^2$

$F = \sum ma = 120 \times 10^3 [\underbrace{(15.5 \times 3)}_{[\sum m_i\phi_i][\Gamma S_a]}] = \underline{\underline{5580\ kN}}$ shear capacity req'd

At $T = 0.7$

Elastic Design Spectrum → $S_a = 2.71g$.

At $\mu = 4$ Ductility $S_a \sim 0.9g$

Need $5580 \times \frac{0.9}{2.71} = \underline{\underline{1850\ kN}}$ (ie ~ 1/3rd of elastic)

4D6 Q4.

a) Flutter derivatives.

Marks will be awarded for clear description of coupled heave-torsion of bridge deck, with coupling occurring via the aerodynamic forcing terms, expressed using flutter derivatives. Further marks will be awarded for clear description of section-model tests and how aerodynamic forces need to be measured as the section undergoes prescribed periodic motions at a range of frequencies (whilst the wind tunnel applies the incident airstream). 20%

b) Marks will be awarded for a clear description of the velocity → force → response

$u \rightarrow f \rightarrow x$

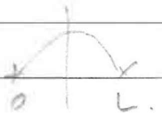
cascade, with transfer functions of aerodynamic and mechanical admittance at each stage. This should include a description of how aerodynamic admittance retains the frequency content of the overall forcing, and acts as a sort of areal reduction factor to take account of the spatial decorrelation of local pressures across the structure.

For full marks, the answer should outline how this then needs to be mode-generalised. 40%

$$I = \frac{\pi D^3 t}{8} = \frac{\pi (0.457)^3 (0.0143)}{8} = \underline{\underline{536 \times 10^{-6}\ m^4}}$$

Mass per unit length = ρπDt = (7850)π(0.457)(0.0143) = 161.2 kg/m

$$\text{Say } \phi = a \sin\left(\frac{\pi x}{L}\right)$$



$$M_{eq} = \int_0^L m \phi^2 \, dx = \frac{mL}{2} = \frac{(\overset{161.2}{\cancel{152.4}})\, 12}{2} = \overset{967\text{ kg}}{\cancel{912.6\text{ kg}}}$$

$$K_{eq} = \int_0^L EI \left(\frac{d^2 \phi}{dx^2}\right)^2 dx = EI \int_0^L \left(\frac{\pi^2}{L^2}\right)^2 \sin^2 \frac{\pi x}{L} \, dx$$

$$= \frac{\pi^4 EI}{L^4} \frac{L}{2} = \frac{\pi^4 EI}{2L^3} = \frac{\pi^4 (210 \times 10^9)(536 \times 10^{-6})}{2(12)^3}$$

$$= \cancel{\pi^4 (210)(0.5)} \quad 3172 \times 10^3 \quad \underline{N/m}. \qquad \frac{kg \cdot m s^{-2}}{m} = kg/s^{-2}$$

$$w = \sqrt{\frac{K}{m}} = \sqrt{\frac{3172 \times 10^3}{\cancel{912.6}\ 967}} = \overset{57.3}{\cancel{59}} \text{ rads/s}$$

$$w = 2\pi f \quad \therefore \quad \boxed{f = \frac{\cancel{59}\,57.3}{2\pi} = \overset{9.12\text{ Hz}}{\cancel{9.4\text{ Hz}}}}$$

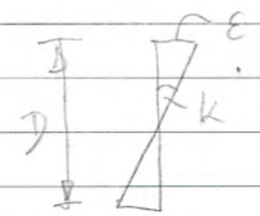
$$U = \frac{f_n D}{S_r} = \frac{(\overset{9.12}{\cancel{9.4}})(0.457)}{0.2} = \boxed{\overset{20.8\text{ m/s}}{\cancel{21.4}}\text{ m/s}}$$

$$S_t = \frac{2(2\pi(0.005))(\overset{161.2}{\cancel{152}})}{1.25(0.457)^2} = \cancel{36.5}.\ 38.8$$

$$y_{max} \sim \frac{1.5 D}{S_t} = \frac{1.5(0.457)}{\cancel{36.5}\ 38.8} = \overset{0.0177\text{ m}}{\cancel{0.0187}} \approx \underline{\underline{2\text{ cm}}}$$

amplitude

Curvature, $K = a\dfrac{d^2\phi}{dx^2} = \dfrac{0.02\,\pi^2}{L^2} = \dfrac{0.02\,\pi^2}{12^2}$.



$\varepsilon = K\dfrac{D}{2}$

$\sigma = E\varepsilon = EK\dfrac{D}{2} = (210\times10^9)\left(\dfrac{0.02\,\pi^2}{12^2}\right)\left(\dfrac{0.457}{2}\right)$

$= \underline{\underline{65.8\ \text{MPa}}}$