Version FAM/3

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 2 May 2023 14.00 to 15.40

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachment: 4D6 Data Sheet (6 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 Fig. 1(a) shows the dynamic model used as the basis for designing the tower of an offshore wind turbine. This comprises a cantilever beam of length L, flexural rigidity $2EI$ and effective mass per unit length $2m$, representing the subsea structure, to which is attached a second beam of length L, flexural rigidity EI and mass per unit length m, representing the upper structure. A point mass $M = mL$ is attached to the tip of the upper beam to represent the generator and blades of the turbine. An approximation to the fundamental sway mode shape of the tower is

$$u = 1 - \cos\frac{\pi x}{4L}$$

where x is the vertical coordinate measured from the seabed.

(a) Use Rayleigh's method to estimate the fundamental natural frequency of the tower. Assume the mass of the entrained water is included in the effective mass of the subsea structure. [40%]

(b) Wave loading may be idealised as a horizontal point force acting at sea level ($x = L$), with the triangular force pulse shown in Fig. 1(b). Estimate the peak displacement of the tower at sea level ($x = L$) and at its tip ($x = 2L$), assuming the response is dominated by the fundamental mode of a tower with parameter values $L = 30$ m, $EI = 2 \times 10^{11}$N m^2 and $m = 3000$ kg m^{-1}. [40%]

(c) Improved estimates of the tower displacements may be obtained by considering a polynomial function for the mode shape.

(i) What order of polynomial would be an appropriate choice and why? [10%]

(ii) Suggest an additional change to the model that may further improve the estimates. [10%]

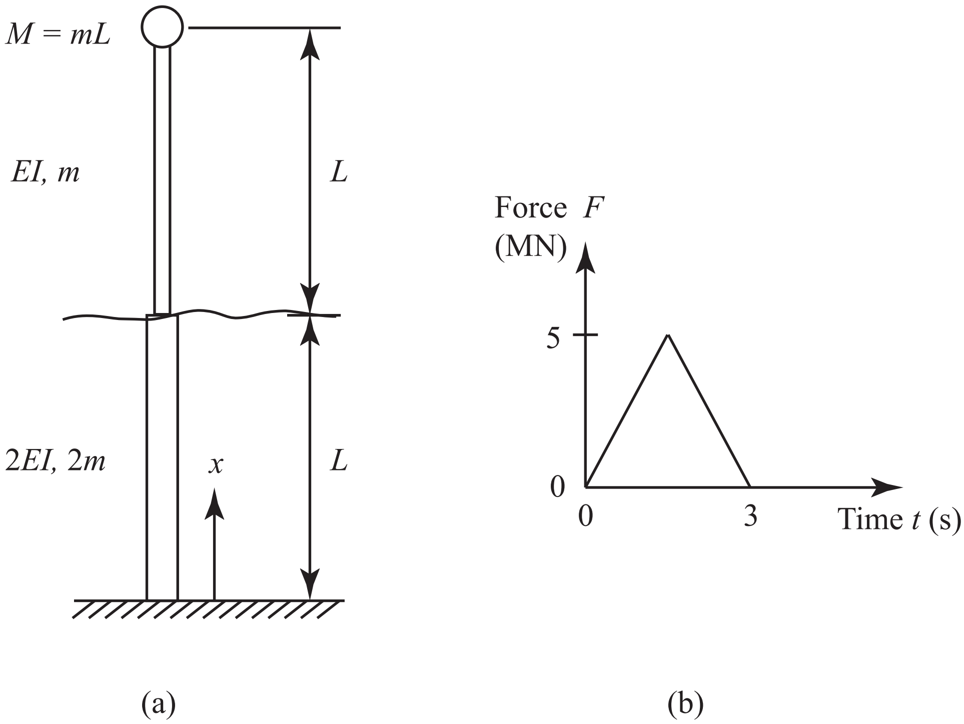

Fig. 1

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2 An onshore wind turbine is to be built in an area prone to earthquakes. The design consists of a square concrete base 4 m thick with a density of 2400 kg m^{-3} embedded such that its surface is level with the soil onto which the base of the wind turbine tower is centrally attached. The wind turbine can be modelled as a 90 m high tower with a mass of 4 tonnes m^{-1} on top of which is a lumped mass of 200 tonnes representing the nacelle and blades of the turbine as shown in Fig. 2. The turbine is subjected to a peak design horizontal wind load of 8 MN applied at the top of the tower.

(a) Assuming the soil to be rigid, estimate the width of the foundation B required to avoid toppling of the turbine system due to the wind load. [25%]

(b) The soil beneath the foundation is found to have an average shear wave velocity of 150 m s^{-1}, a density of 2000 kg m^{-3} and a Poisson's ratio of 0.3. If the foundation is constructed with B = 25 m and assuming the tower to be rigid and the mass of soil participating in the rocking to be negligible, estimate the natural frequency of the system in rocking and comment on whether this would be an appropriate design. [50%]

(c) Give three reasons why the natural frequency of the system may in practice be lower than this value during strong earthquakes. [25%]

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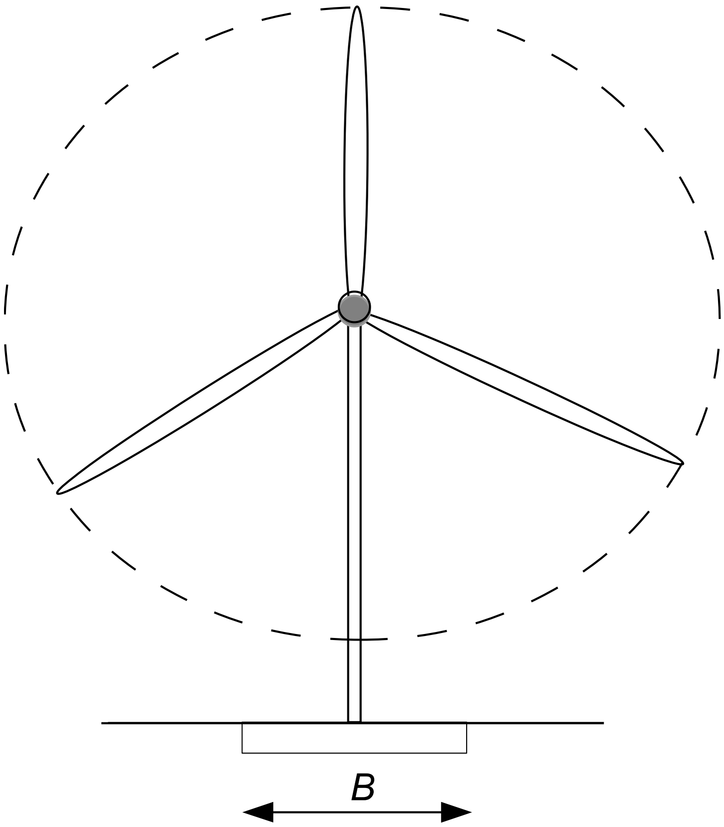

Fig. 2

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3 (a) Explain what is meant by a modal participation factor in earthquake engineering. [20%]

(b) Explain the concept of an inelastic response spectrum, and describe the procedure by which such spectra may be constructed. Define the ductility factor, μ. [30%]

(c) A building has five floors above the ground floor, with constant inter-storey height. It is to be designed to withstand a Peak Ground Acceleration of $0.54g$. The first lateral mode has a natural period of 0.7 s and the shape may be assumed to vary linearly with height. Each floor has a mass of 120 tonnes.

(i) Using the elastic design spectrum of Fig. 3, estimate the peak displacement of the top floor, and determine the total shear capacity required at the ground floor if the structure is to remain elastic. [30%]

(ii) Using the inelastic design spectrum of Fig. 4, determine the required shear capacity at the ground floor if columns can be provided which have a ductility factor $\mu = 4$. [20%]

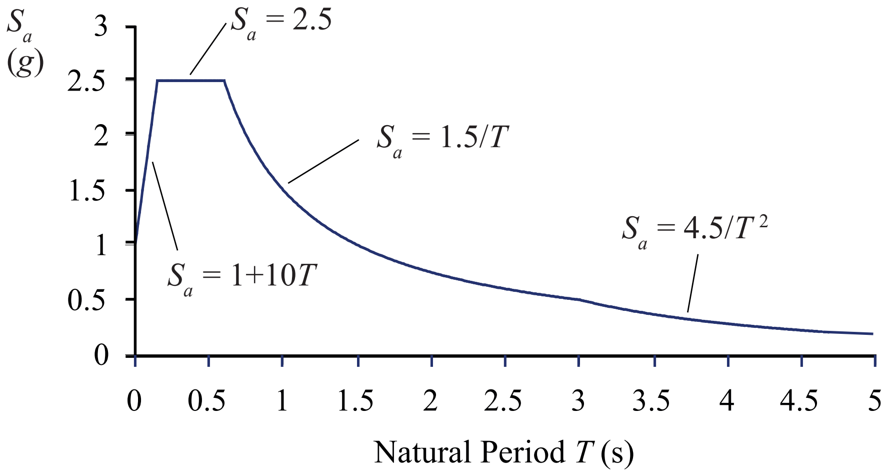

Fig. 3

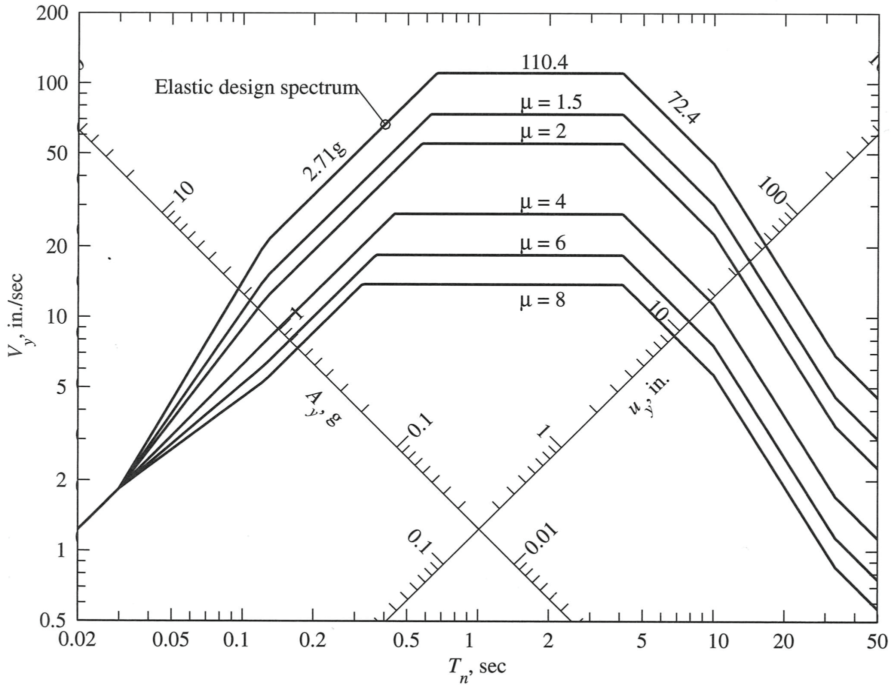

Fig. 4

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4 (a) Describe how flutter derivatives may be used in long span bridge design, and explain how they may be measured in a wind tunnel. [20%]

(b) Describe briefly a procedure for estimating the peak response of a large structure to wind buffeting. [30%]

(c) A steel pipe of length 12 m with circular cross-section acts as a simply-supported beam spanning a river. The pipe has an external diameter of 0.457 m and a wall thickness of 14.3 mm. The damping ratio of the fundamental bending mode is 0.5% of critical.

(The second moment of area of a thin-walled circular cross-section of radius r and wall thickness t is $\pi r^3 t$ about any diameter.)

Assuming the pipe is empty:

(i) determine the natural frequency of the fundamental bending mode; [20%]

(ii) determine the critical wind velocity for vortex-induced vibrations; [10%]

(iii) estimate the amplitude of the resulting oscillatory displacements; [10%]

(iv) determine the amplitude of the dynamic stresses induced. [10%]

END OF PAPER

Module 4D6: Dynamics in Civil Engineering

Data Sheets

Equivalent SDOF Systems

For an *n*-DOF system, with mass matrix $\underline{\underline{M}}$ and stiffness matrix $\underline{\underline{K}}$, responding in mode shape $\underline{\overline{u}}$ to an applied force $\underline{F}$, the parameters of the equivalent SDOF system are:

$$M_{eq} = \underline{\overline{u}}^T \underline{\underline{M}}\, \underline{\overline{u}} \qquad K_{eq} = \underline{\overline{u}}^T \underline{\underline{K}}\, \underline{\overline{u}} \qquad F_{eq} = \underline{F}^T \underline{\overline{u}}$$

For a continuous beam, of length L, mass per unit length m and bending stiffness EI, responding in mode shape $\overline{u}(x)$ to an applied force $f(x)$, the parameters of the equivalent SDOF system are:

$$M_{eq} = \int_0^L m\overline{u}^2 dx \qquad K_{eq} = \int_0^L EI \left(\frac{d^2\overline{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\overline{u}\, dx$$

The corresponding natural frequency is $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}}$

Modal Analysis of a Simply-Supported Beam

$$\overline{u}_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\,eq} = \frac{mL}{2} \qquad K_{i\,eq} = \frac{(i\pi)^4 EI}{2L^3}$$

Ground Motion Participation Factor

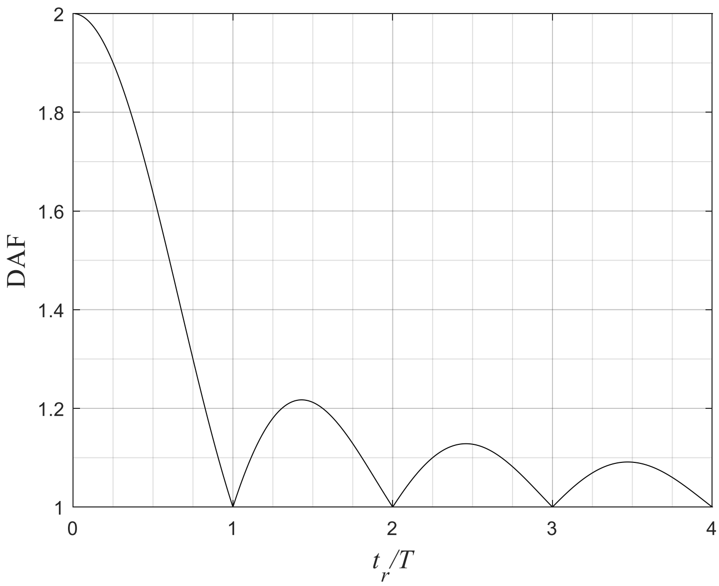
For an *n*-DOF system, with mass matrix $\underline{\underline{M}}$ and stiffness matrix $\underline{\underline{K}}$, responding to ground acceleration $\ddot{u}_g$, the parameters of the equivalent SDOF system are:

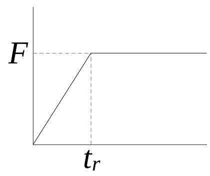
$$M_{eq} = \underline{\overline{u}}^T \underline{\underline{M}}\, \underline{\overline{u}} \qquad K_{eq} = \underline{\overline{u}}^T \underline{\underline{K}}\, \underline{\overline{u}} \qquad F_{eq} = -\Gamma M_{eq}\ddot{u}_g$$

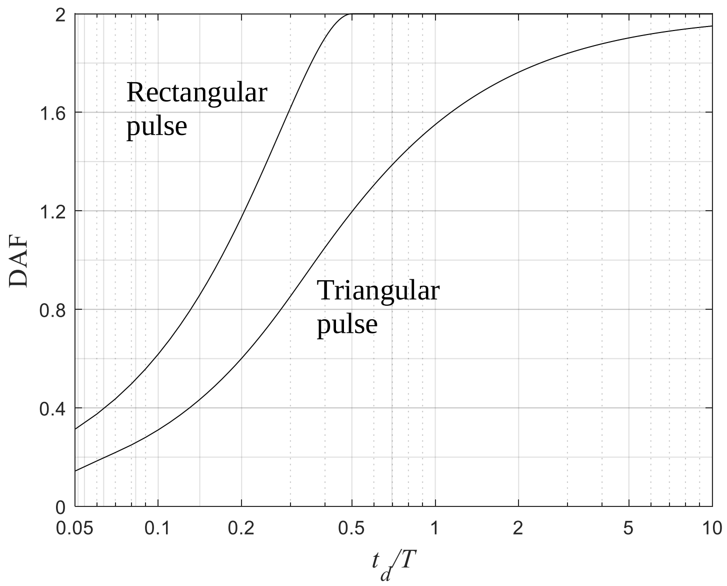
where $\underline{\overline{u}}$ is defined relative to ground and Γ is the modal participation factor

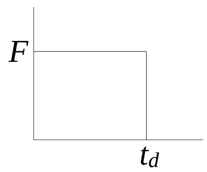
$$\Gamma = \frac{M_1\overline{u}_1 + M_2\overline{u}_2 + \cdots + M_n\overline{u}_n}{M_{eq}}$$

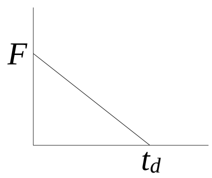
Dynamic Amplification Factors

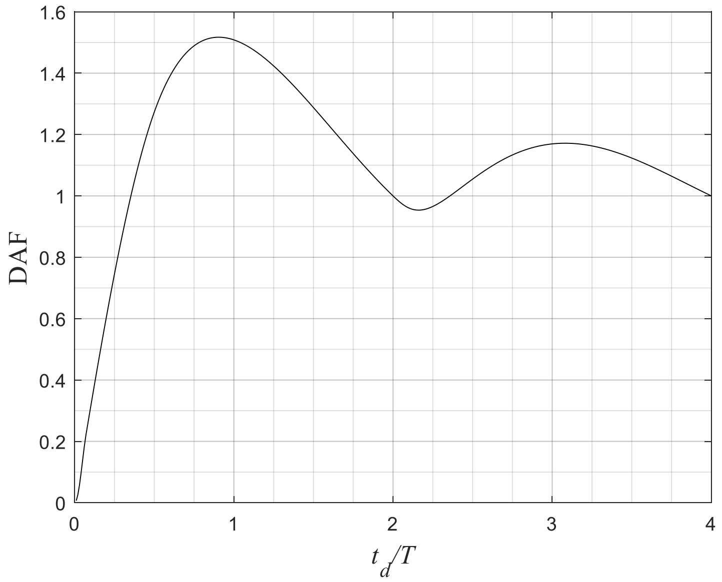


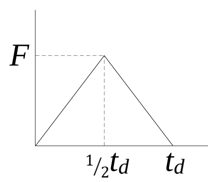






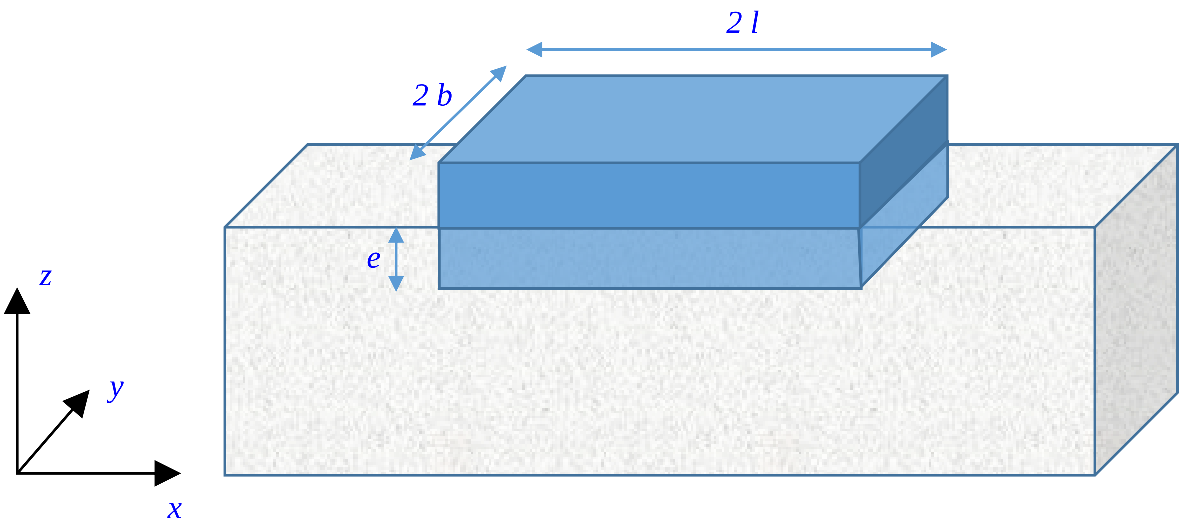






Soil Stiffness for Embedded Footings

Approximate relations for evaluating the soil stiffness for an embedded, prismatic footing of dimensions *2l* and *2b*, embedded to a depth *e*, assuming horizontal shaking in the direction parallel to the *x* axis (i.e. 2*l* of the prismatic structure) are:



$$K_{hx} = \frac{G\,b}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left\{0.33 + \frac{1.34}{1+\frac{l}{b}}\right\}\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{G\,b}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\,\frac{l}{b} + 1.6\right]\left[1 + \left\{0.33 + \frac{1.34}{1+\frac{l}{b}}\right\}\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{v} = \frac{G\,b}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left\{0.25 + \frac{0.25\,b}{l}\right\}\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{G\,b^3}{1-\nu}\left[3.2\,\frac{l}{b} + 0.8\right]\left[1 + \frac{e}{b} + \left(\frac{1.6}{0.35+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{2}\right]$$

$$K_{ry} = \frac{G\,b^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[1 + \frac{e}{b} + \left(\frac{1.6}{0.35+\left(\frac{l}{b}\right)^{4}}\right)\left(\frac{e}{b}\right)^{2}\right]$$

$$K_{tor} = G\,b^3\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[1 + \left(1.3 + 1.32\,\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right]$$

Properties of Soil

Unit weight of soil:

$$\gamma = \frac{(G_s + S_r e)}{1 + e} \gamma_w$$

where e is the void ratio, S_r is the degree of saturation and G_s is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_d = \frac{G_s}{1 + e} \gamma_w$$

Effective mean confining stress:

$$p' = \sigma_v' \frac{(1 + 2 K_o)}{3}$$

where σ_v' is the effective vertical stress and K_o is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$K_o = \frac{\nu}{1 - \nu}$$

Effective stress Principle:

$$p' = p - u$$

The shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{max} = 100 \frac{(3 - e)^2}{(1 + e)} (p')^{0.5}$$

where p' is the effective mean confining pressure (in MPa), e is the void ratio and G_{max} is the small-strain shear modulus (in MPa).

Shear modulus correction for strain may be carried out using the following expressions:

$$\frac{G}{G_{max}} = \frac{1}{1 + \gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a\, e^{-b\left(\frac{\gamma}{\gamma_r}\right)}\right]$$

and *a* and *b* are constants depending on soil type. For sandy soil deposits:

$$a = -0.2\ \ln N$$

$$b = 0.16$$

where *N* is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is the reference shear strain given by

$$\gamma_r = \frac{\tau_{max}}{G_{max}}$$

where

$$\tau_{max} = \sqrt{\left[\left(\frac{1 + K_o}{2}\ \sigma_v'\ sin\varphi'\right)^2 - \left(\frac{1 - K_o}{2}\ \sigma_v'\right)^2\right]}$$

Shear modulus is also related to the shear wave velocity ***v_s*** as

$$v_s = \sqrt{\frac{G}{\rho}}$$

where *G* is the shear modulus and ρ is the mass density of the soil.

Wind Engineering

Vortex-induced vibrations

Strouhal Number for flow past a circular cylinder: $S_t = \frac{n_u D}{U}$

where n_u is the vortex-shedding frequency (in Hz), D is the cylinder diameter (in m) and U is the flow velocity (in m/s).

For circular cylinders, $S_t \approx 0.2$

Scruton Number: $S_c = \frac{2\delta_s m}{\rho D^2}$

where m is the actual mass per unit length of the structure, δ_s is the logarithmic decrement of structural damping (= 2π × fraction of critical damping), D is the diameter of the cylinder and ρ is the density of air (≈ 1.25kg/m^3).

A rough estimate of the amplitude y_{max} of vortex-induced vibrations at resonance can be obtained from

$$\frac{y_{max}}{D} = \frac{1.5}{S_c}$$

Classical flutter

The critical wind velocity v_f for classical flutter of a bridge may be estimated from

$$\frac{v_f}{f_T b} = 1.8\left[1 - 1.1\left(\frac{f_B}{f_T}\right)^2\right]^{1/2}\left(\frac{mr}{\rho b^3}\right)^{1/2}$$

where

f_B, f_T are the natural frequencies of the first vertical bending and torsion modes respectively;

b is the deck width;

m is the mass per unit length;

r is the mass radius of gyration of the cross-section;

ρ is the density of air.

FAM, JPT, SPGM

January, 2022