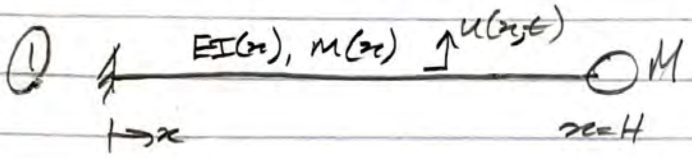
Part IIB 4D6 : Dynamics in Civil Engineering

1)



$$EI(x) = B(x) = B_0\left(1 - \frac{7x}{8H}\right)$$

$$m(x) = m_0\left(1 - \frac{x}{2H}\right)$$

a). Assume $\bar{u}(x) = \left(\frac{x}{H}\right)^2$; $\bar{u}' = \frac{2x}{H^2}$; $\bar{u}'' = \frac{2}{H^2}$

$$M_{eq} = \int_0^H m\bar{u}^2\,dx = \int_0^H m_0\left(1 - \frac{x}{2H}\right)\cdot\frac{x^4}{H^4}\,dx + M\left(\bar{u}(H)\right)^2$$

$$= \frac{m_0}{H^4}\int_0^H x^4 - \frac{x^5}{2H}\,dx + M = \frac{m_0}{H^4}\left[\frac{H^5}{5} - \frac{H^6}{12H}\right] + M = \frac{7m_0H}{60} + M$$

$$K_{eq} = \int_0^H EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx = \int_0^H B_0\left(1 - \frac{7x}{8H}\right)\left(\frac{2}{H^2}\right)^2 dx = \frac{4B_0}{H^4}\left[H - \frac{7H^2}{16H}\right] = \frac{9B_0}{4H^3}$$

b). $H = 60\,m$, $B_0 = 6\times10^{10}\,Nm^2$, $m_0 = 10^4\,kg/m$, $M = 10^5\,kg$

$$\Rightarrow M_{eq} = \frac{7\times10^4\times60}{60} + 10^5 = 1.7\times10^5\,kg$$

$$K_{eq} = \frac{9\times6\times10^{10}}{4\times60^3} = 6.25\times10^5\,N/m \qquad \therefore f_1 = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} = \frac{1}{2\pi}\sqrt{\frac{6.25}{1.7}} = 0.31\,Hz$$

c). $T_1 = \frac{1}{f_1} = \frac{1}{0.31} = 3.2\,s$ $\therefore \frac{t_d}{T_n} = \frac{2}{3.2} \simeq 0.6 \Rightarrow DAF \simeq 1.4$

Point force F applied at $x = x_F = 48\,m$

$\therefore F_{eq} = F\bar{u}(x_F) = 0.5\times10^6\times\left(\frac{48}{60}\right)^2 = 0.32\,MN$

$\therefore u_{max} \simeq 1.4\times\frac{0.32\times10^6}{6.25\times10^5} = 0.72\,m$

d). Increasing K_{eq} would reduce the natural period and hence increase the value of t_d/T_n, assuming the force pulse remains unchanged. This would increase the DAF (according to the DAF curve) and may actually increase the structural stresses. In practice, the force pulse is also likely to change (t_d reduce and magnitude increase, according to conservation of momentum

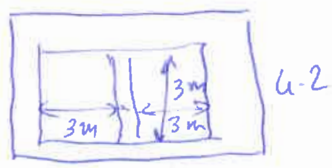
Q2

~~Q3~~ a) Dynamic soil structure interaction occurs when a dynamic load such as earthquake event occurs. The vibration of the structure influences the soil stiffness and the soil stiffness changes the structural vibrations in turn. [10%]

b) Using FEM for 2-phase medium like soil, we need to create two separate meshes, one for solid phase and other for fluid phase, that are overlain on top of one another, ie nodal co-ordinates will be same. The equations of motions are then solved for the two phases iteratively, with a coupling matrix that links the solid and fluid phases. [10%]

c) Consider the tunnel section - 1m into plane of paper.

(i)



$\therefore 2l = 7.8$ $\quad l/b = 7.8$

$2b = 1$ $\quad e/b = 8.4$

$e_s = 4.2$

Shear modulus on reference plane =

Tunnel weight $= (7.8 \times 4.2 - 2 \times 3^2)\, 24 = 354.24$ kN/m.

Bearing pressure $= \frac{354.25}{7.8 \times 1} = 45.41$ kPa.

$\gamma_{dry} = \frac{G_s \gamma_w}{1+e} = \frac{2.65 \times 9.81}{1.85} = 14.05$ kN/m^3.

$\therefore \sigma_v = \sigma_v' = 45.41 + 14.05 \times 1 = 59.46$ kPa. ; $K_0 = \frac{\nu}{1-\nu} = \frac{0.3}{0.7} = 0.428$

$p' = \left(\frac{1+2K_0}{3}\right)\sigma_v' = 0.6191 \times 59.46 = 36.808$ kPa

$G = 100 \frac{[3-e]^2}{1+e}\sqrt{p'} = 100 \frac{[3-0.85]^2}{1.85} \times \sqrt{\frac{36.808}{1000}} = \underline{\underline{47.938 \text{ MPa}}}$

From data sheets; $K_{hx} = \frac{Gb}{2-\nu}\left[6.8 (l/b)^{0.65} + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{1+l/b}\right)(e/b)^{0.8}\right]$

$= \frac{G \times 0.5}{2-0.3} \underbrace{[6.8 \times 7.8^{0.65} + 2.4]}_{16.068} \underbrace{[1 + 0.33 + \frac{1.34}{1+7.8}\ 8.4^{0.8}]}_{2.5479}$

$= G \times 12.0418 = \underline{\underline{1452.262}}$ MN/m

$K_v = \frac{Gb}{1-\nu}\left[3.1\, (l/b)^{0.75} + 1.6\right]\left(1 + \left(0.25 + \frac{0.25}{l/b}\right)(e/b)^{0.8}\right)$

$= \frac{G \times 0.5}{2-0.3} \underbrace{(3.1 \times 7.8^{0.75} + 1.6)}_{16.0683} \underbrace{\left(1 + \left(0.25 + \frac{0.25}{7.8}\right) 8.4^{0.8}\right)}_{2.54} = G \times 12.0418$

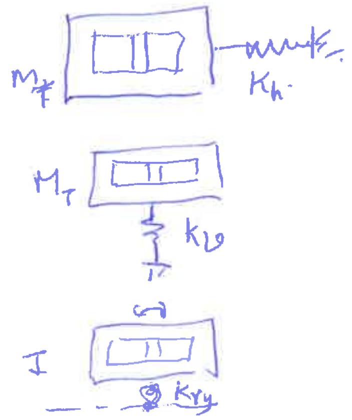
$= \underline{\underline{577.26 \text{ MN/m}}}$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left(3.73\, e/b^{2.4} + 0.27\right)\left(1 + e/b + \frac{1.6}{0.35 + e/b^4}(e/b)^2\right)$$

$$= \frac{G \times 0.5^3}{1-0.3} \times \underbrace{\left(3.73 \times 7.8^4 + 0.27\right)}_{516.3724} \underbrace{\left(1 + 8.4 + \frac{1.6}{0.35 + 7.8^4} \times 8.4^2\right)}_{9.4305}$$

$$K_{ry} = G \times 869.58 = \underline{41685.93} \text{ MN-m/rad}$$ [25%]

C ii) ~~When the rains come and water table rises to soil surface~~
Natural frequence – Small earthquake $G_{max} = G$.



Mass of tunnel $= 354.24/9.81 = 36110.1$ kg/m

$I_{tunnel} = Ma^2 = 36110.1 \times \left[1 + \frac{4.2}{2}\right]^2 = 347018.1$ kg M^2/m

$$\therefore f_{n\text{-}h} = \frac{1}{2\pi}\sqrt{\frac{K_h}{M_T}} = \frac{1}{2\pi}\sqrt{\frac{1452.26 \times 10^6}{36110.1}} = \underline{31.9} \text{ Hz}$$

$$f_{n\text{-}v\theta} = \frac{1}{2\pi}\sqrt{\frac{K_{v\theta}}{M_T}} = \frac{1}{2\pi}\sqrt{\frac{577.26 \times 10^6}{36110.1}} = \underline{20.1} \text{ Hz}$$

$$f_{n\text{-}rock} = \frac{1}{2\pi}\sqrt{\frac{K_r}{I}} = \frac{1}{2\pi}\sqrt{\frac{41685.93 \times 10^6}{347018.1}} = \underline{55.16} \text{ Hz}$$

[20%]

C iii) When rains come and water table rises to surface – tunnel is buoyant

Weight of tunnel $= 354.24$ kN/m

Buoyancy force $= 7.8 \times 4.2 \times 9.81 = 321.38$

$\therefore$ Net weight of tunnel $= 32.8 \Rightarrow$ Bearing pressure $= \frac{32.8}{7.8 \times 1} = \underline{4.21}$ kPa

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e} = \left(\frac{2.65 + 0.85}{1.85}\right) 9.81 = 18.56 \text{ kN/m}^3.$$

$$\gamma' = \gamma_{sat} - \gamma_w = 8.75 \text{ kN/m}^3.$$

$\therefore$ Effect vertical stress on ref plane $= \sigma_v' = 4.21$ kPa $+ 8.75 \times 1 = 12.96$ kPa. $p' = 0.619 \times 12.96 = 8.02$ kPa

$$\therefore G_{max} = 100\, \frac{[3 - 0.85]^2}{1.85} \sqrt{\frac{8}{1000}} = 22.38 \text{ MPa}$$

$\therefore K_h = 30.2946 \times 22.38 = 677.99$ MN/m

$K_{v\theta} = 12.0418 \times 22.38 = 269.49$ MN/m

$K_{rock} = 869.58 \times 22.38 = 19461.2$ MN-m/rad

$\therefore$ New natural frequencies are:

$$f_{n-h} = \frac{1}{2\pi}\sqrt{\frac{677.99\times10^6}{36110.1}} = \underline{21.8} \text{ Hz}$$

$$f_{n-\theta} = \frac{1}{2\pi}\sqrt{\frac{269.49\times10^6}{36110.1}} = \underline{13.75} \text{ Hz}$$

$$f_{n-rock} = \frac{1}{2\pi}\sqrt{\frac{19461.2\times10^6}{347018.1}} = \underline{37.7} \text{ Hz}$$

All natural frequencies have reduced, but away from 1-5 Hz of earthquake frequency range. [25%]

(iv) During a strong earthquake, if the sandy soil liquefies, the tunnel is under serious risk of floation! Not sinking in/settlement. As the unit weight of liquefied soil $\simeq$ 20 kN/m³, the tunnel will float upwards. [10%]

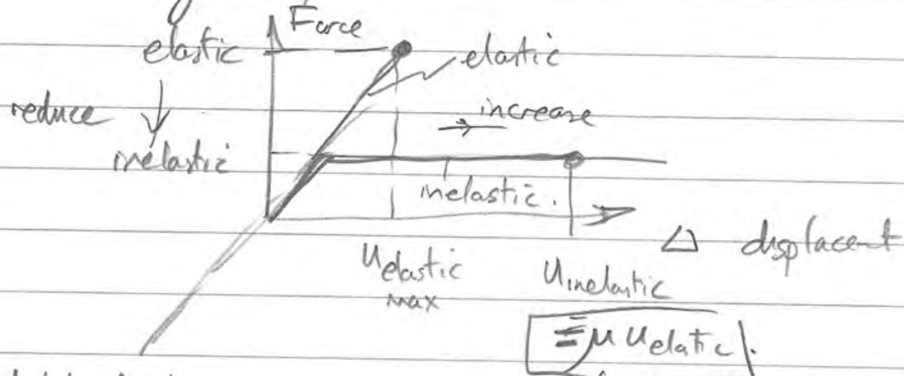
4D6 (Q3.a)
2025

a) i) Modal Participation Factor. Marks will be awarded for clear explanations of it measures the ease with which a mode may be excited by a ground acceleration. For example, purely vertical modes cannot be excited by purely horizontal ground motions. To be excited, the centre of the modal mass needs to move in the excitation direction.

ii) An inelastic response spectrum is a graph giving a measure of how much lower the inelastic response (of a structure to an earthquake) is compared to the elastic response. Increased ductility acts somewhat like increased damping, and so gives a reduced strength requirement.

The "reduction" is actually in the forces (the strength). The displacements actually increase for the inelastic structure, and the required increase in displacement is given by μ.

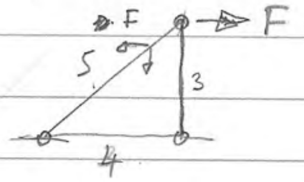
Marks will be given for a sketch

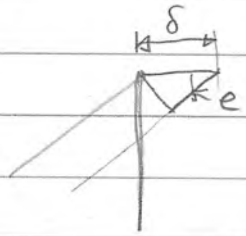


The ductility factor $\mu = \dfrac{\text{required (inelastic) displacement cap.}}{\text{required (elastic) displacement cap.}}$

4D6 Q3 b)

2025 i) Structure lateral behaviour $\simeq$





$L = 5$.

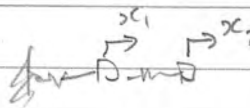
$$e = \text{"}\frac{FL}{EA}\text{"} = \frac{5}{4}\frac{FL}{EA}$$

$$\delta = \frac{5}{4}e = \frac{25}{16}\frac{FL}{EA}$$

"$F = K\delta$" $\therefore$ $K = \frac{16\,EA}{25L} = \frac{16(210\times10^9\,N/m^2)(120\times10^{-6}\,m^2)}{25(5)\,m}$

$$= \frac{16(210\times10^3)(120)}{25(5)} \quad [N/m]$$

$$= \underline{3.226\times10^6\ N/m}$$



$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

for modes.

$$\left[\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\Omega^2 \quad - \omega^2 I\right]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Omega^2 = K/M$

$$\begin{vmatrix} 2-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix} = 0 \qquad \lambda = \frac{\omega^2}{\Omega^2}$$

$$= \frac{3.226\times10^6}{40\times10^3}$$

$\Omega^2 = 80.6$

$\Omega = \cancel{8.98}$ 9 rads/s

$f_{nat} = \frac{9}{2\pi} = 1.429\,Hz$.

$(2-\lambda)(1-\lambda) - 1 = 0$

$2 - \lambda - 2\lambda + \lambda^2 - 1 = 0$

$\lambda^2 - 3\lambda + 1 = 0$

$$\lambda = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3\pm\sqrt{5}}{2} = \underline{0.382} = \frac{3-\sqrt{5}}{2}$$

$\omega^2 = \lambda\Omega^2$ " $= 0.382(80.6) = 30.79 \implies \omega = 5.54$ rads/sec

$f = 0.8831\,Hz$ Nat freq.

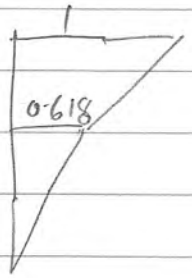
Q3 (c) Need mode shape.

$$(2-\lambda)x_1 - x_2 = 0$$

$$x_2 = (2-\lambda)x_1$$

$$\frac{x_2}{x_1} = 2-\frac{(3-\sqrt{5})}{2}x_1 = \left(\frac{4-3+\sqrt{5}}{2}\right)^{*}_{x_1} = \left(\frac{1+\sqrt{5}}{2}\right)x_1$$

$$x_1 = \frac{2}{1+\sqrt{5}}x_2 = \underline{\underline{0.618}}\ x_2$$



Mode shape.

GIVE THIS TO STUDENTS.

ii) Modal participation factor.

$$M_{eq} = 40\times10^3\left[0.618^2+1^2\right] = 55.3\times10^3\ \underline{\underline{kg}}.$$

$$\Gamma = \frac{\Sigma m\phi}{\Sigma m\phi^2} = \frac{0.618+1}{0.618^2+1^2} = \frac{1.618}{1.382} = \boxed{\underline{\underline{1.17}}}$$

[check Frequency: $K = (\phi_1\ \phi_2)\begin{bmatrix}2 & -1\\ -1 & 1\end{bmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} k$

[students DO NOT NEED TO DO THIS]

$$= \begin{bmatrix}2\phi_1-\phi_2\\ -\phi_1+\phi_2\end{bmatrix}\begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix} = \left[2\phi_1^2-\phi_1\phi_2-\phi_1\phi_2+\phi_2^2\right]k$$

$$= 2\phi_1^2-2\phi_1\phi_2+\phi_2^2 \quad \times k$$

$$= 2(0.618)^2-2(0.618)+1$$

$$= (0.5278)\,k$$

$$= (0.5278)(3.226\times10^6\ N/m)$$

$$\omega = \sqrt{\frac{0.5278(3.226\times10^6)}{55.3\times10^3}} = 5.55\ rads/s = 0.883\,Hz.$$ ✓

Q3 ii)

$$\text{Nat Period} = \frac{1}{f} = \frac{1}{0.883\,Hz} = 1.13 \text{ seconds}$$

so $S_a = \dfrac{1.5}{T}$ from Fig 4

so $S_a = \dfrac{1.5}{1.13} = 1.32$

Peak Ground Accel. $= 0.25g$.

$$\text{Peak Modal Acceleration} = \Gamma S_a (PGA)$$
$$= (1.17)(1.32)(0.25g)$$
$$= 0.387\,g$$
$$= \underline{\underline{3.79\ m/s^2}}$$

$$\text{Peak Modal Displacement} = \frac{(3.79\ m/s^2)}{\omega^2}$$

$d = X \sin \omega t$

$A = \omega^2 X \sin \omega t$

$$= \frac{3.79\ m/s^2}{30.79\ /s^2} = \underline{\underline{0.123\ m}}$$

0.123 m

$0.618(0.123) = \underline{\underline{0.076\,m}}$ ← max interstorey drift.

iii) Force in bracing, if elastic

$$\text{Horiz force} = K\delta$$
$$= [3.226 \times 10^6\ N/m][0.076\ m]$$
$$\approx \underline{\underline{245\ kN}}$$

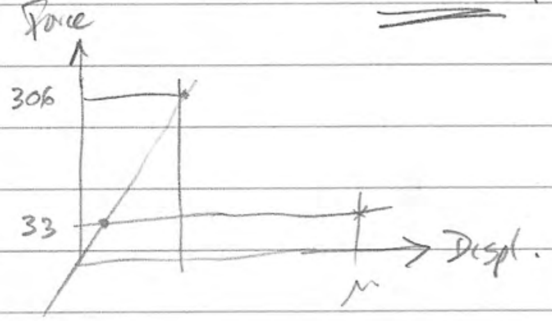
$$\text{Brace force} = \frac{5}{4} \times (245) = \underline{\underline{306.5\ kN}}$$

Q3iii) Strength of brace

Strength = $120\,mm^2 \times 275\ N/mm^2$

$= 275 \times 120\ N$

$= \underline{\underline{33\ kN}}$! But 306 kN needed elastically



$= \dfrac{33}{306} = 0.1078$

Inelastic spectrum. at $T = 1.13$ seconds

If elastic displacement = 21 inches. from Fig 5.

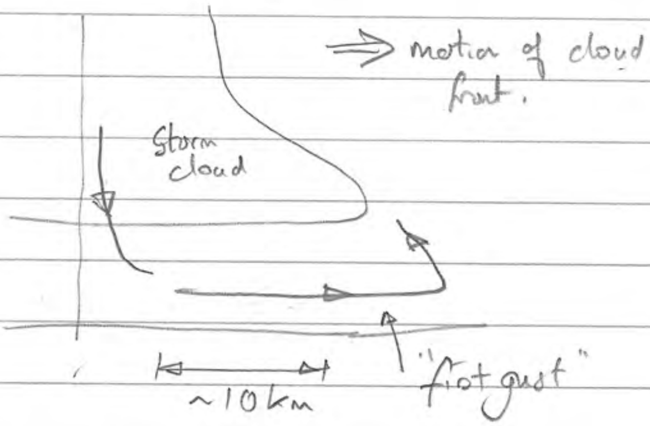
need $(0.1078 \times 21 \text{ inches}) = 2.26$ inches.

$\Rightarrow \underline{\underline{\mu = 8}}$ at. $T = 1.13$ secs. (Fig 5)

Required.

4D6 Q4 a)
2025.

Tropical Thunderstorms

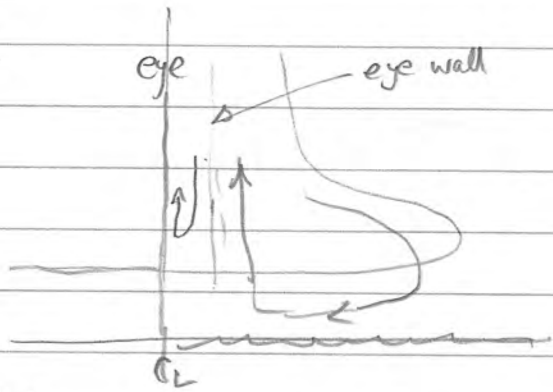


High velocity down draft inside cloud (exacerbated by weight of falling raindrops) then blows outwards, creating phenomenon of "first gust".

This is a long-lasting blast of air at high ~~..~~, near-constant, velocity (ie. not much turbulence).

So buffetting theory may not apply

Tropical cyclones.



Form over oceans.

Solar heating → moist air rises → cools → releases latent heat of evap.

Very high wind speeds, blowing inwards (and around due to Coriolis)

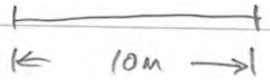
Much larger. Much longer lasting. (~ hours to pass)

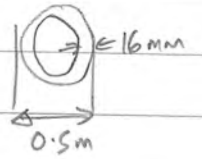
Turbulent Buffetting is a danger (→ dynamics, as well as static).

4D6 Q4 2025.

b)

i)





$\zeta = 0.006$ of critical.

$$I = \pi r^3 t \qquad r = 0.25 - 0.008 = 0.242$$

$$= \pi (0.242)^3 (0.016) = 712.4 \times 10^{-6}\ m^4$$

$$EI = 210 \times 10^9 (712.4 \times 10^{-6}) = \underline{149.6 \times 10^6\ Nm^2}$$

mass per unit length $m = A\rho = (2\pi r t)(7850\ kg/m^3)$

$$= [2\pi (0.242)(0.016)][7850]$$

$$= \underline{191\ kg/m}$$

Assume half sine wave. Effective mass $= \int_0^L \phi^2 m\, dx = \frac{1}{2} m (10)$

$$= 5(191) = \underline{955\ kg}$$

Effective stiffness $= \int_0^L EI(\phi'')^2\, dx = \left(\frac{\pi}{L^2}\right)^2 (EI) \frac{L}{2}$

$\phi = \sin\left(\frac{\pi x}{L}\right)$

$$= \frac{\pi^4}{2L^3} EI = 0.0487\ EI$$

$$= \underline{7.29 \times 10^6\ N/m} = 7290 \times 10^3\ N/m.$$

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{7.29 \times 10^6\ N/m}{955\ kg}} = 87.3 \text{ radians/s} \qquad \sqrt{\frac{N/m}{kg}} = \sqrt{\frac{kg\,m\,s^{-2}/m}{kg}} = s^{-1}$$

$$f = \frac{87.3}{2\pi} = \underline{13.9\ Hz}$$

ii) Critical Strouhal $St = \frac{f_v H_D}{U}$

$$U_{crit} = \frac{f_v H_D}{St} = \frac{(13.9)(0.5)}{0.2} = 34.75\ m/s.$$

4D6 Q4 (2015) b) cont'd.

iii) Scruton Number: $S_c = \frac{2 \delta_s m}{\rho H_D^2} = \frac{2(0.006)(191)}{1.2(0.5)^2} = 48$

2π

(large ∴ VIV problematic)

iv) $\frac{h_{viv}}{H_D} = \frac{1}{4\pi} \frac{1}{St^2} \frac{1}{S_c} \sqrt{2}\, C_{lat}$

$Re = \frac{\rho u L}{\mu} = \frac{(1.2)(34.75)(0.5)}{3.178 \times 10^{-5} \text{ kg m}^{-1}\text{s}^{-1}}$

$= 6.5 \times 10^5$

so $C_{lat,0} = 0.2$

$\frac{h_{viv}}{H_D} = \frac{1}{4\pi} \frac{1}{(0.2^2)} \frac{1}{(48)} \sqrt{2}\,(0.2)$

$= 0.0117$

so $h_{viv} = (0.0117)(0.5) = 0.006 = \underline{\underline{6\text{mm}}}$

v) Amplitude of dynamic stresses.

Mode shape: $\phi = 6\text{mm} \times \sin\left(\frac{\pi x}{L}\right)$

At centre $K = \phi''$ at centre $= \frac{\pi^2}{L^2}(6\text{mm}) = \frac{\pi^2 (0.006)}{(10)^2}$ $\text{m}^{-1} = 0.0006\ \text{m}^{-1}$

$M = EIK = (149.6 \times 10^6)(0.0006)$

$= 88.6$ kNm

$\sigma = \frac{My}{I} = \frac{(88.6 \times 10^3)(0.25\text{m})}{712.4 \times 10^{-6}\text{ m}^4} = 31.1 \times 10^6 \text{ N/m}^2$

$= \underline{\underline{31.1 \text{ MPa}}}$