Version FAM/2

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 6 May 2025 14.00 to 15.40

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachment: 4D6 Data Sheet (8 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 The substation for an offshore windfarm is supported at a height H above the seabed by the structure shown in Fig. 1(a). To estimate its bending response, the structure is modelled as a non-uniform cantilever beam (see Fig. 1(b)) whose bending stiffness $B(x)$ decreases linearly with height x above the seabed, being B_0 at the seabed and $B_0/8$ at height H. Similarly, the effective mass $m(x)$ per unit height of the support tower is assumed to decrease linearly from m_0 at the seabed to $m_0/2$ at the top. The substation is modelled as a point mass M located at the top of the cantilever.

(a) Using Rayleigh's principle and assuming a parabolic form for the fundamental mode shape, find expressions for the equivalent mass and stiffness of the substation mounted on the jacket structure. Assume the mass of the entrained water is included in the effective mass of the structure. [45%]

(b) If H = 60 m, $B_0 = 6 \times 10^{10}$ Nm^2, $m_0 = 1 \times 10^4$ kg m^{-1} and $M = 1 \times 10^5$ kg, estimate the natural frequency of the fundamental sway mode. [15%]

(c) A ship collides with the tower. The resulting impact load is idealised as a concentrated horizontal force at height x = 48 m, with the triangular force pulse shown in Fig. 1(c). Estimate the peak displacement of the substation, assuming the response is dominated by the fundamental sway mode. [25%]

(d) To mitigate the effects of future ship impacts, it is proposed that additional bracing is added to the support tower. Comment on the potential for success of this proposal. [15%]

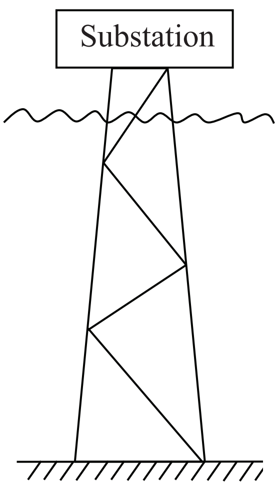


Fig. 1(a)

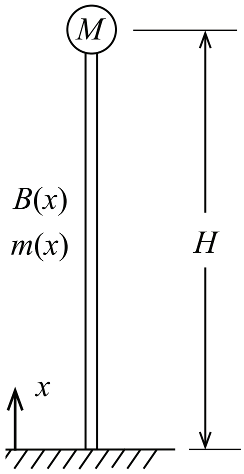


Fig. 1(b)

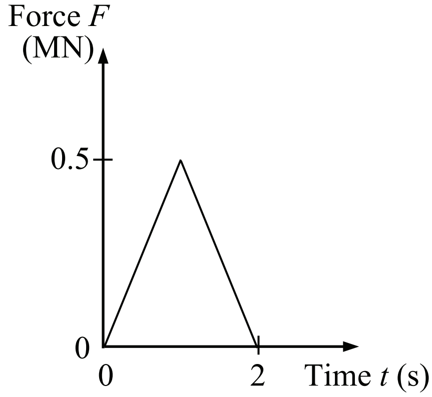


Fig. 1(c)

2 (a) Explain what is meant by dynamic soil-structure interaction. [10%]

(b) How do you use the finite element method to model a two-phase material like saturated sand? [10%]

(c) A tunnel is constructed using the cut-and-cover method in a dry sandy soil layer overlying bedrock. The cross-section of the tunnel is shown in Fig. 2. The slab and wall thicknesses are all 0.6 m. The unit weight of concrete is 24 kN m^{-3}. The sandy soil has a void ratio of 0.85, a Poisson's ratio of 0.3 and the sand particles have a specific gravity of 2.65. The reference plane can be taken as 1 m below the base slab. Consider a tunnel section of 1 m into the plane of the paper.

(i) Calculate the horizontal, vertical and rocking stiffnesses of the tunnel section. You may assume that the tunnel itself is rigid. [25%]

(ii) Suggest simple discrete models for the tunnel structure and estimate the natural frequency for the horizontal, vertical and rocking modes, assuming small amplitude motions. [20%]

(iii) Following heavy rains, the water table rises to the ground surface. Calculate new values for the natural frequencies estimated in part (c) ii). [25%]

(iv) What other risks does the tunnel face if an earthquake causes the sand layer to liquefy? [10%]

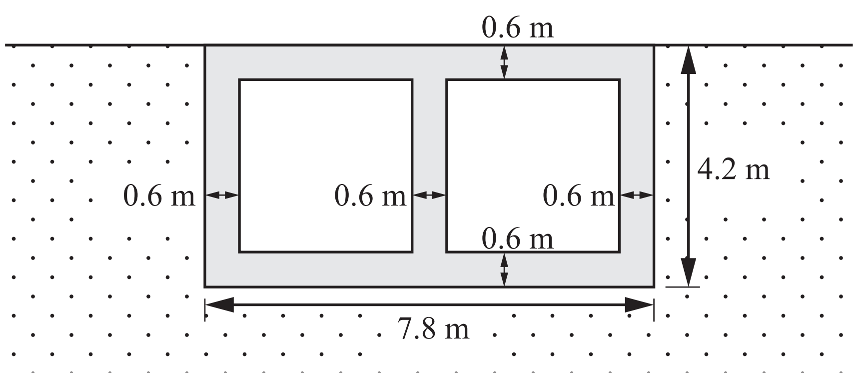


Fig. 2

3 (a) In the context of the design of buildings in earthquake engineering, explain what is meant by:

(i) modal participation factor; [10%]

(ii) inelastic response spectrum. (Include a definition of the ductility factor, μ). [20%]

(b) One bay of a two-storey building is idealised by the structure drawn in Fig. 3. Assume that the floors are rigid and that each floor has a mass of 40 tonnes. All columns and braces are pin-ended, and are made of steel with 275 MPa yield stress. Each brace has a cross-sectional area of 120 mm^2. Each brace is effective in tension only, and columns may be assumed to be axially-rigid.

(i) Estimate the natural frequencies of the structure for lateral sway vibrations. [20%]

(ii) Consider only the first mode, and assume it has a mode shape $\phi = (0.618, 1.0)$. Using the elastic design spectrum of Fig. 4, estimate the maximum interstorey drift for an event with a Peak Ground Acceleration of 0.25g. [25%]

(iii) Using the inelastic design spectrum of Fig. 5, estimate the required ductility factor if yield of the braces governs. [25%]

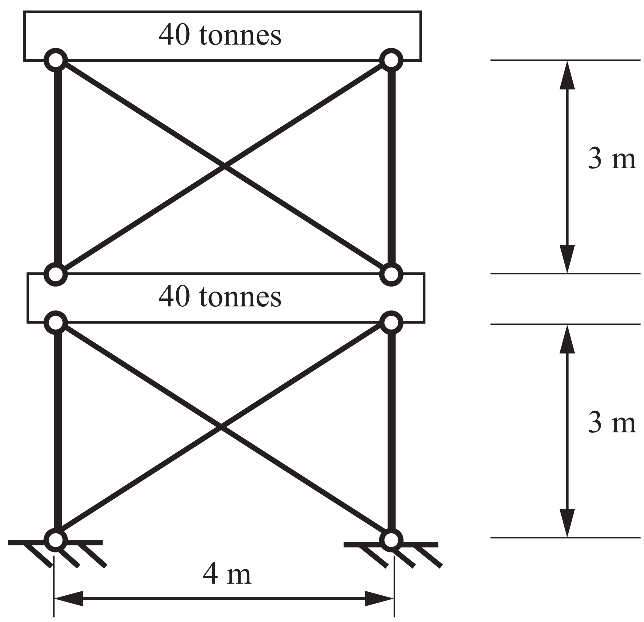
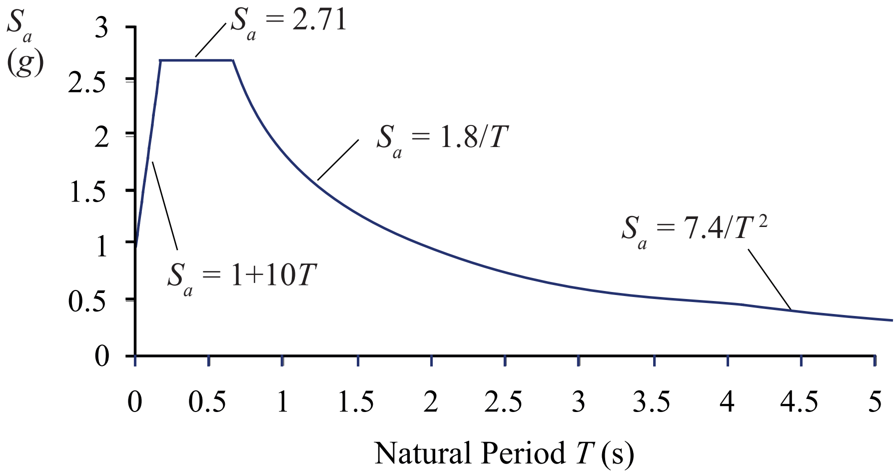
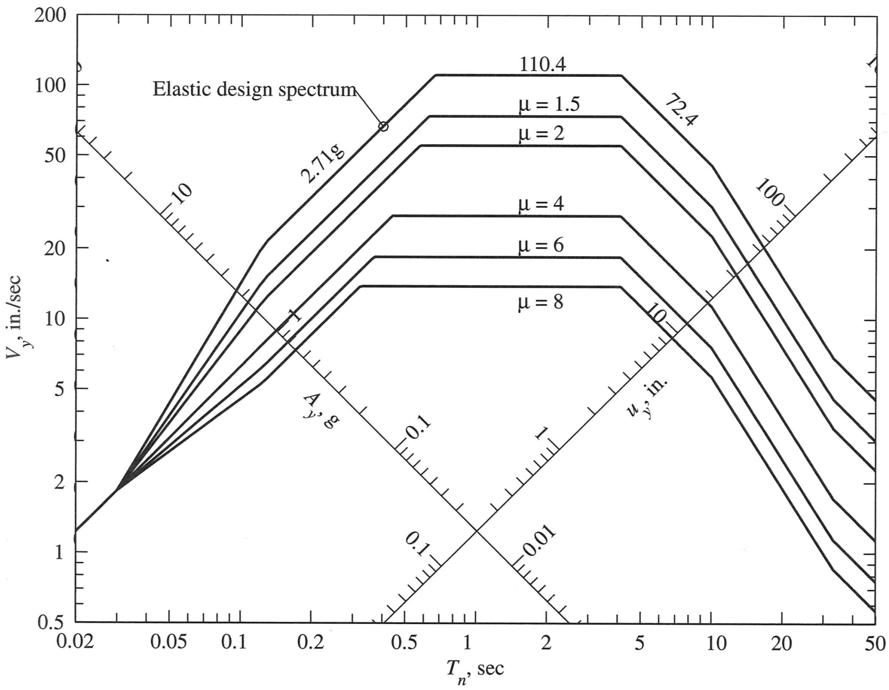


Fig. 3



S_a (g)
$S_a = 2.71$
$S_a = 1.8/T$
$S_a = 7.4/T^2$
$S_a = 1+10T$
Natural Period T (s)

Fig. 4



Elastic design spectrum
110.4
72.4
2.71g
μ = 1.5
μ = 2
μ = 4
μ = 6
μ = 8
V_y, in./sec
A_y, g
u_y, in.
T_n, sec

Fig. 5

4 (a) Briefly describe the nature of the extreme winds generated by tropical thunderstorms and tropical cyclones. [20%]

(b) A pipe bridge consists of a simply-supported straight steel pipe of length 10 m. It has a circular cross-section of external diameter 0.5 m and wall thickness 16 mm. The damping ratio of the fundamental bending mode is 0.6% of critical.

(The second moment of area of a thin-walled circular cross-section of radius r and wall thickness t is $\pi r^3 t$ about any diameter.)

Assuming the pipe is empty, estimate:

(i) the natural frequency of the fundamental bending mode; [20%]

(ii) the critical wind velocity for vortex-induced vibrations; [20%]

(iii) the amplitude of the resulting oscillatory displacements; [20%]

(iv) the amplitude of the dynamic stresses induced. [20%]

END OF PAPER

Engineering Tripos Part IIB/EIST Part II FOURTH YEAR

Module 4D6: Dynamics in Civil Engineering

Data Sheets

Equivalent SDOF Systems

For an *n*-DOF system, with mass matrix $\underline{\underline{M}}$ and stiffness matrix $\underline{\underline{K}}$, responding in mode shape $\underline{\overline{u}}$ to an applied force $\underline{F}$, the parameters of the equivalent SDOF system are:

$$M_{eq} = \underline{\overline{u}}^T \underline{\underline{M}}\, \underline{\overline{u}} \qquad K_{eq} = \underline{\overline{u}}^T \underline{\underline{K}}\, \underline{\overline{u}} \qquad F_{eq} = \underline{F}^T \underline{\overline{u}}$$

For a continuous beam, of length L, mass per unit length m and bending stiffness EI, responding in mode shape $\overline{u}(x)$ to an applied force $f(x)$, the parameters of the equivalent SDOF system are:

$$M_{eq} = \int_0^L m\overline{u}^2 dx \qquad K_{eq} = \int_0^L EI \left(\frac{d^2\overline{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\overline{u}\, dx$$

The corresponding natural frequency is $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}}$

Modal Analysis of a Simply-Supported Beam

$$\overline{u}_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\,eq} = \frac{mL}{2} \qquad K_{i\,eq} = \frac{(i\pi)^4 EI}{2L^3}$$

Ground Motion Participation Factor

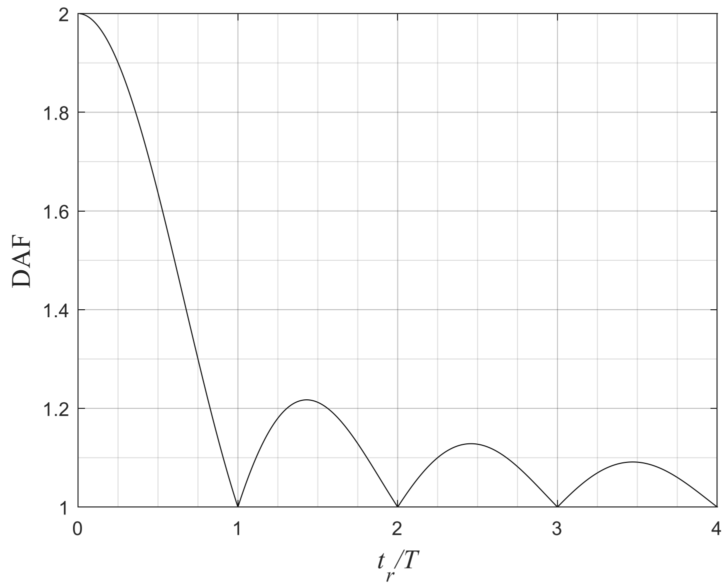
For an *n*-DOF system, with mass matrix $\underline{\underline{M}}$ and stiffness matrix $\underline{\underline{K}}$, responding to ground acceleration $\ddot{u}_g$, the parameters of the equivalent SDOF system are:

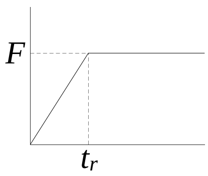
$$M_{eq} = \underline{\overline{u}}^T \underline{\underline{M}}\, \underline{\overline{u}} \qquad K_{eq} = \underline{\overline{u}}^T \underline{\underline{K}}\, \underline{\overline{u}} \qquad F_{eq} = -\Gamma M_{eq}\ddot{u}_g$$

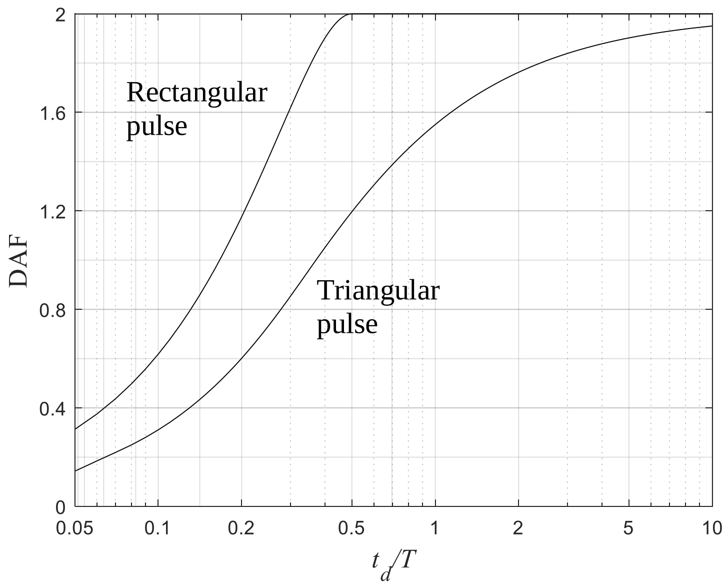
where $\underline{\overline{u}}$ is defined relative to ground and Γ is the modal participation factor

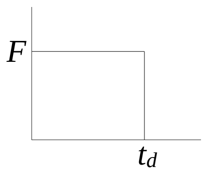
$$\Gamma = \frac{M_1\overline{u}_1 + M_2\overline{u}_2 + \cdots + M_n\overline{u}_n}{M_{eq}}$$

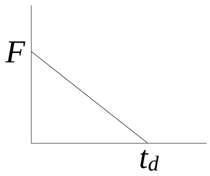
Dynamic Amplification Factors

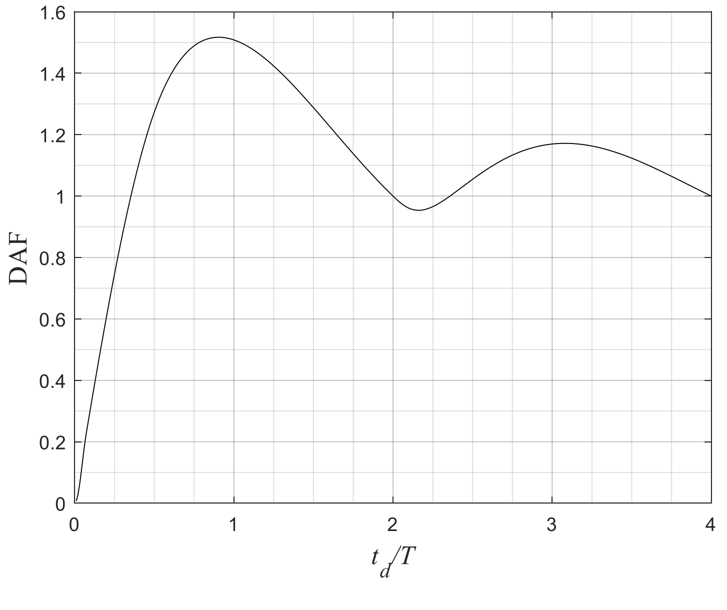


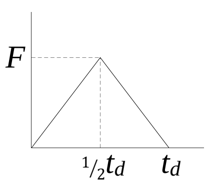




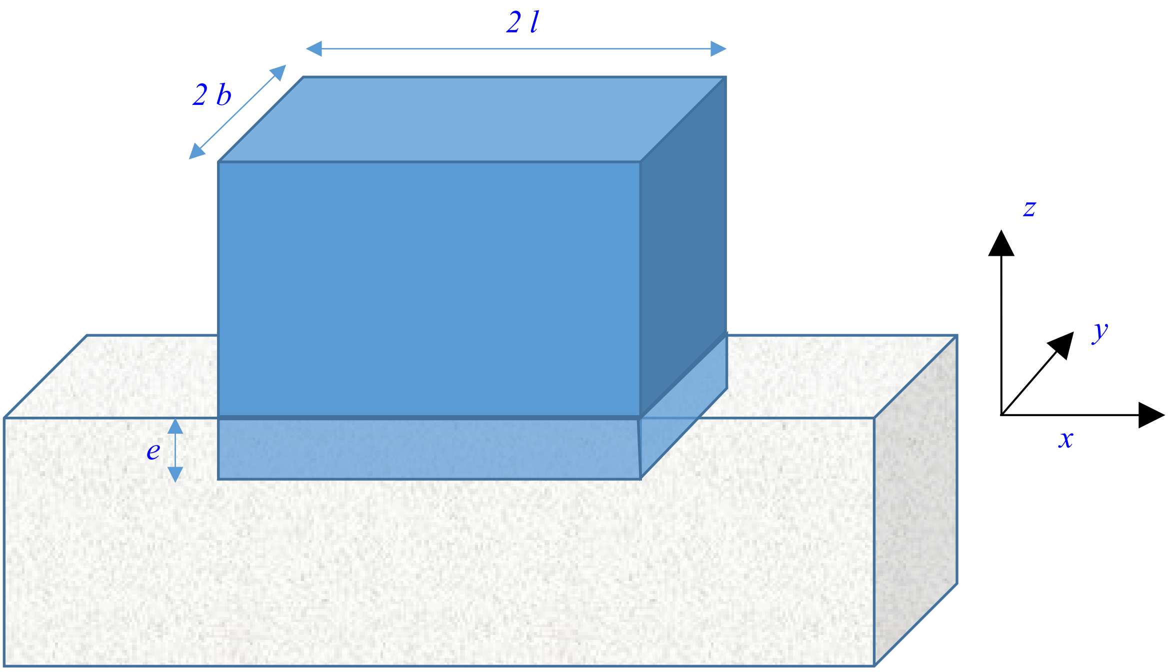








Approximate relations for evaluating soil stiffness for an embedded prismatic structure of dimensions *2l* and *2b*, embedded to a depth *e* are:



$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\,\frac{l}{b} + 1.6\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{v} = \frac{Gb}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left(0.25 + \frac{0.25b}{l}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{Gb^3}{1-\nu}\left[3.2\,\frac{l}{b} + 0.8\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35 + \frac{l}{b}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35 + \left(\frac{l}{b}\right)^{4}}\left(\frac{e}{b}\right)^{2}\right)\right]$$

$$K_{tor} = Gb^3\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[\left(1 + \left(1.3 + 1.32\,\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right)\right]$$

Unit weight of soil:

$$\gamma = \frac{(G_s + eS_r)\gamma_w}{1+e}$$

where e is the void ratio, S_r is the degree of saturation, G_s is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_d = \frac{G_s\gamma_w}{1+e}$$

Effective mean confining stress

$$p' = \sigma'_v \frac{(1+2K_o)}{3}$$

where σ'_v is the effective vertical stress, K_o is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$K_o = \frac{\nu}{1-\nu}$$

Effective stress Principle:

$$p' = p - u$$

Shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{max} = 100\,\frac{(3 - e)^2}{(1 + e)}\,(p')^{0.5}$$

where p′ is the effective mean confining pressure in **MPa,** e is the void ratio and G_{max} is the small strain shear modulus in **MPa**

Shear modulus correction for strain may be carried out using the following expressions;

$$\frac{G}{G_{max}} = \frac{1}{1 + \gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a.\, e^{-b\left(\frac{\gamma}{\gamma_r}\right)}\right]$$

'a' and 'b' are constants depending on soil type; for sandy soil deposits we can take

$$a = -0.2 \ln N$$

$$b = 0.16$$

where N is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is reference shear strain given by

$$\gamma_r = \frac{\tau_{\max}}{G_{\max}}$$

where

$$\tau_{\max} = \left[\left(\frac{1 + K_o}{2}\sigma'_v \sin\phi'\right)^2 - \left(\frac{1 - K_o}{2}\sigma'_v\right)^2\right]^{0.5}$$

Shear Modulus is also related to the shear wave velocity $\boldsymbol{v_s}$ as follows;

$$v_s = \sqrt{\frac{G}{\rho}}$$

where G is the shear modulus and ρ is the mass density of the soil.

Wind Engineering Data Sheet

1. *Extreme value statistics*

Empirical distribution:

$$P(X \leq U_i) = \frac{n_i}{N+1}$$

where: n_i – ordered sample i; N – number of samples; U_i –wind speed of sample i (samples here are maximum annual mean wind speeds).

Gumbel distribution:

$$P(X \leq U) = \exp\left(-\exp(-y)\right)$$

$$y = a(m - U)$$

where: a, m – distribution parameters; U – mean wind speed.

Return period:

$$P(X \leq U) = 1 - \frac{1}{\mathrm{T}}$$

where: $T -$ return period.

2. *Longitudinal wind buffeting response*

Turbulence intensity:

$$I_u = \frac{\sigma_u}{U}$$

where: σ_u – standard deviation of longitudinal turbulence; U – mean wind speed.

Mean force:

$$F_D = \frac{1}{2}\rho U^2 C_D A$$

where: ρ – fluid density ($\approx 1.2\ \mathrm{kg/m^3}$); C_d – drag coefficient; A – representative dimension (for bridges typically deck width B) .

Peak response:

$$x_p = x_m + q_p \sigma_x$$

where: x_p – peak response; x_m – mean response; q_p – peak factor; σ_x – fluctuating response.

Fluctuating response:

$$\sigma_x^2 = 4x_m^2 \frac{\sigma_u^2}{U^2}(B_g{}^2 + R^2)$$

where: B_g − background response; R − resonant response.

Background response:

$$B_g^2 = \frac{1}{2\pi}\int_0^{\infty} \chi_D^2(\omega)\frac{S_{uu}(\omega)}{\sigma_u^2}\mathrm{d}\omega$$

where: χ_D^2 − drag admittance due to longitudinal fluctuations; S_{uu} – longitudinal wind spectrum.

Resonant response:

$$R^2 = \chi_D^2(\omega_n)\frac{S_{uu}(\omega_n)}{\sigma_u^2}\frac{\omega_n}{8\xi}$$

where: ω_n − natural frequency; ξ – damping ratio.

Aerodynamic damping:

$$\xi_a = \frac{\rho U C_D B}{2m\omega_n}$$

where: m − mass per metre.

3. *Bridge flutter*

Coupled equation of motion for a 2 dimensional system (h - vertical; α – torsional):

$$m_h\ddot{h} + c_h\dot{h} + k_h h = \frac{1}{2}\rho U^2 B\left(KH_1^*\frac{\dot{h}}{U} + KH_2^*\frac{B\dot{\alpha}}{U} + K^2H_3^*\alpha + KH_4^*\frac{h}{B}\right),$$

$$m_\alpha\ddot{\alpha} + c_\alpha\dot{\alpha} + k_\alpha \alpha = \frac{1}{2}\rho U^2 B^2\left(KA_1^*\frac{\dot{h}}{U} + KA_2^*\frac{B\dot{\alpha}}{U} + K^2A_3^*\alpha + KA_4^*\frac{h}{B}\right)$$

where: m, c, k – mass, damping coefficient, and stiffness of the corresponding degree of freedom; ρ − fluid density; B − deck width; K − reduced frequency; H_i^*, A_i^* − aerodynamic derivatives.

Reduced velocity:

$$V_r = \frac{2\pi U}{\omega B} = \frac{2\pi}{K}$$

where:

ω = oscillating frequency (rads/s)

4. Vortex-induced vibrations

Strouhal number:

$$\mathrm{St} = \frac{f_v H_D}{U}$$

where: f_v – vortex shedding frequency (Hz); H_D – referent dimension (depth for bridge decks; diameter for cylinder). U – flow velocity.

For circular cylinders $\mathrm{St} \approx 0.2$.

Scruton number:

$$\mathrm{Sc} = \frac{2\delta_s m}{\rho H_D^2}$$

where: δ_s – logarithmic decrement (=2π × fraction of critical damping);
m – actual mass per metre;
H_D – reference dimension (depth for bridge decks; diameter for cylinder).
U – flow velocity.
air density (= 1.2 kg/m^3)

Peak response for vortex-induced vibrations:

$$\frac{h_{viv}}{H_D} = \frac{1}{4\pi}\frac{1}{\mathrm{St}^2}\frac{1}{\mathrm{Sc}}\sqrt{2}\ c_{lat}$$

where: c_{lat} – lateral force coefficient.

For rectangular cross sections $c_{lat} \approx 1.1$.

For circular cross-sections at typical civil engineering Reynolds Numbers, $c_{lat} \approx 0.2$
(Eurocode 1 Appendix E has more information)