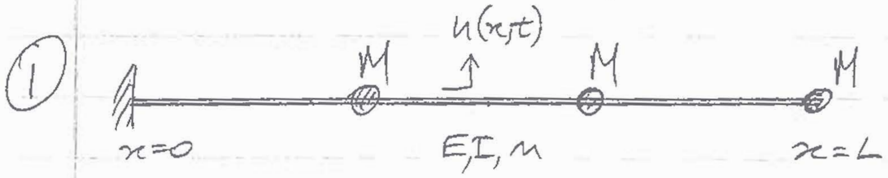
Post exam 2021

Part IIB 4D6: Dynamics in Civil Engineering

1.



← Z in the question!

a). Assume $\bar{u}(x) = \left(\frac{x}{L}\right)^2$. Given $m(x) = m_0\left(1-\frac{x}{L}\right)$ and $B(x) = B_0\left(1-\frac{x}{L}\right)$

$$M_{eq} = \int_0^L m\bar{u}^2\,dx = \int_0^L m_0\left(1-\frac{x}{L}\right)\cdot\frac{x^4}{L^4}\,dx + \sum_{j=1}^{3} M\left(\bar{u}\left(x=\frac{jL}{3}\right)\right)^2$$

$$= \frac{m_0}{L^4}\int_0^L x^4 - \frac{x^5}{L}\,dx + M\left(\left(\frac{1}{9}\right)^2 + \left(\frac{4}{9}\right)^2 + 1\right)$$

$$= \frac{m_0}{L^4}\left(\frac{L^5}{5} - \frac{L^5}{6}\right) + \frac{98M}{81} = \frac{m_0 L}{30} + \frac{98M}{81}$$

$$K_{eq} = \int_0^L EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx = \int_0^L B_0\left(1-\frac{x}{L}\right)\left(\frac{2}{L^2}\right)^2 dx = \frac{4B_0}{L^4}\left(L-\frac{L}{2}\right) = \frac{2B_0}{L^3}$$

b). $m_0 = 2.5\times10^6$ kg/m, $M = 11\times10^3$ $\Rightarrow$ $M_{eq} = \frac{2.5\times10^6\times240}{30} + \frac{98\times11\times10^3}{81}$

$L = 240$ m

$B_0 = 5\times10^{14}$ Nm^2

$$= 20.0\times10^6 \text{ kg}$$

$$K_{eq} = \frac{2\times5\times10^{14}}{240^3} = 72.3\times10^6 \text{ N/m}$$

$$\therefore f_1 = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} = \frac{1}{2\pi}\sqrt{\frac{72.3}{20.0}} = 0.30\text{Hz}$$

c).

$$F_{eq} = \int_0^L f\bar{u}\,dx = \int_0^L F\frac{x}{L}\left(\frac{x}{L}\right)^2 dx = \frac{F}{L^3}\int_0^L x^3\,dx = \frac{FL}{4}$$

$$= \frac{100\times10^3\times240}{4} = 6\times10^6 \text{ N}$$

$$\therefore u_{total} = \frac{F_{eq}}{K_{eq}}\bar{u} = \frac{6\times10^6}{72.3\times10^6} = 0.083 \text{ m}$$

① cont.

$T = \frac{1}{0.3} = 3.3\,s \quad \therefore \quad \frac{t_d}{T} = \frac{1}{3.3} = 0.3 \quad \Rightarrow \quad DAF = 1.6$ (datasheet)

$\therefore \ u_{max} = 1.6 \times 0.083 = 0.13\,m$

d). Resonance of the fundamental mode may best be controlled by the addition of one or more tuned-mass dampers, tuned to the 0.3Hz frequency of the mode.

Q1 Dynamics of a tower block

A straightforward question about the dynamical characteristics of a tall building (which just happened to have a passing resemblance to the Bahrain World Trade Center on the front of the 2009 Databook). Most attempts were reasonable and most errors were algebraic during the various integrations.

a) When sandy soils are shaken cyclic shear stresses cause soils to tend to contract, especially for loose soils. If pores are saturated and water cannot escape due to low permeability, the high bulk modulus of water will mean the soil cannot compress and the tendency to contract will be exhibited as an increase in pore pressure.

High pore pressures reduce effective stresses.

Low effective stresses give low strength + stiffness ~~etc~~, hence liquefaction.

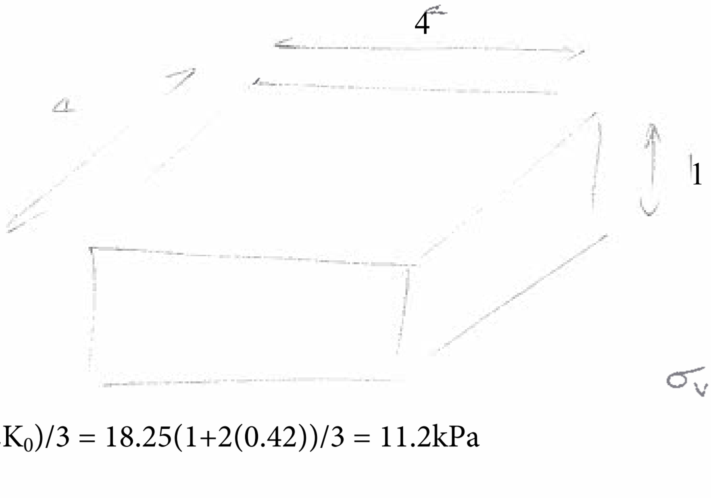
We hence need — SAND
SATURATED
LOOSE

b) Densification is most popular method, either drop weights on surface or force in probe to compact soil.

Stone columns can improve drainage reducing e.p.p.

or other techniques such as MICP.

c) i)



$$G_{max} = \frac{100\,(3-e)^2}{1+e}\sqrt{p'}\quad \text{MPa}$$

$e = 0.8$

$\sigma_v = 24 \times 1 + 18.5 \times 0.5$

$= 33.25$ kPa

$u = 15$ kPa at 1.5m depth

$\sigma_v' = 18.25$ kPa

p' = $\sigma_v'(1+2K_0)/3 = 18.25(1+2(0.42))/3 = 11.2$kPa

$K_0 = \frac{0.3}{0.7} = 0.42$

$\nu = 0.3$

$$G_{max} = 100 \times \frac{2.2^2}{1.8}\sqrt{\frac{11.2}{1000}}$$

$$= \underline{28.46\ MPa}$$

from databook $L = b = 2m$ $\frac{L}{b} = 1$ $\frac{e}{b} = \frac{1}{2}$

$e = 1m$

$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{L}{b}\right)^{0.65} + 2.4\right]\left[1 + \left\{0.33 + \frac{1.34}{1+L/b}\right\}\left(\frac{e}{b}\right)^{0.8}\right]$$

=1

$$= \frac{Gb}{1.7}\,[9.2][1.574] = 17.0\,G$$

$$= 485\ \frac{MN}{m}$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73\left(\frac{L}{b}\right)^{2.4} + 0.27\right]\left[1 + \frac{e}{b} + \frac{1.6}{0.35 + (L/b)^4}\left(\frac{e}{b}\right)^2\right]$$

$$= \frac{Gb^3}{1-\nu}\left[4.0\right]\left[1.796\right] = 82.1G$$

$$= 2337\ \frac{MN\,m}{rad}$$

c) ii)

Concrete block weighs $16 \times 24 =$ 384 kN

= 38,400 kg

machine mas 5,000 kg

H mode $\omega_n = \sqrt{\frac{K_h}{m_T}} = \sqrt{\frac{485 \times 10^6}{43{,}400}}$ = 106 rads/s = 16.8Hz

R mode $\omega_n = \sqrt{\frac{K_r}{I}} = \sqrt{\frac{2337 \times 10^6}{38400 \times 1^2 + 5000 \times 3.5^2}}$ = 153.1 rads/s = 24.4 Hz

Q2 Liquefaction and foundation vibration

Most scored highly on the first parts, describing liquefaction and its possible remedies. On the numerical part, there were quite a few errors in the effective stress calculation. A common error was thinking the pore water had only half a meter (rather than 1.5m) of hydrostatic pressure beneath the slab. There were, of course, quite a few arithmetic errors when evaluating those long Wolff formulae.

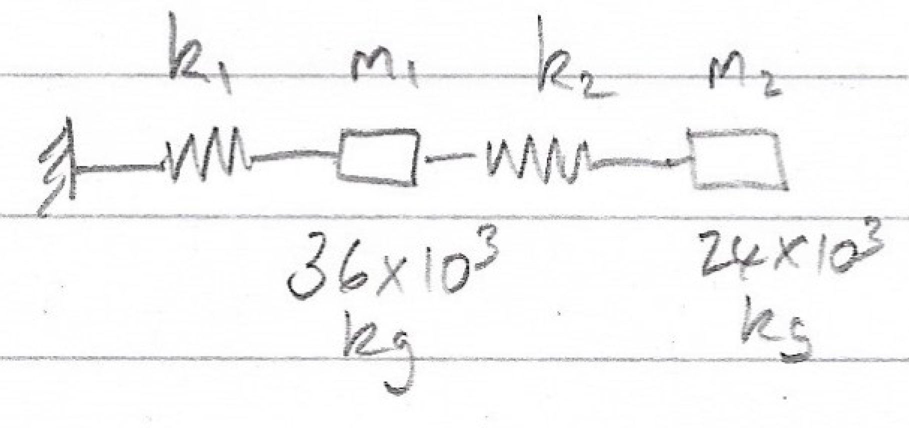
For the final part, a few people actually ignored the mass and rotational inertia of the foundation, and considered the machine alone.

4D6 Q3, 2021.

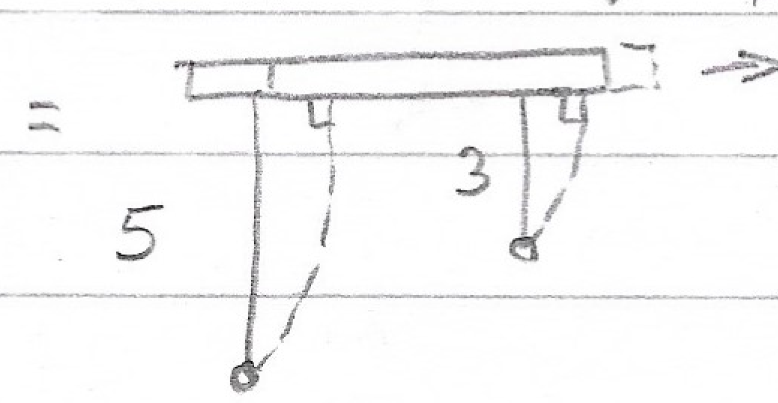
a) In Response Spectrum Analysis, as used in earthquake engineering, the response spectrum is defined as the response of a set of s.d.o.f oscillators of various frequencies to given input, such as the ground motions of a particular earthquake, or an idealised, averaged earthquake.

Spectral analysis in wind engineering theory of buffetting is when random vibration theory is applied to determine the response of a structure to buffetting by incident turbulence. Unlike in earthquake engineering this takes account of spatial as well as temporal variations in the applied pressures. It involves many "spectra", each being the Fourier transforms of some signal, (one of which might be the displacement at some point on the structure, i.e. the "response").

b) Background calculations (NOT REQUIRED FROM THE STUDENTS)



$k_1 =$

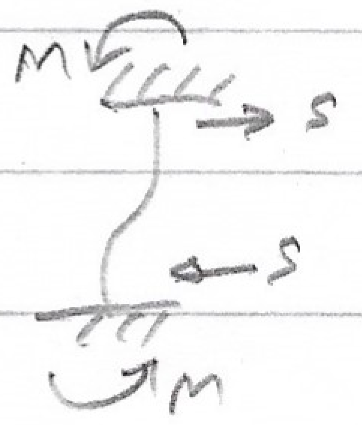


Cantilever $\delta = \dfrac{WL^3}{3EI}$

$W = \left(\dfrac{3EI}{L^3}\right)\delta \leftarrow k$

$k_1 = 3(2000\,\mathrm{kNm^2})\left(\dfrac{1}{5^3} + \dfrac{1}{3^3}\right) = 3(2000)(0.045)$
$= \underline{270\ \mathrm{kN/m}}$

$k_2 =$



$M = \dfrac{6EI}{L^2}$, $SL = 2M$, $S = \left(\dfrac{12EI}{L^3}\right)\delta \leftarrow k$

$k_2 = 2\left(\dfrac{12EI}{L^3}\right) = \dfrac{24(2000)}{4^3} = \underline{750\ \mathrm{kN/m}}$

2 of them

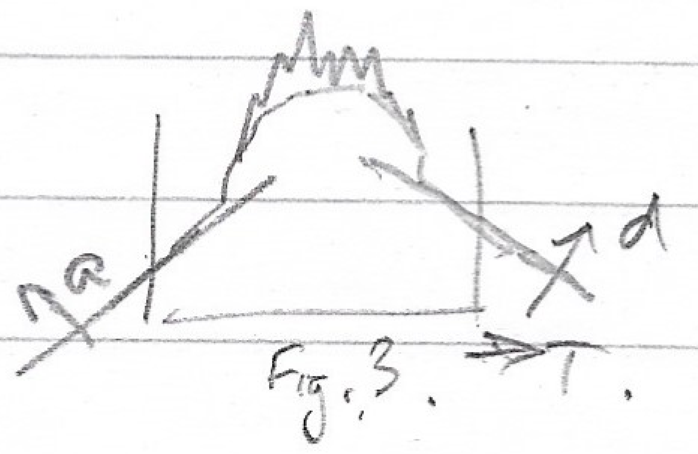
$$\underline{\underline{M}} = \begin{array}{c} \text{TOP} \\ \begin{bmatrix} 24000 & 0 \\ 0 & 36000 \end{bmatrix} \\ \text{BOT} \end{array} \mathrm{kg} \qquad K = \begin{bmatrix} 750 & -750 \\ -750 & 750+270 \end{bmatrix} \times 10^3\ \mathrm{N/m}.$$

Matlab, $\omega^2 = \mathrm{eig}[\mathrm{inv}(M) * K] \rightarrow f = 0.3275\ \mathrm{Hz},\ 1.1841\ \mathrm{Hz}$

$$\text{evecs} = \begin{bmatrix} 1 \\ 0.865 \end{bmatrix}, \begin{bmatrix} 1 \\ -0.7712 \end{bmatrix}$$

4D6 Q3, 2021

b) i) $a = 0.3g$, $d \approx 8 \text{ inches} = 203\,mm$



b) ii)

$f_1 = 0.33\,Hz$	$\omega_1 = 2.07$ rads/s	$T_1 = 1/f_1 = 3.03$ secs	$S_a \approx 0.1g$
$f_2 = 1.18\,Hz$	$\omega_2 = 7.41$ rads/s	$T_2 = 1/f_2 = 0.85$ secs	$S_a \approx 0.5g$

from Fig 3.

$$M_{eq1} = \left[24(1)^2 + 36(0.86)^2\right] \text{ tonne} = 50.6 \text{ tonne}$$
$$M_{eq2} = \left[24(1)^2 + 36(0.77)^2\right] \text{ tonne} = 45.4 \text{ tonne}$$

$$\Gamma_1 = \frac{M_1 u_1 + M_2 u_2}{M_{eq}} = \frac{24 + (0.86)36}{50.6} = \frac{55.0}{50.6} = 1.086$$

$$\Gamma_2 = \quad '' \quad = \frac{24 + (-0.77)36}{45.4} = \frac{-3.72}{45.4} = -0.082$$

Mode 1: $S_{d_1} = \dfrac{S_{a_1}}{\omega_1^2} = \dfrac{(0.1)(9.81)}{(2.07)^2} = 0.23\,m$ ($\sim$ 9 inches)

Fig 3 ✓

Mode 2: $S_{d_2} = \dfrac{S_{a2}}{\omega_2^2} = \dfrac{(0.5)(9.81)}{(7.41)^2} = 0.09\,m$ ($\sim$ 3.5 inches)

$$u_1 = \Gamma_1 S_{d_1} \phi_1 = (1.086)(0.23m)\begin{bmatrix}1\\0.86\end{bmatrix} = \begin{bmatrix}0.25\\0.21\end{bmatrix} m$$

$$u_2 = \Gamma_2 S_{d_2} \phi_2 = (0.082)(0.09m)\begin{bmatrix}1\\-0.77\end{bmatrix} = \begin{bmatrix}0.01\\-0.01\end{bmatrix} m \quad \text{(tiny)}$$

Bottom floor: $SRSS = \sqrt{(0.21)^2 + (-0.01)^2} = \underline{0.21\,m}$

$$\text{Shear } S = \left(\frac{3EI}{L^3}\right)\delta = \frac{3(2000)\,kNm^2}{3^3 \quad m^3}(0.21m) = \underline{\underline{46.7\,kN}}$$

on 3m column.

(Top storey interstorey drift is clearly less critical)

Alternatively: Base shear $= \Gamma^2 M_{eq} S_a = (1.086)^2 (50.6 \times 10^3\,kg)(0.1 \times 9.81\,m/s^2) = 58.5\,kN$

(for mode 1 – ignore mode 2)

Pro rata by stiffness to get fraction on 3m leg: $\dfrac{1/3^3}{1/5^3 + 1/3^3} = \dfrac{0.037}{0.08 + 0.037} = 0.821 \rightarrow (0.821)(58.5\,kN)$

$= 48\,kN$

for 3m column

4D6 2021, Q3.

c). First mode $T_1 = 3.03$ seconds.

Column shear capacity = 20 kN
but from b) we need 46.7 kN if elastic,
based on a deflection of 0.21 m of first floor.

So we only want 1st floor to have an elastic deflection of

$$0.21\text{m} \times \left(\frac{20\text{ kN}}{46.7\text{ kN}}\right) = \underline{\underline{0.09\text{m}}}$$

$$\therefore \quad \delta_1 = \Gamma_1 S_{d_1} \Phi_{\substack{1st\\floor}} = \Gamma_1 \left(\frac{S_{a_1}}{\omega_1^2}\right) \Phi_{\substack{1st\\floor}} = 0.09\text{m}$$

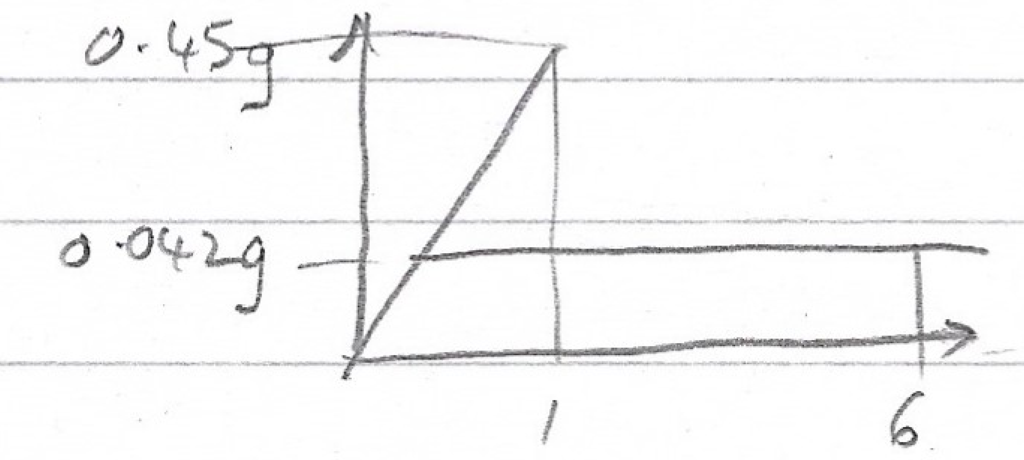
$$\therefore \quad S_a = \frac{\delta_1 \omega_1^2}{\Gamma_1 \Phi_{1st\,floor}} = \frac{(0.09\text{m})(2.07)^2/s^2}{(1.086)(0.86)} = 0.41\ \text{m/s}^2 = \underline{\underline{0.042g}}$$

The PGA at the site = 0.45g.

so require S_a in Fig 4 plot (which is for PGA = 1g) $= \dfrac{0.042g}{0.45} = \underline{\underline{0.09g}}$

$$\left(\text{i.e.} \quad \frac{0.09g}{1g} = \frac{0.042g}{0.45g}\right)$$

From Fig. 4, $T = 3.03$ secs, $S_a = 0.09g \rightarrow \underline{\underline{\mu \simeq 6}}$ req'd.

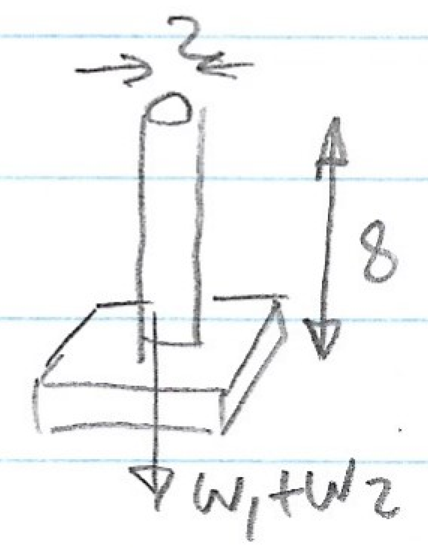


Q3 Response Spectrum Analysis

Most students could explain the difference between response spectrum analysis in earthquake engineering and the spectral analysis used in wind engineering. Many students did not read the correct PGA and PGD from the graph. Estimates of the peak column shear were generally reasonable. A common error was omitting the floor masses from the modal participation factor (which is only possible if all floor masses are equal, and thus cance). Another error was to assume that both base columns carried the same shear. A few students managed to obtain the correct ductility factor from the inelastic response spectra, and many followed the method through correctly, but with earlier numerical errors (which were not penalised twice).

4D6 Q4 2021.

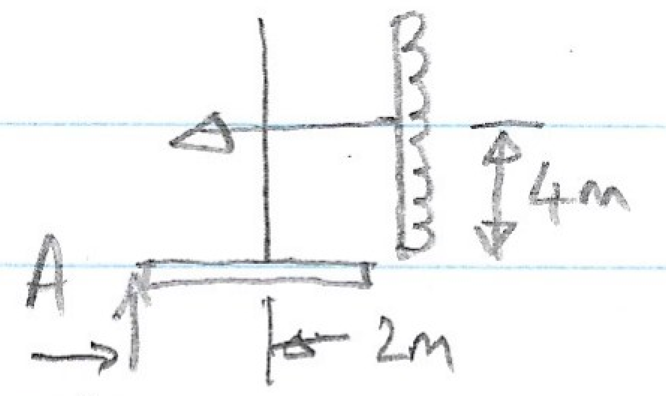
a)



$$\text{Force per unit height} = \tfrac{1}{2}\rho V^2 C_D D = \tfrac{1}{2}(1.25)(60)^2(1.2)2$$

(1.25 kg/m³ say)

$$= \underline{5.4\ kN/m}.$$



$$\text{Moments about A} = (5.4\,kN/m \times 8m) \times 4m \quad \leftarrow \text{or } 4.5m\text{, say.}$$

$$= \underline{173\ kNm} \qquad \left(= \underset{\text{for } 4.5m}{\underline{195\ kNm}}\right)$$

$$\text{Restoring moment required} = \underset{\text{F.o.S.}}{2.5} \times (195\,kNm) = \underline{488\,kNm}$$

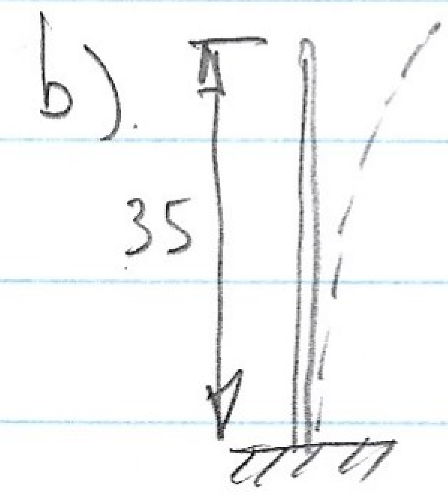
Tank weighs 4 tonne = 40kN (assume empty - worst case)

$$\text{Slab} = (4 \times 4 \times t)\,m^3 \times 24\,kN/m^3 = \underline{384t\ kN}$$

$$\text{Moment equilibrium:}\quad (40 + 384t) \times 2m = 488\,kNm.$$

$$\therefore\ t = \left(\frac{488}{2} - 40\right)/384 = \underline{0.53m} \quad \text{slab thickness req'd.}$$

b)



2.5m dia.
t = 10mm.

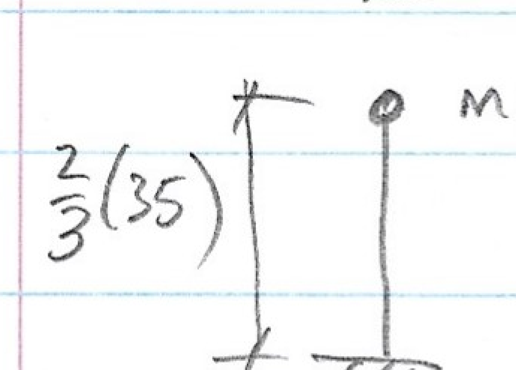
$$I = \pi r^3 t = \pi (1.25)^3 (0.01)\,m^4$$

$$= 0.0614\,m^4$$

$$m = 7860\,kg/m^3 \times \pi(2.5)(0.01)\,m^2 = \underline{617\,kg/m}$$

Nat freq? Assume a mode shape - eg 1-cosine

or use "2/3 mass at 2/3 height" approximation.



$$m = \tfrac{2}{3}(617\,kg/m \times 35m) = 14{,}400\,kg.$$

$$\text{Cantilever stiffness} = \frac{3EI}{L^3} = \frac{3(210\times10^9\,N/m^2)(0.0614\,m^4)}{(23.3)^3\,m^3}$$

$$= 3045 \times 10^3\,N/m.$$

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{3045\times10^3}{14.4\times10^3}} = \underline{14.5\ rads/s.} \qquad \underline{f = \frac{\omega}{2\pi} = 2.3\,Hz}$$

[Exact analytical answer is (students not expected to remember this)

$$\omega = \frac{3.52}{L^2}\sqrt{\frac{EI}{\rho A}} = \frac{3.52}{(35)^2}\sqrt{\frac{210\times10^9(0.0614)}{617}}$$

$$\omega = 13.13\,rads/s \rightarrow \underline{f = 2.1\,Hz}$$]

4D6 Q4 2021

4b ii) Strouhal number of cylinder = 0.2 for high Re.

$$St = \frac{f.b}{u} \quad \text{(data sheet)} \qquad b \leftarrow \text{diameter}$$

$$\therefore u = \frac{f.b}{St} = \frac{(2.3\,Hz)(2.5\,m)}{0.2} = \underline{\underline{29\ m/s}}$$

iii) Scruton number $Sc = \dfrac{2\delta_s m}{\rho b^2}$ $\quad \delta_s = $ log. decrement $= 2\pi\xi$ fraction of critical.

Need to assume a value for damping. A steel structure could be as low as $\xi = 0.5\%$ critical, but foundation may push this up significantly. In lieu of further info, assume $\xi = 0.5\%$.

$$\therefore Sc = \frac{4\pi\xi m}{\rho b^2} = \frac{4\pi(0.005)(617\ kg/m)}{(1.25\ kg/m^3)(2.5^2)\,m^2}$$

$$= \underline{5.2}$$

$$y_{max} \approx \frac{1.5D}{Sc} = \frac{1.5(2.5)}{5.2} \simeq \underline{\underline{0.7\,m}} \quad \text{(large!)}$$

(at 2.3 Hz, for 29 m/s windspeed).

c) Helical strakes disrupt the heightwise coherence of the shed vortices, creating a more disordered, highly 3-dimensional wake. Forces are therefore not in synchrony up the height of the structure, thus amplitudes are greatly reduced. (There is a consequent increase in static drag).

There were only a few attempts, and these were generally good. Estimating the frequency of the cantilever proved the greatest difficulty as no method was suggested: almost every student tried a different method, with the two thirds mass at two thirds height being perhaps the simplest reasonable approximation. Similarly, students needed to assume the damping for the Scruton number calculation: most picked 5% of critical but without any justification. The damping for a steel structure like this could be almost an order of magnitude smaller, but perhaps soil damping could push this figure up, all of which could have been mentioned.