EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 4 May 2021   1.30 to 3.10

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet and at the top of each answer sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachment: 4D6 Data Sheet (6 pages)
You are allowed access to the electronic version of the Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.

The time taken for scanning/uploading answers is 15 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

1 Figure 1 shows the dynamic model that was used as the basis for designing a twin-tower 'skyscraper' that incorporates its own wind turbines. For the purposes of predicting the bending of the building about its minor axis x-x, the whole primary structure is modelled as a non-uniform cantilever beam of height H with a bending stiffness $B(z)$ and a mass per unit height $m(z)$. At the base $B(0) = B_0$ and $m(0) = m_0$ with each falling linearly to zero at the top of the tower. Each turbine is modelled as a point mass M. These are located at heights $z = H/3$, $z = 2H/3$ and $z = H$.

(a) Assuming a parabolic form for the fundamental mode shape, use Rayleigh's Principle to find expressions for the equivalent mass and stiffness of the building. [40%]

(b) The height $H = 240$ m, $B_0 = 5 \times 10^{14}$ Nm^2, $m_0 = 2.5 \times 10^6$ kg m^{-1} and a turbine mass $M = 11 \times 10^3$ kg. Estimate the fundamental natural frequency of the building. [10%]

(c) A wind gust in the y direction generates a spatially-distributed time-varying force $F(z,t) = f(z)q(t)$ where $f(z)$ is a linearly-distributed force per unit height that rises from zero at the base to a magnitude of 100 kN m^{-1} at the top of the building. Estimate the peak displacement of the building to a 1 s gust assuming $q(t)$ may be treated as a rectangular pulse of unit amplitude. [40%]

(d) Under certain wind conditions, there is a risk that the fundamental mode of the building may be excited into resonance. Suggest one means by which this risk may be mitigated. [10%]

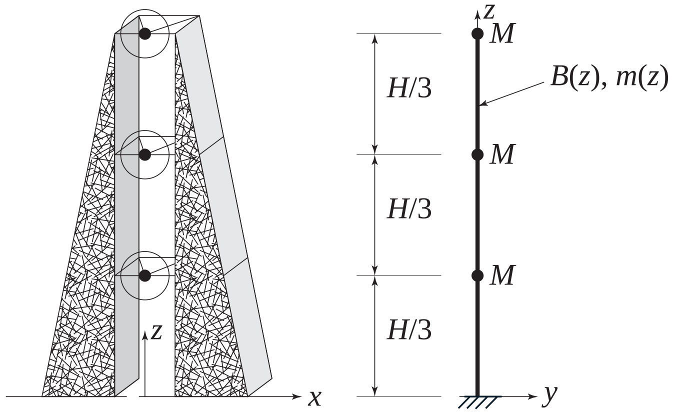


Fig. 1

2 (a) Explain why soils may liquefy during earthquakes, and explain which types of soil are susceptible and why. [30%]

(b) If a site is classified as potentially liquefiable, how might its performance be improved? [20%]

(c) A concrete foundation block for a heavy machine has dimensions of 4 m × 4 m in plan and a depth of 1 m. The foundation block is embedded into a saturated sand layer such that the top surface of the block is level with the sand surface. The unit weight of concrete is 24 kN m^{-3} and that of the saturated sand is 18.5 kN m^{-3}. The voids ratio and the Poisson's ratio of the sand may be taken as 0.8 and 0.3 respectively. The water table is at the surface of the sand layer. Liquefaction effects may be ignored.

(i) By considering a reference plane 0.5 m below the foundation block, calculate the horizontal stiffness of the foundation-sand system, and its rotational stiffness about a horizontal axis. [25%]

(ii) The concrete block supports a machine of mass 5000 kg, and this mass may be assumed to be concentrated at a point 2 m above the top surface of the foundation block, and located centrally in plan. Assuming the whole system will rock about an axis in the reference plane considered in part (c) (i) above, calculate the natural frequencies of the horizontal and rocking modes of vibration. [25%]

3 (a) Briefly explain the concepts of response spectrum analysis as used in earthquake engineering and spectral analysis as used in the wind engineering theory of buffeting, highlighting any differences between the two. [20%]

(b) Figure 2 shows a two-storey sway frame. The masses of the upper and lower floors are 24000 kg and 36000 kg respectively. Each column has flexural rigidity EI = 2000 kN m^2. The column feet are pinned at ground level, and all other connections are fixed. The in-plane sway modes have natural frequencies of 0.33 Hz and 1.18 Hz with associated mode shapes [1, 0.86] and [1, -0.77] respectively, these having been normalised to have unit displacement at the upper floor. The structure has 5% damping, and experiences an earthquake with the response spectrum shown in Fig. 3.

(i) Determine the maximum ground acceleration and displacement. [10%]

(ii) Determine the maximum column shear force. [40%]

(c) The structure described in part (b) is being designed using the inelastic design spectra in Fig. 4 for an earthquake with peak ground acceleration of 0.45g. Considering only the first mode, determine the required ductility factor μ if each column has a shear capacity of 20 kN. [30%]

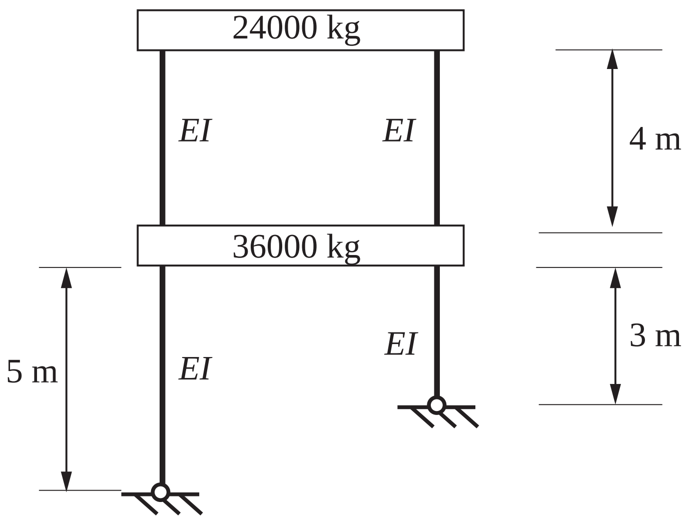


Fig. 2

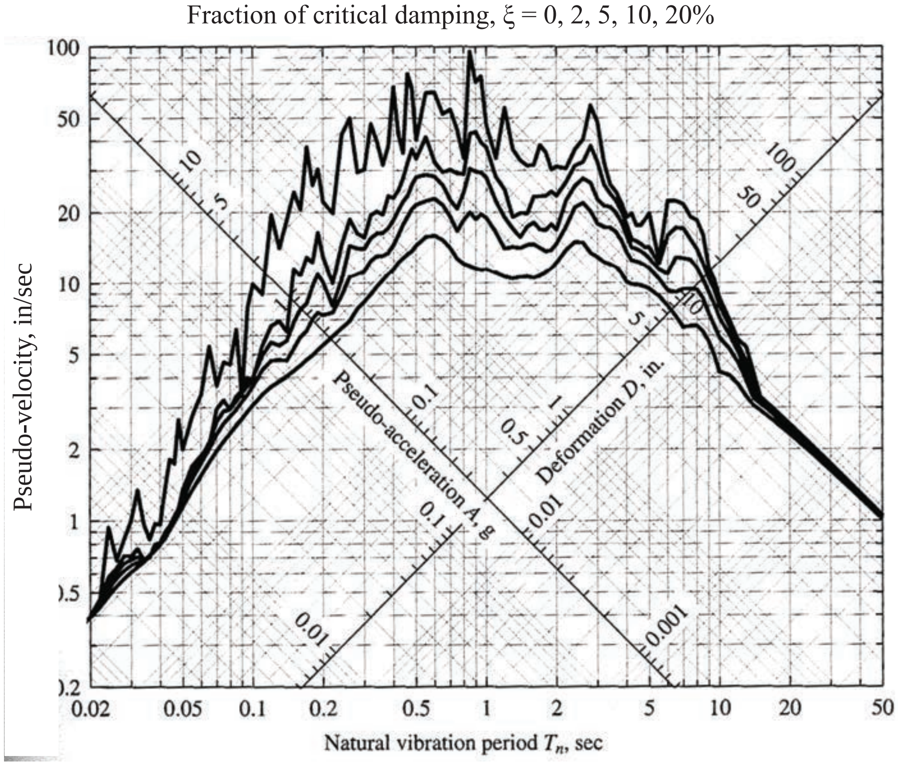

Fraction of critical damping, ξ = 0, 2, 5, 10, 20%
Pseudo-velocity, in/sec
Pseudo-acceleration A, g
Deformation D, in.
Natural vibration period T_n, sec

Fig. 3

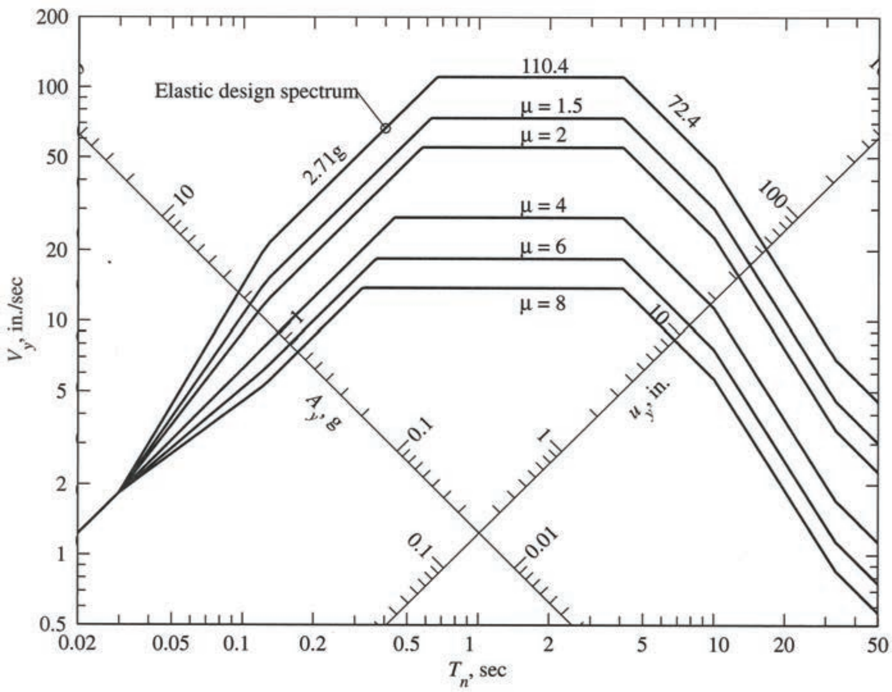

Elastic design spectrum
110.4
72.4
2.71g
μ = 1.5
μ = 2
μ = 4
μ = 6
μ = 8
V_y, in./sec
A_y, g
u_y, in.
T_n, sec

Fig. 4

4 (a) A cylindrical propane tank is situated in a region prone to severe tropical cyclones. The tank is 8 m tall and 2 m diameter, as shown in Fig. 5. The mass of the empty tank is 4000 kg. The tank is located centrally above a concrete foundation 4 m × 4 m in plan. The tank is bolted securely to the slab. Assuming suitable densities, design a slab thickness to provide a Factor of Safety against overturning of 2.5 at a uniform steady windspeed of 60 m s^{-1}. The drag coefficient for a circular cylinder may be taken as 1.2 at the windspeed in question. [20%]

(b) A steel chimney is a 35 m high cantilever with a diameter of 2.5 m and wall thickness 10 mm. Stating any assumptions, estimate:

(i) the natural frequency of the fundamental mode; [30%]

(ii) the critical wind velocity for vortex-induced vibrations; [20%]

(iii) the amplitude of vortex-induced vibrations at the critical wind velocity. [20%]

(c) Briefly explain why helical strakes reduce vortex-induced vibrations. [10%]

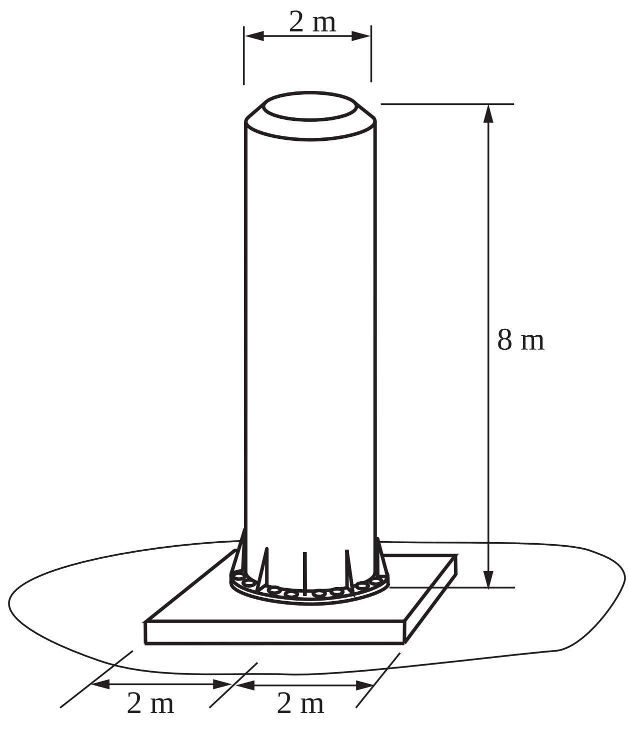


Fig. 5

END OF PAPER

Module 4D6: Dynamics in Civil Engineering

Data Sheets

Equivalent SDOF Systems

For an *n*-DOF system, with mass matrix $\underline{\underline{M}}$ and stiffness matrix $\underline{\underline{K}}$, responding in mode shape $\underline{\overline{u}}$ to an applied force $\underline{F}$, the parameters of the equivalent SDOF system are:

$$M_{eq} = \underline{\overline{u}}^T \underline{\underline{M}}\, \underline{\overline{u}} \qquad K_{eq} = \underline{\overline{u}}^T \underline{\underline{K}}\, \underline{\overline{u}} \qquad F_{eq} = \underline{F}^T \underline{\overline{u}}$$

For a continuous beam, of length *L*, mass per unit length *m* and bending stiffness *EI*, responding in mode shape $\overline{u}(x)$ to an applied force $f(x)$, the parameters of the equivalent SDOF system are:

$$M_{eq} = \int_0^L m\overline{u}^2 dx \qquad K_{eq} = \int_0^L EI \left(\frac{d^2\overline{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\overline{u}\, dx$$

The corresponding natural frequency is $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}}$

Modal Analysis of a Simply-Supported Beam

$$\overline{u}_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\,eq} = \frac{mL}{2} \qquad K_{i\,eq} = \frac{(i\pi)^4 EI}{2L^3}$$

Ground Motion Participation Factor

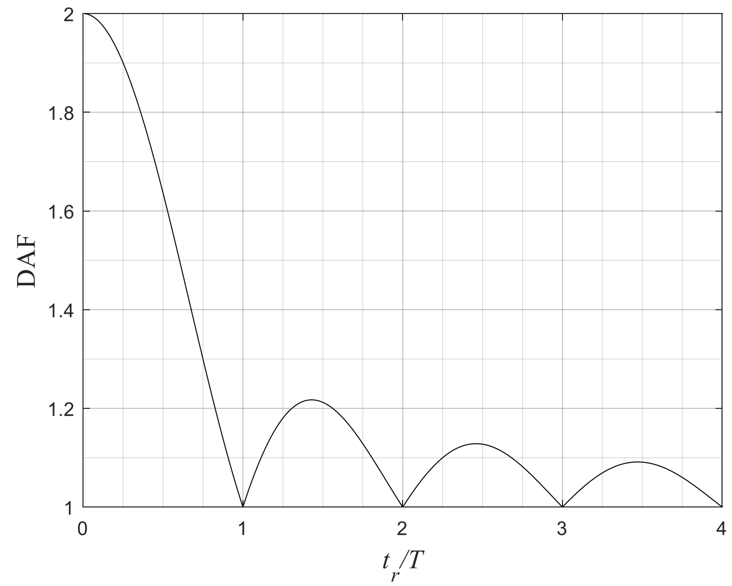
For an *n*-DOF system, with mass matrix $\underline{\underline{M}}$ and stiffness matrix $\underline{\underline{K}}$, responding to ground acceleration $\ddot{u}_g$, the parameters of the equivalent SDOF system are:

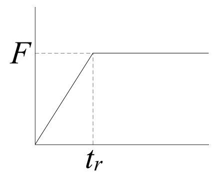
$$M_{eq} = \underline{\overline{u}}^T \underline{\underline{M}}\, \underline{\overline{u}} \qquad K_{eq} = \underline{\overline{u}}^T \underline{\underline{K}}\, \underline{\overline{u}} \qquad F_{eq} = -\Gamma M_{eq}\ddot{u}_g$$

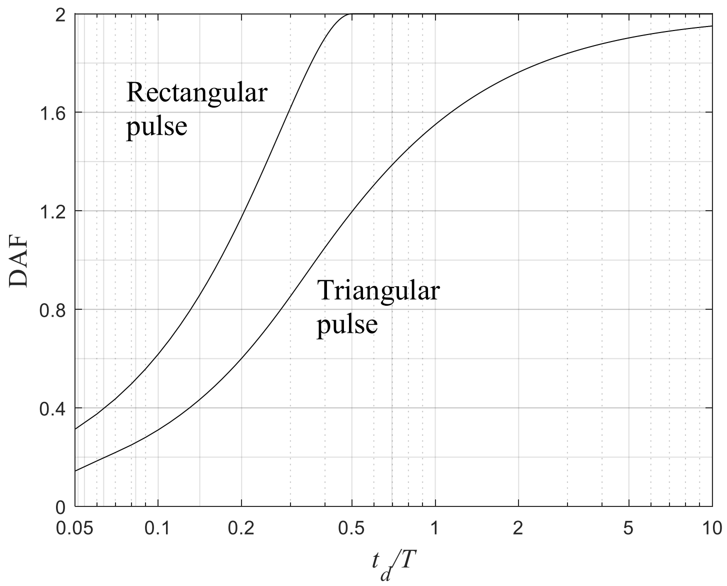
where $\underline{\overline{u}}$ is defined relative to ground and Γ is the modal participation factor

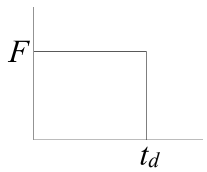
$$\Gamma = \frac{M_1\overline{u}_1 + M_2\overline{u}_2 + \cdots + M_n\overline{u}_n}{M_{eq}}$$

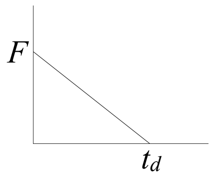
Dynamic Amplification Factors

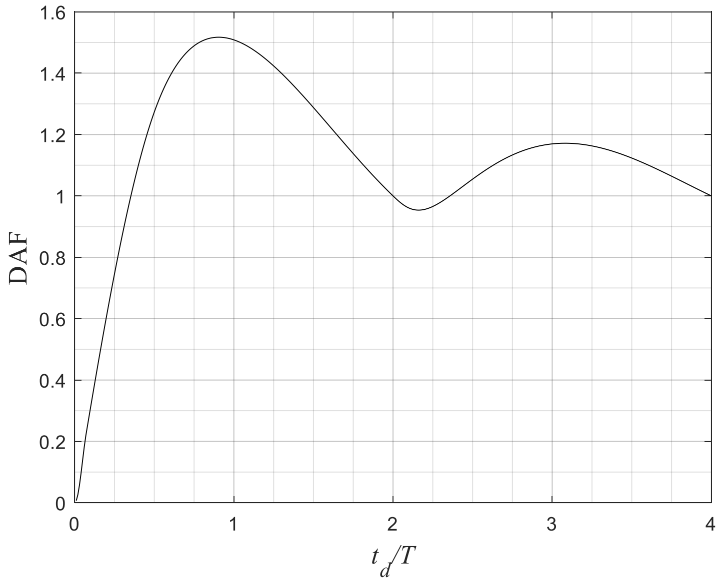


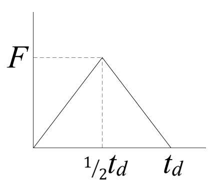






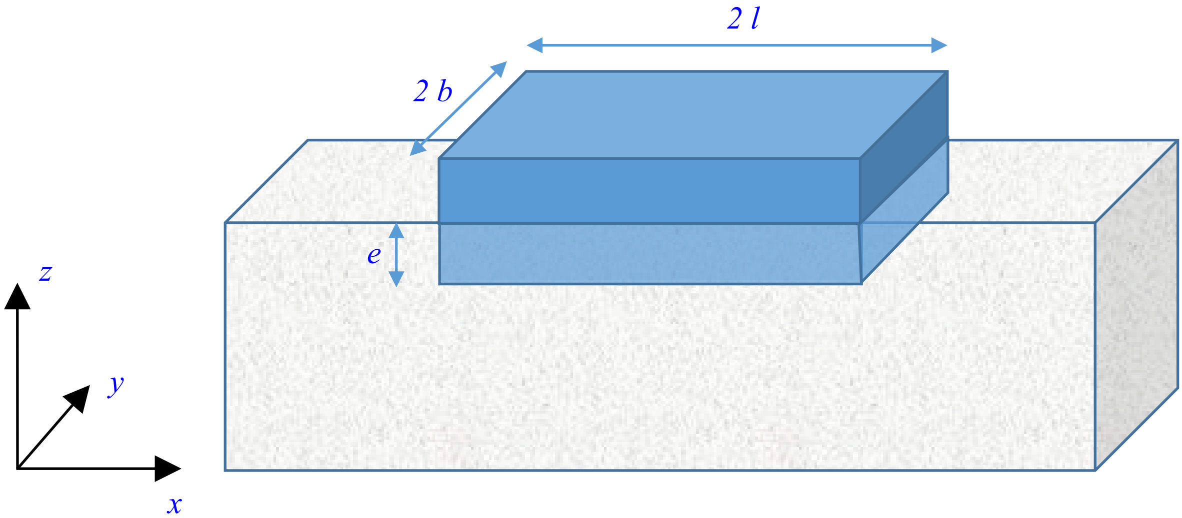






Soil Stiffness for Embedded Footings

Approximate relations for evaluating the soil stiffness for an embedded, prismatic footing of dimensions *2l* and *2b*, embedded to a depth *e*, assuming horizontal shaking in the direction parallel to the *x* axis (i.e. 2*l* of the prismatic structure) are:



$$K_{hx} = \frac{G\,b}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left\{0.33 + \frac{1.34}{1+\frac{l}{b}}\right\}\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{G\,b}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\,\frac{l}{b} + 1.6\right]\left[1 + \left\{0.33 + \frac{1.34}{1+\frac{l}{b}}\right\}\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{v} = \frac{G\,b}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left\{0.25 + \frac{0.25\,b}{l}\right\}\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{G\,b^3}{1-\nu}\left[3.2\,\frac{l}{b} + 0.8\right]\left[1 + \frac{e}{b} + \left(\frac{1.6}{0.35+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{2}\right]$$

$$K_{ry} = \frac{G\,b^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[1 + \frac{e}{b} + \left(\frac{1.6}{0.35+\left(\frac{l}{b}\right)^{4}}\right)\left(\frac{e}{b}\right)^{2}\right]$$

$$K_{tor} = G\,b^3\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[1 + \left(1.3 + 1.32\,\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right]$$

Properties of Soil

Unit weight of soil:

$$\gamma = \frac{(G_s + S_r e)}{1 + e} \gamma_w$$

where e is the void ratio, S_r is the degree of saturation and G_s is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_d = \frac{G_s}{1 + e} \gamma_w$$

Effective mean confining stress:

$$p' = \sigma'_v \frac{(1 + 2 K_o)}{3}$$

where σ'_v is the effective vertical stress and K_o is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$K_o = \frac{\nu}{1 - \nu}$$

Effective stress Principle:

$$p' = p - u$$

The shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{max} = 100 \frac{(3 - e)^2}{(1 + e)} (p')^{0.5}$$

where p' is the effective mean confining pressure (in MPa), e is the void ratio and G_{max} is the small-strain shear modulus (in MPa).

Shear modulus correction for strain may be carried out using the following expressions:

$$\frac{G}{G_{max}} = \frac{1}{1 + \gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r}\left[1 + a\, e^{-b\left(\frac{\gamma}{\gamma_r}\right)}\right]$$

and *a* and *b* are constants depending on soil type. For sandy soil deposits:

$$a = -0.2 \ \ln N$$

$$b = 0.16$$

where *N* is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is the reference shear strain given by

$$\gamma_r = \frac{\tau_{max}}{G_{max}}$$

where

$$\tau_{max} = \sqrt{\left[\left(\frac{1 + K_o}{2} \ \sigma_v' \ sin\varphi'\right)^2 - \left(\frac{1 - K_o}{2} \ \sigma_v' \right)^2\right]}$$

Shear modulus is also related to the shear wave velocity $\boldsymbol{v_s}$ as

$$v_s = \sqrt{\frac{G}{\rho}}$$

where *G* is the shear modulus and ρ is the mass density of the soil.

Wind Engineering

Vortex-induced vibrations

Strouhal Number for flow past a circular cylinder: $S_t = \frac{n_u D}{U}$

where n_u is the vortex-shedding frequency (in Hz), D is the cylinder diameter (in m) and U is the flow velocity (in m/s).

For circular cylinders, $S_t \approx 0.2$

Scruton Number: $S_c = \frac{2\delta_s m}{\rho D^2}$

where m is the actual mass per unit length of the structure, δ_s is the logarithmic decrement of structural damping (= 2π × fraction of critical damping), D is the diameter of the cylinder and ρ is the density of air (≈ 1.25kg/m^3).

A rough estimate of the amplitude y_{max} of vortex-induced vibrations at resonance can be obtained from

$$\frac{y_{max}}{D} = \frac{1.5}{S_c}$$

JPT
SPGM
FAM

January, 2021