Version JJO/4

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 26 April 2022   14.00 to 15.40

Module 4D7

CONCRETE AND PRESTRESSED CONCRETE

*Answer **all** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4D7 Data Sheet (5 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 A prefabricated prestressed concrete beam has an inverse T-shaped cross section as shown in Fig. 1. The beam is supported by two rectangular reinforced concrete columns with a reinforcement layout as specified in Fig. 2.

(a) The beam is loaded by a sagging moment which varies between 30 kNm and 200 kNm.

(i) Determine the position of the centroid, the second moment of area, and the elastic moduli for the top and bottom fibre (the effect of the small area of the prestressing tendon can be ignored). [10%]

(ii) If the prestressing tendon carries an effective prestress of 800 MPa, determine the top and bottom fibre stresses under the applied moments. The tendon has an area of 1000 mm^2. [10%]

(iii) If the permissible stresses in the concrete are 12 MPa in compression and 1 MPa in tension, draw a Magnel diagram and determine the maximum permissible prestress if the tendon could be placed anywhere in the section. [25%]

(iv) Is it possible to fit the tendon into the section if exactly 60 mm cover must be left between the centre of the tendons and the bottom edge of the beam? If so, suggest a corresponding prestress force. If not, which design choices are available to satisfy this requirement? [15%]

(b) The column has a design concrete compressive strength f_{cd} of 25 MPa and all longitudinal reinforcement has a diameter of 20 mm, a design yield strength f_{yd} of 400 MPa and Young's modulus of E_s = 200 GPa.

(i) Determine the maximum axial load P_0 the column can withstand under a centric uni-axial compression loading. [5%]

(ii) Determine the maximum bending moment M_0 the column can withstand under uni-directional bending around the x-x axis. Start your calculations assuming the neutral axis runs through the centroid of the compression reinforcement. [20%]

(iii) Sketch an interaction diagram for this column using the above calculated capacities (there is no need to calculate any additional points on the diagram). Comment on what the different zones of the diagram represent. [15%]

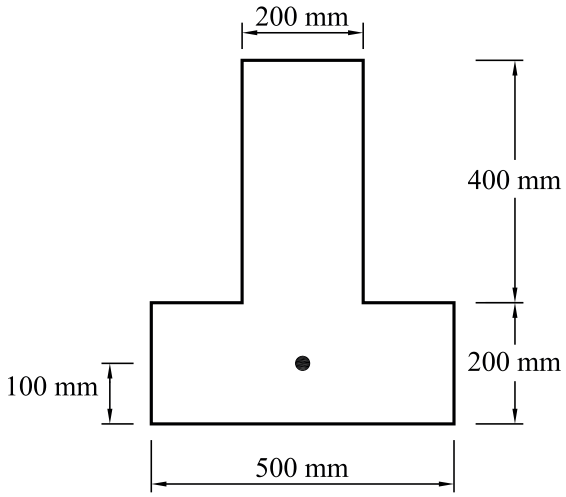

Fig. 1

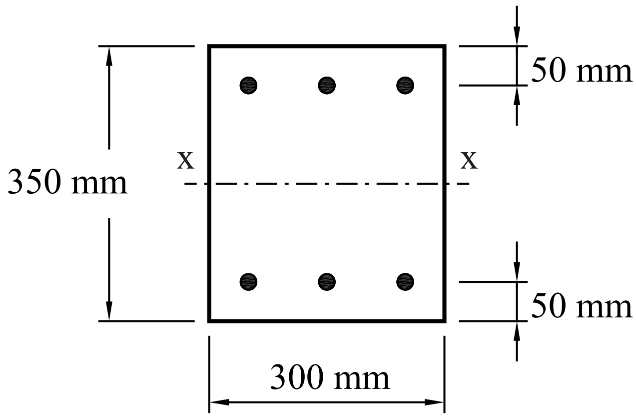

Fig. 2

2 (a) Explain the basic principles of limit state design and discuss the importance of limit states to the safety and whole life performance of concrete structures. [10%]

(b) Codified design sets minimum performance criteria, allowing designers to produce materially inefficient structures. How might this be addressed in the design of concrete structures, to ensure that we consume enough material, and no more. [20%]

(c) A factory in Bristol produces precast reinforced concrete beams for use in building construction. The beams have a span of 10 m. The flexural strength of the beam (including self weight) has a mean value of 100 kNm and coefficient of variation of 0.15. When placed in a building, they are loaded at mid span by a single point load P which has a mean value of 10 kN and coefficient of variation of 0.15. At ULS, the material partial safety factor $\gamma_m = 1.5$ and the load partial safety factor $\gamma_f = 1.4$. Both load and resistance are initially assumed to be normally distributed.

(i) Determine the *characteristic* load effect and *design* load effect. [10%]

(ii) Determine the *characteristic* bending strength and *design* bending strength of the beam. [10%]

(iii) Would this beam be considered safe in flexure? [5%]

(iv) Determine the reliability index β and the probability of failure in bending for this beam. [30%]

(v) The design is altered such that the characteristic strength against applied load is exactly equal to the characteristic value of the maximum load applied during the design life of the structure. Calculate a revised probability of failure and comment on your results. [15%]

3 (a) A reinforced concrete sway frame structure built in 1960 in Birmingham is to be extended and refurbished. In order to justify retaining the existing structure, an assessment must be made of the likely future lifespan of the existing concrete frame from the effects of carbonation. All elements in the frame have a cover to reinforcement of 20 mm. In 2020, cores were taken from an uncracked section of the frame and a phenolphthalein indicator solution was applied. On average, 14 mm of the core was clear, and the remaining region turned a deep pink colour. Critical threshold for the depassivation of steel is assumed to be when $pH = 12$.

(i) The design team propose that the retained structure should resist the effects of corrosion for a further 60 years (from 2020). By estimating the age at which corrosion of the steel will be first initiated, determine if their ambition is achievable. [40%]

(ii) What other factors affect the time to initiation and rate of corrosion? [15%]

(b) What role does the reuse of concrete assets play in tackling climate change? [25%]

(c) What are the key considerations for a designer who is considering extending the life of an existing asset? [20%]

END OF PAPER

Module 4D7: Data Sheet

Ultimate limit states

For STR and/or GEO it shall be verified that:

$$E_d \leq R_d \tag{1}$$

The design value of the effect of actions, E_d, is given by:

$$E_d = E\left\{\sum_{j\geq 1}\gamma_{G,j}G_{k,j}\text{“}+\text{”}\gamma_p P\text{“}+\text{”}\gamma_{Q,1}Q_{k,1}\text{“}+\text{”}\sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i}\right\} \tag{2}$$

Material partial factors are normally $\gamma_s = 1.15$ for steel and $\gamma_c = 1.5$ for concrete; Partial factors on actions are normally $\gamma_{G,j} = 1.35$ and $\gamma_{Q,1} = 1.5$.

Serviceability limit states

It shall be verified that:

$$E_d \leq C_d \tag{3}$$

The characteristic combination is:

$$E_d = E\left\{\sum_{j\geq 1}G_{k,j}\text{“}+\text{”}P\text{“}+\text{”}Q_{k,1}\text{“}+\text{”}\sum_{i>1}\psi_{0,i}Q_{k,i}\right\} \tag{4}$$

The frequent combination is:

$$E_d = E\left\{\sum_{j\geq 1}G_{k,j}\text{“}+\text{”}P\text{“}+\text{”}\psi_{1,1}Q_{k,1}\text{“}+\text{”}\sum_{i>1}\psi_{2,i}Q_{k,i}\right\} \tag{5}$$

Probability of failure

Design values of actions:

$$F_d = F_k\gamma_f \tag{6}$$

$$F_k = \mu_s + 1.645\sigma_s \tag{7}$$

$$\sigma_s = CoV \times \mu_s \tag{8}$$

Where F_d is the design value; γ_f is the partial safety factor; F_k is the characteristic value, μ_s is the mean value, σ_s is the standard deviation, and CoV is the coefficient of variation.

Design values of product properties:

$$X_d = \frac{X_k}{\gamma_m} \tag{9}$$

$$X_k = \mu_R - 1.645\sigma_R \tag{10}$$

$$\sigma_R = CoV \times \mu_R \tag{11}$$

Where X_d is the design value; γ_m the partial safety factor; X_k the characteristic value, μ_R the mean value, σ_R the standard deviation, and CoV the coefficient of variation.

Reliability index, β

$$\beta = \frac{\mu_R - \mu_s}{\sqrt{\sigma_R^2 + \sigma_s^2}} \quad (12)$$

Probability of failure:

$$P_f = \Phi(-\beta) \quad (13)$$

Where Φ is the standard normal cumulative distribution function.

The difference between two normally distributed variables is itself normally distributed, with mean equal to the difference of the means, and variance the sum of the squares of the standard deviations.

Durability considerations

Uniaxial diffusion into a homogenous material:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} \quad (14)$$

Solution:

$$C_x = C_0\left[1 - \mathrm{erf}\,(z)\right] \quad (15)$$

$$z = \frac{x}{2(Dt)^{0.5}} \quad (16)$$

Table of *erf(z)*

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	∞
erf(z)	0	0.11	0.22	0.33	0.43	0.52	0.60	0.68	0.74	0.80	0.84	0.88	0.91	0.93	0.95	0.97	1.00

Deflections

Interpolated curvature:

$$\alpha = \zeta\alpha_{||} + (1-\zeta)\,\alpha_{|} \quad (17)$$

Where α is a deflection, $\alpha_{/}$ and $\alpha_{//}$ are the values for the uncracked and fully cracked conditions, ζ is a distribution coefficient:

$$\zeta = 1 - \beta\left(\frac{\sigma_{sr}}{\sigma_s}\right)^2 \quad (18)$$

Where σ_{sr} is the stress in the tension reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking; σ_s is the stress in the tension reinforcement calculated on the basis of a cracked section; $\beta = 1.0$ for single short term loading and $\beta = 0.5$ for sustained loads or many cycles of repeated loading.

ULS Flexure

A doubly reinforced concrete section when flexural strength is reached is shown in Figure 1. It is usual to assume that failure occurs when the extreme fibre compressive strain in the concrete reaches a limiting value of 0.0035. Forces are found by equilibrium of the section.

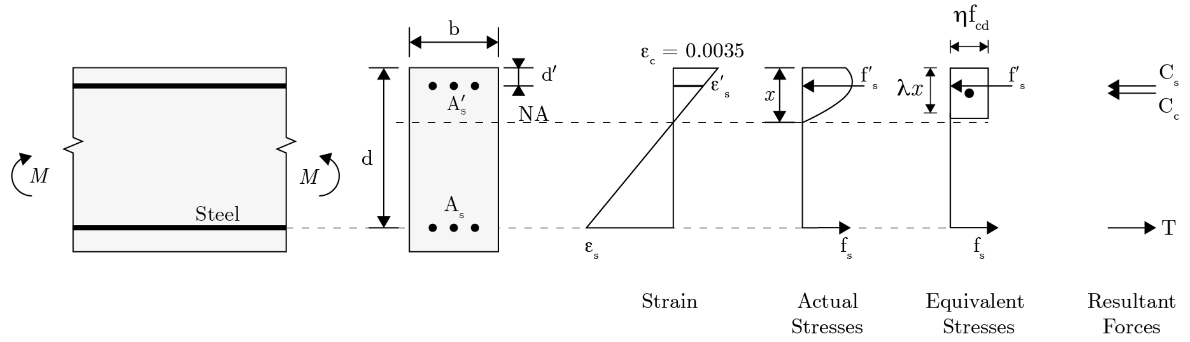

Figure 1

	$f_{ck} \leqslant 50$ MPa	50 MPa $< f_{ck} \leqslant 90$ MPa
λ	0.8	0.8 – (f_{ck} -50)/400
η	1.0	1.0 – (f_{ck} -50)/200

ULS Shear and Torsion

For unreinforced webs at ULS:

$$V_{Rd,c} = \left[\frac{0.18}{\gamma_c} k \left(100\rho_1 f_{ck}\right)^{\frac{1}{3}} + 0.15\sigma_{cp}\right] b_w d \qquad (19)$$

$$\geq \left(v_{min} + 0.15\sigma_{cp}\right) b_w d$$

$k = 1 + (200/d)^{0.5} \leq 2.0$

$\gamma_c = 1.5$

$\rho_1 = A_s/b_w d$ ($\rho_1 \leq 0.02$)

$v_{min} = 0.035 k^{3/2} f_{ck}^{1/2}$

For reinforced webs at ULS:

$$V_{Rd,s} = \frac{A_{sw}}{s} z f_{ywd} cot\theta \qquad (20)$$

$$V_{Rd,max} = \alpha_{cw} b_w z \nu_1 f_{cd} / \left(cot\theta + tan\theta\right) \qquad (21)$$

$\alpha_{cw} = 1$ for non-prestressed structures

$\nu_1 = 0.6$ for $f_{ck} \leq 60$ MPa and $\nu_1 = 0.9 - f_{ck}/200 > 0.5$ for $f_{ck} \geq 60$MPa

The shear stress in a wall of a section subject to pure torsion:

$$\tau_{t,i} t_{ef,i} = \frac{T_{Ed}}{2A_k} \qquad (22)$$

$\tau_{t,i}$ = torsional stress in wall i; $t_{ef,i}$ = effective wall thickness (= total area of cross section / outer circumference), A_k = area enclosed by centrelines of the walls including inner hollow areas.

Prestressed concrete

Elastic analysis: compression is positive. Eq.(23) applies for both top and bottom fibres since Z_i has sign:

$$\sigma = \frac{P}{A} + \frac{Pe}{Z_i} - \frac{M}{Z_i} \tag{23}$$

To design prestress, stress inequalities take the form:

$$f_c \geq \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} \geq f_t \tag{24}$$

For fibre 1 (top):

$$-\frac{Z_1}{A} + \frac{f_c Z_1}{P} + \frac{M}{P} \leq e \leq -\frac{Z_1}{A} + \frac{f_t Z_1}{P} + \frac{M}{P} \tag{25}$$

For fibre 2 (bottom):

$$-\frac{Z_2}{A} + \frac{f_c Z_2}{P} + \frac{M}{P} \geq e \geq -\frac{Z_2}{A} + \frac{f_t Z_2}{P} + \frac{M}{P} \tag{26}$$

Cumulative normal distribution function

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} e^{-\frac{x^2}{2}}\,dx \quad \text{FOR} \quad 0{\cdot}00 \le u \le 4{\cdot}99.$$

u	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7703	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
1·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
1·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·90147
1·3	·90320	·90490	·90658	·90824	·90988	·91149	·91309	·91466	·91621	·91774
1·4	·91924	·92073	·92220	·92364	·92507	·92647	·92785	·92922	·93056	·93189
1·5	·93319	·93448	·93574	·93699	·93822	·93943	·94062	·94179	·94295	·94408
1·6	·94520	·94630	·94738	·94845	·94950	·95053	·95154	·95254	·95352	·95449
1·7	·95543	·95637	·95728	·95818	·95907	·95994	·96080	·96164	·96246	·96327
1·8	·96407	·96485	·96562	·96638	·96712	·96784	·96856	·96926	·96995	·97062
1·9	·97128	·97193	·97257	·97320	·97381	·97441	·97500	·97558	·97615	·97670
2·0	·97725	·97778	·97831	·97882	·97932	·97982	·98030	·98077	·98124	·98169
2·1	·98214	·98257	·98300	·98341	·98382	·98422	·98461	·98500	·98537	·98574
2·2	·98610	·98645	·98679	·98713	·98745	·98778	·98809	·98840	·98870	·98899
2·3	·98928	·98956	·98983	·9^{2} 0097	·9^{2} 0358	·9^{2} 0613	·9^{2} 0863	·9^{2} 1106	·9^{2} 1344	·9^{2} 1576
2·4	·9^{2} 1802	·9^{2} 2024	·9^{2} 2240	·9^{2} 2451	·9^{2} 2656	·9^{2} 2857	·9^{2} 3053	·9^{2} 3244	·9^{2} 3431	·9^{2} 3613
2·5	·9^{2} 3790	·9^{2} 3963	·9^{2} 4132	·9^{2} 4297	·9^{2} 4457	·9^{2} 4614	·9^{2} 4766	·9^{2} 4915	·9^{2} 5060	·9^{2} 5201
2·6	·9^{2} 5339	·9^{2} 5473	·9^{2} 5604	·9^{2} 5731	·9^{2} 5855	·9^{2} 5975	·9^{2} 6093	·9^{2} 6207	·9^{2} 6319	·9^{2} 6427
2·7	·9^{2} 6533	·9^{2} 6636	·9^{2} 6736	·9^{2} 6833	·9^{2} 6928	·9^{2} 7020	·9^{2} 7110	·9^{2} 7197	·9^{2} 7282	·9^{2} 7365
2·8	·9^{2} 7445	·9^{2} 7523	·9^{2} 7599	·9^{2} 7673	·9^{2} 7744	·9^{2} 7814	·9^{2} 7882	·9^{2} 7948	·9^{2} 8012	·9^{2} 8074
2·9	·9^{2} 8134	·9^{2} 8193	·9^{2} 8250	·9^{2} 8305	·9^{2} 8359	·9^{2} 8411	·9^{2} 8462	·9^{2} 8511	·9^{2} 8559	·9^{2} 8605
3·0	·9^{2} 8650	·9^{2} 8694	·9^{2} 8736	·9^{2} 8777	·9^{2} 8817	·9^{2} 8856	·9^{2} 8893	·9^{2} 8930	·9^{2} 8965	·9^{2} 8999
3·1	·9^{3} 0324	·9^{3} 0646	·9^{3} 0957	·9^{3} 1260	·9^{3} 1553	·9^{3} 1836	·9^{3} 2112	·9^{3} 2378	·9^{3} 2636	·9^{3} 2886
3·2	·9^{3} 3129	·9^{3} 3363	·9^{3} 3590	·9^{3} 3810	·9^{3} 4024	·9^{3} 4230	·9^{3} 4429	·9^{3} 4623	·9^{3} 4810	·9^{3} 4991
3·3	·9^{3} 5166	·9^{3} 5335	·9^{3} 5499	·9^{3} 5658	·9^{3} 5811	·9^{3} 5959	·9^{3} 6103	·9^{3} 6242	·9^{3} 6376	·9^{3} 6505
3·4	·9^{3} 6631	·9^{3} 6752	·9^{3} 6869	·9^{3} 6982	·9^{3} 7091	·9^{3} 7197	·9^{3} 7299	·9^{3} 7398	·9^{3} 7493	·9^{3} 7585
3·5	·9^{3} 7674	·9^{3} 7759	·9^{3} 7842	·9^{3} 7922	·9^{3} 7999	·9^{3} 8074	·9^{3} 8146	·9^{3} 8215	·9^{3} 8282	·9^{3} 8347
3·6	·9^{3} 8409	·9^{3} 8469	·9^{3} 8527	·9^{3} 8583	·9^{3} 8637	·9^{3} 8689	·9^{3} 8739	·9^{3} 8787	·9^{3} 8834	·9^{3} 8879
3·7	·9^{3} 8922	·9^{3} 8964	·9^{4} 0039	·9^{4} 0426	·9^{4} 0799	·9^{4} 1158	·9^{4} 1504	·9^{4} 1838	·9^{4} 2159	·9^{4} 2468
3·8	·9^{4} 2765	·9^{4} 3052	·9^{4} 3327	·9^{4} 3593	·9^{4} 3848	·9^{4} 4094	·9^{4} 4331	·9^{4} 4558	·9^{4} 4777	·9^{4} 4988
3·9	·9^{4} 5190	·9^{4} 5385	·9^{4} 5573	·9^{4} 5753	·9^{4} 5926	·9^{4} 6092	·9^{4} 6253	·9^{4} 6406	·9^{4} 6554	·9^{4} 6696
4·0	·9^{4} 6833	·9^{4} 6964	·9^{4} 7090	·9^{4} 7211	·9^{4} 7327	·9^{4} 7439	·9^{4} 7546	·9^{4} 7649	·9^{4} 7748	·9^{4} 7843
4·1	·9^{4} 7934	·9^{4} 8022	·9^{4} 8106	·9^{4} 8186	·9^{4} 8263	·9^{4} 8338	·9^{4} 8409	·9^{4} 8477	·9^{4} 8542	·9^{4} 8605
4·2	·9^{4} 8665	·9^{4} 8723	·9^{4} 8778	·9^{4} 8832	·9^{4} 8882	·9^{4} 8931	·9^{4} 8978	·9^{5} 0226	·9^{5} 0655	·9^{5} 1066
4·3	·9^{5} 1460	·9^{5} 1837	·9^{5} 2199	·9^{5} 2545	·9^{5} 2876	·9^{5} 3193	·9^{5} 3497	·9^{5} 3788	·9^{5} 4066	·9^{5} 4332
4·4	·9^{5} 4587	·9^{5} 4831	·9^{5} 5065	·9^{5} 5288	·9^{5} 5502	·9^{5} 5706	·9^{5} 5902	·9^{5} 6089	·9^{5} 6268	·9^{5} 6439
4·5	·9^{5} 6602	·9^{5} 6759	·9^{5} 6908	·9^{5} 7051	·9^{5} 7187	·9^{5} 7318	·9^{5} 7442	·9^{5} 7561	·9^{5} 7675	·9^{5} 7784
4·6	·9^{5} 7888	·9^{5} 7987	·9^{5} 8081	·9^{5} 8172	·9^{5} 8258	·9^{5} 8340	·9^{5} 8419	·9^{5} 8494	·9^{5} 8566	·9^{5} 8634
4·7	·9^{5} 8699	·9^{5} 8761	·9^{5} 8821	·9^{5} 8877	·9^{5} 8931	·9^{5} 8983	·9^{6} 0320	·9^{6} 0789	·9^{6} 1235	·9^{6} 1661
4·8	·9^{6} 2067	·9^{6} 2453	·9^{6} 2822	·9^{6} 3173	·9^{6} 3508	·9^{6} 3827	·9^{6} 4131	·9^{6} 4420	·9^{6} 4696	·9^{6} 4958
4·9	·9^{6} 5208	·9^{6} 5446	·9^{6} 5673	·9^{6} 5889	·9^{6} 6094	·9^{6} 6289	·9^{6} 6475	·9^{6} 6652	·9^{6} 6821	·9^{6} 6981

Example: $\Phi(3{\cdot}57) = {\cdot}9^{3}\,8215 = 0{\cdot}9998215.$

4D7 2022
Numerical Answers

1(a)(i)
Centroid = 233.33mm from base
$I = 5.4 \times 10^9$ mm^4
$Z_1 = -1.47 \times 10^7$ mm^3
$Z_2 = 2.31 \times 10^7$ mm^3

1(a)(ii)
Maximum Moment:
$\sigma_{top} = 10.78$ MPa
$\sigma_{base} = 0.41$ MPa
Minimum Moment:
$\sigma_{top} = -0.76$ MPa
$\sigma_{base} = 7.76$ MPa

1(a)(iii)
$P_{max} = 1352$ kN

1(b)(i)
$P_0 = 2820$ kN

1(b)(ii)
$M_0 = 98$ kNm

2(c)(i)
$F_k = 31.2$ kNm
$F_d = 43.6$ kNm

2(c)(ii)
$X_k = 75.3$ kNm
$X_d = 50.2$ kNm

2(c)(iv)
$\beta = 0.9^{6}39$
$P_f = 6.1 \times 10^{-7}$

3(a)(i)
$t_2 = 122$ years (from 1960)