Question

(1) (a)

- find $\bar{y}$:

$$(500).(200).(100) + (400).(200).(400) = \bar{y}\,((500).(200) + (400).(200))$$

$$\bar{y} = 233{,}33\ mm$$

- Sectional area $A = 180\,000\ mm^2$

- Second moment of area :

$$I = \frac{(500).(200)^3}{12} + \frac{(200).(400)^3}{12} + (500).(200).(133{,}33)^2 + (400).(200).(400 - 233{,}33)^2$$

$$= 5{,}4 . 10^9\ mm^4$$

- $$Z_1 = \frac{-I}{(600 - \bar{y})} = -1{,}47 . 10^7\ mm^3$$

$$Z_2 = \frac{I}{\bar{y}} = 2{,}31 . 10^7\ mm^3$$

(b) $P = 800\ MPa . 1000\ mm^2 = 800\,000\ N$

$e = 233{,}33 - 100 = 133{,}33\ mm$

$P/A = 800\,000 / 180\,000 = 4{,}4\ MPa$

$$\frac{Pe}{Z_1} = -7{,}24\ MPa$$

$$\frac{Pe}{Z_2} = 4{,}61\ MPa$$

stresses under maximum moment.

Top :

$$\sigma = \frac{P}{A} + \frac{Pe}{Z_1} - \frac{M_{max}}{Z_1} = 4{,}4 - 7{,}24 + \frac{2.10^8}{1{,}47.10^7}$$

$$\sigma = 10{,}78\ MPa$$

bottom:

$$\sigma = \frac{P}{A} + \frac{Pe}{Z_2} - \frac{M_{max}}{Z_2} = 4,4 + 4,61 - \frac{2.10^8}{2,31.10^7}$$

$$\sigma = 0,41 \text{ MPa}$$

- Similarly stresses under minimum moment:

top: $\sigma = -0,76$ MPa

bottom: $\sigma = 7,76$ MPa.

(c) To construct Magnel diagram, choose prestressing force:

$$f_c / 2A = 1080 \text{ kN}$$

At top, worst tension when min moment is applied, worst compression when max moment is applied:

$$\frac{f_c Z_1}{P} - \frac{Z_1}{A} + \frac{M}{P} < e < \frac{f_t Z_1}{P} - \frac{Z_1}{A} + \frac{M}{P}$$

So:

$$\frac{(12).(-1,47.10^7)}{1,08.10^6} + \frac{1,47.10^7}{180000} + \frac{2.10^8}{1,08.10^6} < e$$

$$e > 103,37 \text{ mm} \quad (\text{max compr.})$$

similarly

$$e < 123,23 \text{ mm} \quad (\text{max tension})$$

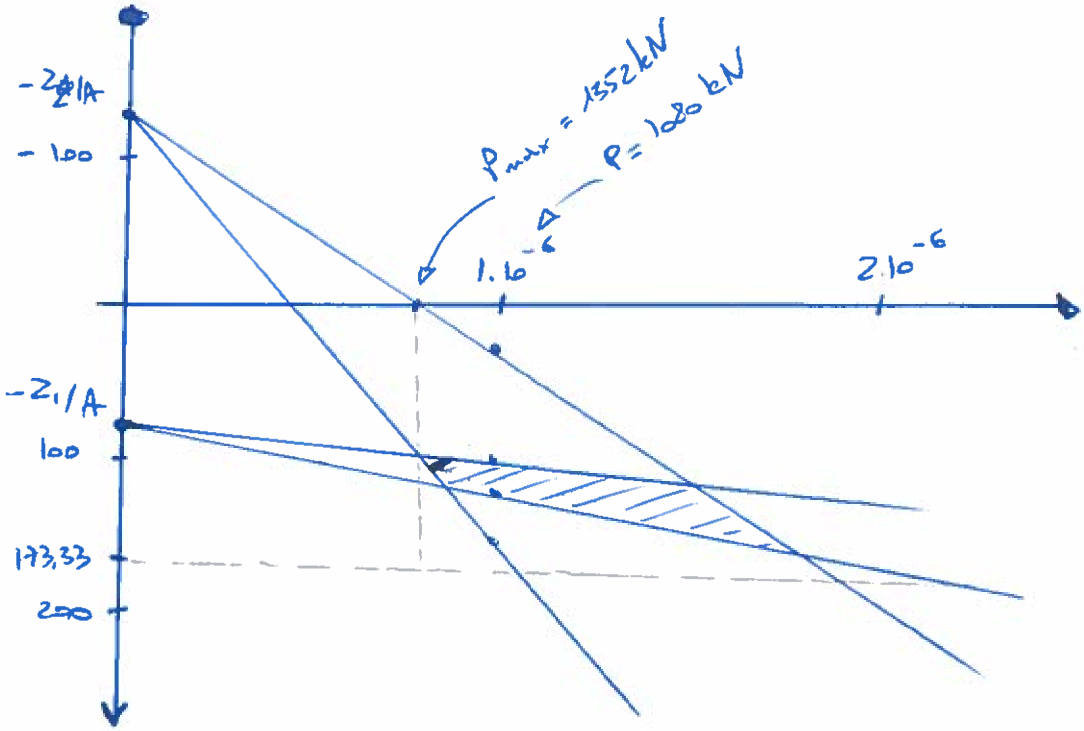
Again at bottom fibre stress limits:

$$\frac{f_c Z_2}{P} - \frac{Z_2}{A} + \frac{M}{P} > e > \frac{f_t Z_2}{P} - \frac{Z_2}{A} + \frac{M}{P}$$

$$\Rightarrow \quad e > 35,19 \text{ mm}$$

$$e < 156,35 \text{ mm}.$$

Magnel diagram



To find P_{max} :

$$\frac{(12).(-1{,}47.10^7)}{P} + \frac{1{,}47.10^7}{180000} + \frac{2.10^8}{P} = \frac{(12).(2{,}31.10^7)}{P} - \frac{2{,}31.10^7}{180000} + \frac{3.10^7}{P}$$

$$\Rightarrow P_{max} = 1351{,}98 \text{ kN}$$

(d) if cover is 60 mm $\Rightarrow$ e = 173,33 mm.

see above, not possible.

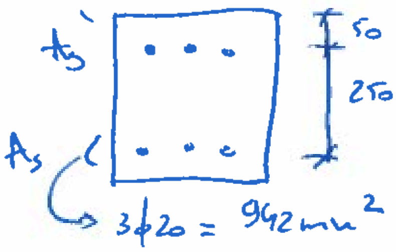
solutions include:

- change geometry
- change load arrangement (esp. min moment)
- change material properties
  ↳ limited effect as -1 MPa stress limit. is concrete grade independent

(ii) (a) $A_{tot} = 400 \cdot 350 = 140\,000 \text{ mm}^2$ $\quad A_s = 6 \cdot A_{s\phi 20} = 1885 \text{ mm}^2$

$$P_o = A_c \cdot f_{cd} + A_s \cdot f_{yd}$$
$$= (A_{tot} - A_s) \cdot f_{cd} + A_s \cdot f_{yd}$$
$$= 2820 \text{ kN}.$$

(b).



doubly reinforced section

$A_s = A_s'$

↳ likely A_s' will not yield

assume $x = 50$ mm and check:

$$\varepsilon_s' = \frac{3{,}5‰}{x}(x - d') = 0 \Rightarrow f_s' = 0 \text{ MPa}$$

$$\lambda x = \frac{(A_s \cdot f_{yd} - A_s' \cdot f_s')}{\eta \, f_{cd} \cdot b} \quad \text{with } \lambda = 0{,}8 \quad \eta = 1{,}0$$

$\Rightarrow x = 59$ mm.

new estimate for $x = 53$ mm and check:

$\varepsilon_s' \Rightarrow f_s' = 39{,}6$ MPa

from equilibrium $x = 53{,}07$ mm

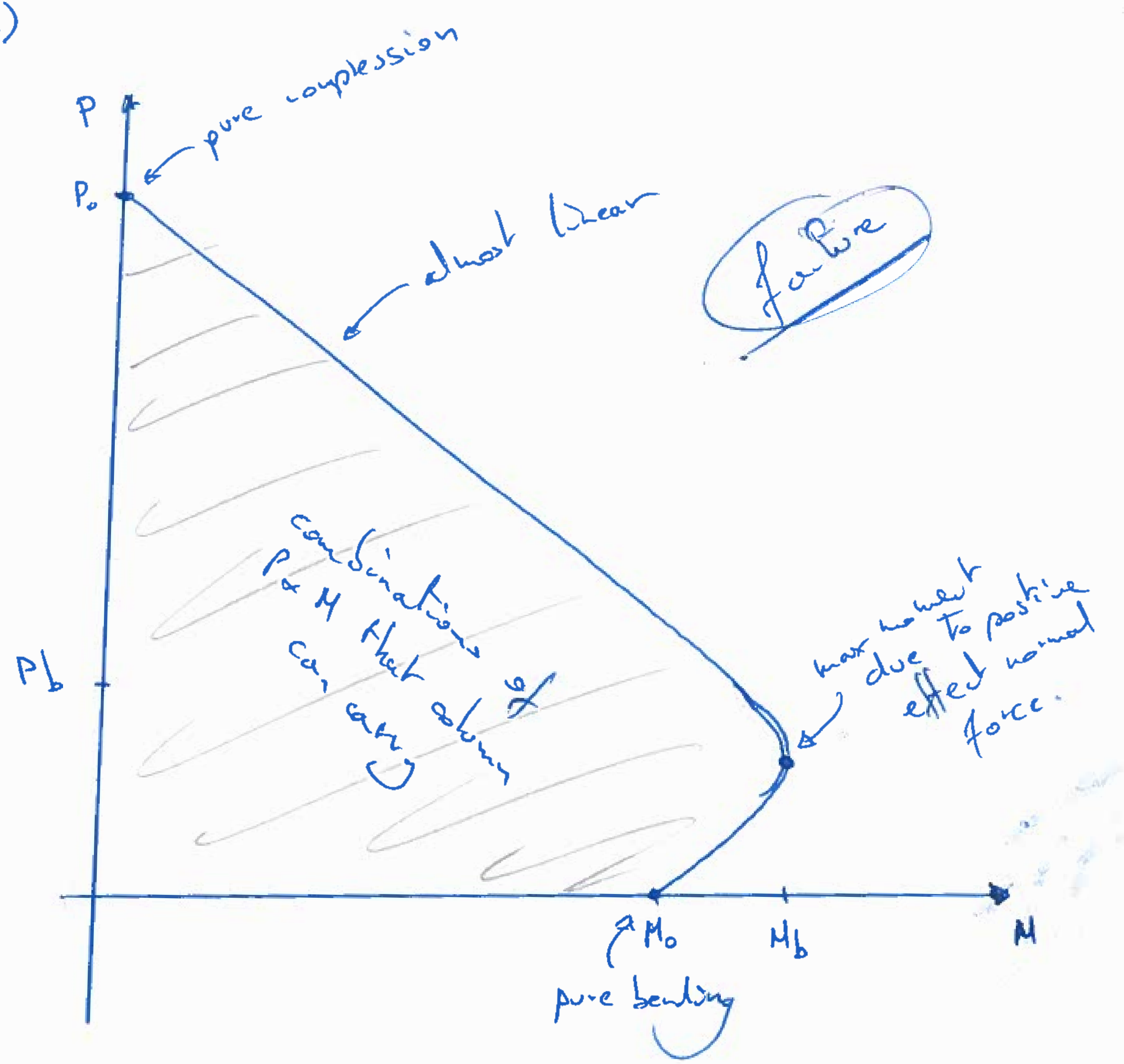
ok ✓

- Hence:

$$C_s = A_s' \cdot f_s' = 942 \text{ mm}^2 \cdot 39{,}6 \text{ MPa} = 37{,}3 \text{ kN}.$$
$$C_c = \eta \, f_{cd} \, \lambda x \, b = 1{,}0 \cdot 25 \cdot 0{,}8 \cdot 53 \cdot 300 = 318 \text{ kN}.$$

$$\Rightarrow M_o = C_c \cdot (d - 0{,}5\,\lambda x) + C_s \cdot (d - d')$$
$$= 318 \cdot (300 - 0{,}5 \cdot 0{,}8 \cdot 53) + 37{,}3\,(300 - 50).$$
$$= 98 \text{ kNm}.$$

(c)



This question was quite poorly answered and had the lowest average mark in the paper. Most students were able to determine parts (a)(i) and (a)(ii), but the latter stages of (a) were poorly answered, in particular drawing a Magnel Diagram. In part (b) most students did not determine the value of Po correctly, as they ignored the contribution of the steel reinforcement.

4D7 2022

Q2(a)

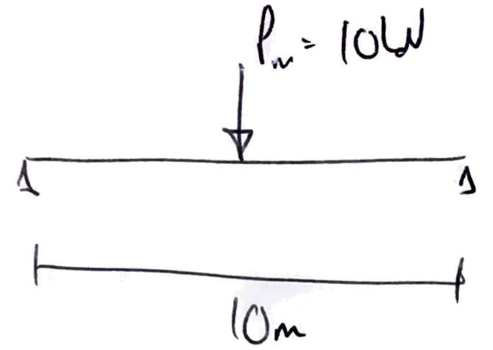
- Philosophy under which structures are designed such that the probability that a number of performance criteria are exceeded is deemed to be acceptably small during the required functional lifetime of the structure.
- When a structure, or element within a structure, ceases to satisfy one or more of these performance criteria it is deemed to have exceeded a limit state and thus now fails to fulfil satisfactorily the design requirements.
- Limit state design may be achieved using probabilistic methods or by the partial factor method. The latter is by far the dominant method in practice. It requires the designer to verify that relevant limit states are not exceeded: ULS is about safety, SLS is about serviceability.
- A full answer would consider some of the design situations (which relate to the limit states, and are either persistent, transient, or accidental design situations).

Q2(b)

- Structural materials are often inefficiently utilised, with resistance rarely set equal to effects of actions (at both ULS and SLS).
- This mitigation against uncertainty, or additional "sleep at night" factor, perhaps reflects a significant downside in getting it wrong, with little upside to be found in being materially efficient.
- Potential ideas for codified design to limit inefficiency:
  - Setting upper bounds on resistance compared to effects of actions. Students should explain how this might work, and what the unintended consequences of such an approach might be – for example, designers might simply always work to the upper bound, rather than the lower bound, of the resistance.
  - Adding "...and no more" to design codes – encouraging designers to specify what is needed, and no more.
  - Adding a "climate limit state" in addition to SLS and ULS criteria.
- Students may propose any approach that could result in limiting material inefficiency, discussing the pros and cons of their idea.

407 2022

Q2(c)



(i) Characteristic Load Effect, f_k

$$f_k = \mu_s + 1.645\sigma_s$$

$$\mu_s = M = PL/4 = 25\,kNm$$

$$\sigma_s = CoV \times \mu_s = 0.15 \times 25 = 3.75\,kNm$$

$$\therefore f_k = 25 + 1.645(3.75) = \underline{\underline{31.2\,kNm}}$$

Design Load Effect, f_d

$$f_d = f_k \times \gamma_f$$

$$= 31.2 \times 1.4 = \underline{\underline{43.6\,kNm}}$$

(ii) Characteristic Bending Strength

$$X_k = \mu_R - 1.645\sigma_R$$

$$\mu_R = 100\,kNm$$

$$\sigma_R = CoV \times \mu_R = 0.15 \times 100 = 15\,kNm$$

$$\therefore X_k = 100 - 1.645(15) = \underline{\underline{75.3\,kNm}}$$

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Q2 (c)

(ii contd..)

Design bending strength $= X_d = \frac{X_k}{\gamma_m}$

$= \frac{75.3}{1.5} = \underline{\underline{50.2}}$ kNm

(iii) $X_d > f_d$ ∴ Design is $\underline{\underline{ok}}$

$(50.2 > 43.6)$

(iv) $\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{100 - 25}{\sqrt{(3.75^2 + 15^2)}} =$

$\beta = 4.851$

Then, from tables:

$\beta = 4.850 = 0.9^6 38$

$\beta = 4.860 = 0.9^6 41$

so $\beta = 4.861 = 0.9^6 39$ (by interpolation)

p(f) = 1 - φ(β)

$= \underline{\underline{6.1 \times 10^{-7}}}$ (low)

4D7 2022

(3)

Q2(c)

(v) Characteristic Strength = Characteristic Load

$X_k = f_k$

$\mu_R - 1.645\,\sigma_R = \mu_s + 1.645\,\sigma_s$

$\mu_R - 1.645(0.15\,\mu_R) = 31.2$

$\therefore \mu_R = \underline{\underline{41.4}}$ kNm

$$\beta = \frac{41.4 - 31.2}{\sqrt{(3.75^2 + 6.21^2)}} = 1.406$$

from tables:

$\beta = 1.40 \rightarrow 0.91924$

$\beta = 1.41 \rightarrow 0.92073$

$\therefore \beta = 1.406 \rightarrow 0.920134$

& $P(f) = \underline{\underline{0.080}}$ (much higher).

This question was generally very well answered, with the only area of difficulty being in the calculation of the reliability index and probability of failure.

407 2022

Q3 (a)

(i) Carbonation $\propto \sqrt{t}$

at $t = 60$ years $c = 14$ mm

ph = 12 when $c = 20$ mm & steel corrodes

$$\frac{20}{14} = \sqrt{\frac{t_2}{60}}$$

$\therefore t_2 = 122$ years, which is from 1960

$\therefore$ remaining life = 62 years.

→ close to the required design life.

→ more cores should be taken to validate rate of corrosion / carbonation

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Q3(a)

(ii) Chloride ingress

- cores should be taken & cement dust analysed.
- Areas of cracked concrete: corrosion may have reached steel already
- Changes in load on new structure: may cause cracking in new areas.

Q3(b)

- Student should explain how whole life carbon is calculated
- "upfront" carbon = 0 for reused elements
- retention of assets → decrease demand for new cement.
- etc.

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Q3 (c)

- Condition assessment
- Impact of corrosion
- Change in loading + arrangement of loading
- understanding actual reinforcement layouts
- accurate assessment of capacity
- understanding real β values.
- etc.

This question was generally quite well answered, with (a)(i) being achievable by all candidates. Answers to following sections were often too brief, and in (b) there was a poor understanding of the carbon content associated with reused elements. Excellent responses were those that demonstrated knowledge beyond repetition of the content of the course handbook.