Version JJO/6

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 23 April 2024 14.00 to 15.40

Module 4D7

CONCRETE AND PRESTRESSED CONCRETE

*Answer **all** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4D7 Data Sheet (5 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 A reinforced concrete office building, built in 1956, is shown in Fig. 1. The existing structure comprises of a car park at ground level, and an office space at first floor level. The office space is supported on tapered concrete beams ($ABCD$), which vary linearly in total height from 250 mm to 500 mm over the 2400 mm tapered section (CD). It is proposed to add one extra storey to the frame, as shown by the dotted line in Fig. 1, and convert the building into residential units.

Eight concrete cores, each 50 mm in diameter and 50 mm long, have been taken from various positions in the structure. Compression testing of these cores gives a mean strength of 55 MPa and standard deviation 4 MPa. The steel is assumed to be deformed bars with a characteristic yield strength of 250 MPa. Investigations show that cover to the steel is 25 mm.

(a) Calculate the characteristic (95th percentile) concrete compressive strength. [5%]

(b) What are the limitations of your estimate of the characteristic concrete strength, and discuss how useful it may be for the purposes of assessing an existing structure. [15%]

(c) Draw shear force and bending moment diagrams for the tapered beam section under the proposed loading. State any assumptions about the load paths in the structure clearly. You may use partial factors of $\gamma_g = 1.35$ and $\gamma_q = 1.5$. [25%]

(d) The beam $ABCD$ has breadth $b = 200$ mm, and is found to have 10 mm diameter links at 150 mm centres in the tapered span. Calculate the shear capacity in the tapered section of the beam at two locations. Comment on your results and assess the viability of the proposed extension. [55%]

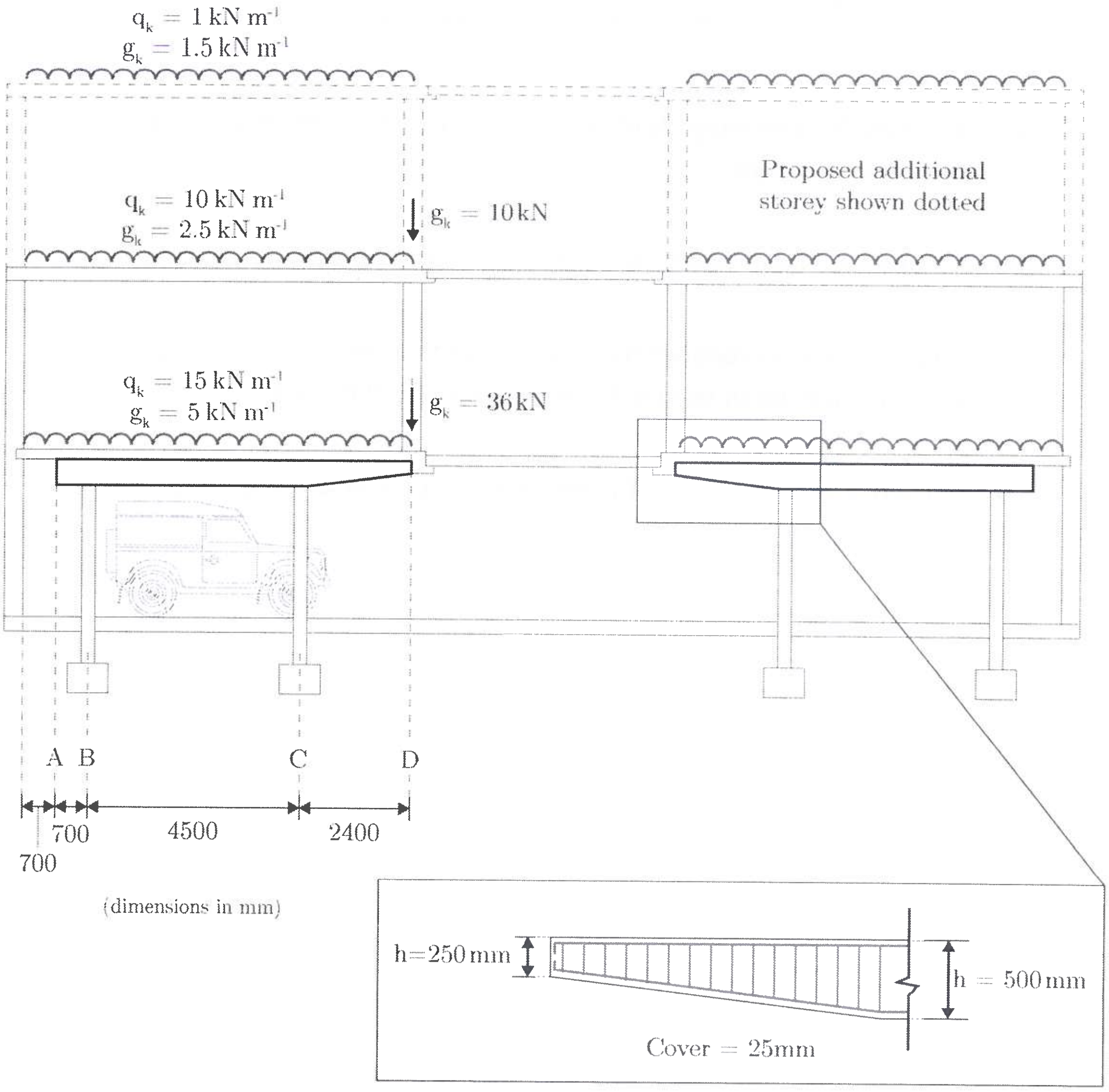
$q_k = 1$ kN m^{-1}
$g_k = 1.5$ kN m^{-1}
$q_k = 10$ kN m^{-1}
$g_k = 2.5$ kN m^{-1}
$g_k = 10$ kN
Proposed additional storey shown dotted
$q_k = 15$ kN m^{-1}
$g_k = 5$ kN m^{-1}
$g_k = 36$ kN
A
B
C
D
700
700
4500
2400
(dimensions in mm)
h=250 mm
h = 500 mm
Cover = 25mm

Fig. 1

2 A reinforced concrete frame built in 1956 is being assessed. All elements to the frame have cover to reinforcement of 25 mm. Carbonation depths were measured at five locations on the frame, with subsequent testing revealing carbonation depths from the surface of 20 mm, 18 mm, 5 mm, 3 mm, and 2 mm. Chloride content was also measured and, on average, chloride contents were 0.60 % by weight of cement at 5 mm from the surface, and 0.30 % by weight of cement at 15 mm from the surface.

(a) Estimate the remaining life of the structure before the steel may be at risk of corrosion. Comment on your results. [50%]

(b) What other factors affect the time to initiation and rate of corrosion? [20%]

(c) Suggest two interventions that will reduce the rate of deterioration of the structure, and enable it to remain in service for at least another 200 years. [15%]

(d) What role does the reuse of concrete assets play in sustainable design? [15%]

3 A prefabricated prestressed concrete beam has a rectangular cross section. The beam is loaded by a sagging moment which varies between 25 kN m and 100 kN m. The moment at transfer is 5 kN m. The characteristic compressive strength of the concrete is 10 MPa at transfer and 15 MPa at 28 days.

(a) Determine the minimum effective depth of the beam if the aspect ratio of the beam (effective depth d to width b) is 2:1 and using the stress limitations in tension and compression given in Table 1. You may assume a loss ratio R of 0.7. [30%]

Table 1

	Transfer	Working load
Tension	$f_{tt} = -1.0$ MPa	$f_{tw} = 0.0$ MPa
Compression	$f_{ct} = 0.40\, f_{ck}$	$f_{cw} = 0.33\, f_{ck}$

(b) Determine the position of the centroid, the second moment of area, and the elastic moduli for the top and bottom fibre using the effective depth calculated in (a) rounded to the nearest multiple of 50 mm. The effect of the small area of the prestressing tendon can be ignored. Assume the total height of the beam $h = d + 50$ mm. [20%]

(c) With the above mentioned permissible stresses in the concrete under the working load, draw a Magnel diagram and determine the maximum permissible prestress if the tendon could be placed anywhere in the section. [40%]

(d) What eccentricity does the cable need to be placed at to result in a zero stress at the top face of the beam due to the prestress? [10%]

END OF PAPER

THIS PAGE IS BLANK

Module 4D7: Data Sheet

Ultimate limit states

For STR and/or GEO it shall be verified that:

$$E_d \leq R_d \tag{1}$$

The design value of the effect of actions, E_d, is given by:

$$E_d = E\left\{\sum_{j\geq 1}\gamma_{G,j}G_{k,j}\text{“}+\text{”}\gamma_p P\text{“}+\text{”}\gamma_{Q,1}Q_{k,1}\text{“}+\text{”}\sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i}\right\} \tag{2}$$

Material partial factors are normally $\gamma_s = 1.15$ for steel and $\gamma_c = 1.5$ for concrete; Partial factors on actions are normally $\gamma_{G,j} = 1.35$ and $\gamma_{Q,1} = 1.5$.

Serviceability limit states

It shall be verified that:

$$E_d \leq C_d \tag{3}$$

The characteristic combination is:

$$E_d = E\left\{\sum_{j\geq 1}G_{k,j}\text{“}+\text{”}P\text{“}+\text{”}Q_{k,1}\text{“}+\text{”}\sum_{i>1}\psi_{0,i}Q_{k,i}\right\} \tag{4}$$

The frequent combination is:

$$E_d = E\left\{\sum_{j\geq 1}G_{k,j}\text{“}+\text{”}P\text{“}+\text{”}\psi_{1,1}Q_{k,1}\text{“}+\text{”}\sum_{i>1}\psi_{2,i}Q_{k,i}\right\} \tag{5}$$

Probability of failure

Design values of actions:

$$F_d = F_k\gamma_f \tag{6}$$

$$F_k = \mu_s + 1.645\sigma_s \tag{7}$$

$$\sigma_s = CoV \times \mu_s \tag{8}$$

Where F_d is the design value; γ_f is the partial safety factor; F_k is the characteristic value, μ_s is the mean value, σ_s is the standard deviation, and CoV is the coefficient of variation.

Design values of product properties:

$$X_d = \frac{X_k}{\gamma_m} \tag{9}$$

$$X_k = \mu_R - 1.645\sigma_R \tag{10}$$

$$\sigma_R = CoV \times \mu_R \tag{11}$$

Where X_d is the design value; γ_m the partial safety factor; X_k the characteristic value, μ_R the mean value, σ_R the standard deviation, and CoV the coefficient of variation.

Reliability index, β

$$\beta = \frac{\mu_R - \mu_s}{\sqrt{\sigma_R^2 + \sigma_s^2}} \tag{12}$$

Probability of failure:

$$P_f = \Phi(-\beta) \tag{13}$$

Where Φ is the standard normal cumulative distribution function.

The difference between two normally distributed variables is itself normally distributed, with mean equal to the difference of the means, and variance the sum of the squares of the standard deviations.

Durability considerations

Uniaxial diffusion into a homogenous material:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} \tag{14}$$

Solution:

$$C_x = C_0\left[1 - \mathrm{erf}\,(z)\right] \tag{15}$$

$$z = \frac{x}{2(Dt)^{0.5}} \tag{16}$$

Table of *erf(z)*

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	∞
erf(z)	0	0.11	0.22	0.33	0.43	0.52	0.60	0.68	0.74	0.80	0.84	0.88	0.91	0.93	0.95	0.97	1.00

Deflections

Interpolated curvature:

$$\alpha = \zeta\alpha_{||} + (1 - \zeta)\,\alpha_{|} \tag{17}$$

Where α is a deflection, $\alpha_{/}$ and $\alpha_{//}$ are the values for the uncracked and fully cracked conditions, ζ is a distribution coefficient:

$$\zeta = 1 - \beta\left(\frac{\sigma_{sr}}{\sigma_s}\right)^2 \tag{18}$$

Where σ_{sr} is the stress in the tension reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking; σ_s is the stress in the tension reinforcement calculated on the basis of a cracked section; $\beta = 1.0$ for single short term loading and $\beta = 0.5$ for sustained loads or many cycles of repeated loading.

ULS Flexure

A doubly reinforced concrete section when flexural strength is reached is shown in Figure 1. It is usual to assume that failure occurs when the extreme fibre compressive strain in the concrete reaches a limiting value of 0.0035. Forces are found by equilibrium of the section.

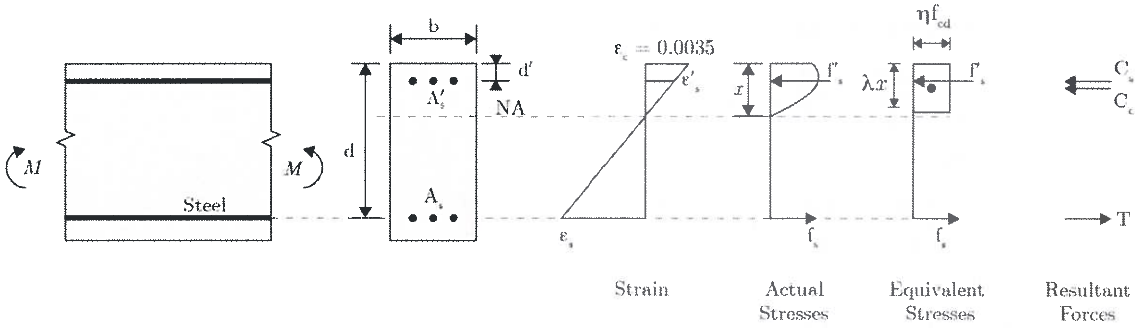

Figure 1

	$f_{ck} \leqslant 50$ MPa	50 MPa $< f_{ck} \leqslant 90$ MPa
λ	0.8	$0.8 - (f_{ck} - 50)/400$
η	1.0	$1.0 - (f_{ck} - 50)/200$

ULS Shear and Torsion

For unreinforced webs at ULS:

$$V_{Rd,c} = \left[\frac{0.18}{\gamma_c} k \left(100 \rho_1 f_{ck}\right)^{\frac{1}{3}} + 0.15\sigma_{cp}\right] b_w d \qquad (19)$$

$$\geq \left(v_{min} + 0.15\sigma_{cp}\right) b_w d$$

$k = 1 + (200/d)^{0.5} \leq 2.0$

$\gamma_c = 1.5$

$\rho_l = A_s/b_w d \ (\rho_l \leq 0.02)$

$v_{min} = 0.035 k^{3/2} f_{ck}^{1/2}$

For reinforced webs at ULS:

$$V_{Rd,s} = \frac{A_{sw}}{s} z f_{ywd} \cot\theta \qquad (20)$$

$$V_{Rd,max} = \alpha_{cw} b_w z \nu_1 f_{cd} / \left(\cot\theta + \tan\theta\right) \qquad (21)$$

$\alpha_{cw} = 1$ for non-prestressed structures

$\nu_l = 0.6$ for $f_{ck} \leq 60$ MPa and $\nu_l = 0.9 - f_{ck}/200 > 0.5$ for $f_{ck} \geq 60$MPa

The shear stress in a wall of a section subject to pure torsion:

$$\tau_{t,i} t_{ef,i} = \frac{T_{Ed}}{2A_k} \qquad (22)$$

$\tau_{t,i}$ = torsional stress in wall i; $t_{ef,i}$ = effective wall thickness (= total area of cross section / outer circumference), A_k = area enclosed by centrelines of the walls including inner hollow areas.

Prestressed concrete

Elastic analysis: compression is positive. Eq.(23) applies for both top and bottom fibres since Z_i has sign:

$$\sigma = \frac{P}{A} + \frac{Pe}{Z_i} - \frac{M}{Z_i} \tag{23}$$

To design prestress, stress inequalities take the form:

$$f_c \geq \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} \geq f_t \tag{24}$$

For fibre 1 (top):

$$-\frac{Z_1}{A} + \frac{f_c Z_1}{P} + \frac{M}{P} \leq e \leq -\frac{Z_1}{A} + \frac{f_t Z_1}{P} + \frac{M}{P} \tag{25}$$

For fibre 2 (bottom):

$$-\frac{Z_2}{A} + \frac{f_c Z_2}{P} + \frac{M}{P} \geq e \geq -\frac{Z_2}{A} + \frac{f_t Z_2}{P} + \frac{M}{P} \tag{26}$$

Cumulative normal distribution function

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} e^{-\frac{x^2}{2}}\,dx \quad \text{FOR} \quad 0{\cdot}00 \le u \le 4{\cdot}99.$$

u	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7703	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
1·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
1·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·90147
1·3	·90320	·90490	·90658	·90824	·90988	·91149	·91309	·91466	·91621	·91774
1·4	·91924	·92073	·92220	·92364	·92507	·92647	·92785	·92922	·93056	·93189
1·5	·93319	·93448	·93574	·93699	·93822	·93943	·94062	·94179	·94295	·94408
1·6	·94520	·94630	·94738	·94845	·94950	·95053	·95154	·95254	·95352	·95449
1·7	·95543	·95637	·95728	·95818	·95907	·95994	·96080	·96164	·96246	·96327
1·8	·96407	·96485	·96562	·96638	·96712	·96784	·96856	·96926	·96995	·97062
1·9	·97128	·97193	·97257	·97320	·97381	·97441	·97500	·97558	·97615	·97670
2·0	·97725	·97778	·97831	·97882	·97932	·97982	·98030	·98077	·98124	·98169
2·1	·98214	·98257	·98300	·98341	·98382	·98422	·98461	·98500	·98537	·98574
2·2	·98610	·98645	·98679	·98713	·98745	·98778	·98809	·98840	·98870	·98899
2·3	·98928	·98956	·98983	$\cdot9^{2}0097$	$\cdot9^{2}0358$	$\cdot9^{2}0613$	$\cdot9^{2}0863$	$\cdot9^{2}1106$	$\cdot9^{2}1344$	$\cdot9^{2}1576$
2·4	$\cdot9^{2}1802$	$\cdot9^{2}2024$	$\cdot9^{2}2240$	$\cdot9^{2}2451$	$\cdot9^{2}2656$	$\cdot9^{2}2857$	$\cdot9^{2}3053$	$\cdot9^{2}3244$	$\cdot9^{2}3431$	$\cdot9^{2}3613$
2·5	$\cdot9^{2}3790$	$\cdot9^{2}3963$	$\cdot9^{2}4132$	$\cdot9^{2}4297$	$\cdot9^{2}4457$	$\cdot9^{2}4614$	$\cdot9^{2}4766$	$\cdot9^{2}4915$	$\cdot9^{2}5060$	$\cdot9^{2}5201$
2·6	$\cdot9^{2}5339$	$\cdot9^{2}5473$	$\cdot9^{2}5604$	$\cdot9^{2}5731$	$\cdot9^{2}5855$	$\cdot9^{2}5975$	$\cdot9^{2}6093$	$\cdot9^{2}6207$	$\cdot9^{2}6319$	$\cdot9^{2}6427$
2·7	$\cdot9^{2}6533$	$\cdot9^{2}6636$	$\cdot9^{2}6736$	$\cdot9^{2}6833$	$\cdot9^{2}6928$	$\cdot9^{2}7020$	$\cdot9^{2}7110$	$\cdot9^{2}7197$	$\cdot9^{2}7282$	$\cdot9^{2}7365$
2·8	$\cdot9^{2}7445$	$\cdot9^{2}7523$	$\cdot9^{2}7599$	$\cdot9^{2}7673$	$\cdot9^{2}7744$	$\cdot9^{2}7814$	$\cdot9^{2}7882$	$\cdot9^{2}7948$	$\cdot9^{2}8012$	$\cdot9^{2}8074$
2·9	$\cdot9^{2}8134$	$\cdot9^{2}8193$	$\cdot9^{2}8250$	$\cdot9^{2}8305$	$\cdot9^{2}8359$	$\cdot9^{2}8411$	$\cdot9^{2}8462$	$\cdot9^{2}8511$	$\cdot9^{2}8559$	$\cdot9^{2}8605$
3·0	$\cdot9^{2}8650$	$\cdot9^{2}8694$	$\cdot9^{2}8736$	$\cdot9^{2}8777$	$\cdot9^{2}8817$	$\cdot9^{2}8856$	$\cdot9^{2}8893$	$\cdot9^{2}8930$	$\cdot9^{2}8965$	$\cdot9^{2}8999$
3·1	$\cdot9^{3}0324$	$\cdot9^{3}0646$	$\cdot9^{3}0957$	$\cdot9^{3}1260$	$\cdot9^{3}1553$	$\cdot9^{3}1836$	$\cdot9^{3}2112$	$\cdot9^{3}2378$	$\cdot9^{3}2636$	$\cdot9^{3}2886$
3·2	$\cdot9^{3}3129$	$\cdot9^{3}3363$	$\cdot9^{3}3590$	$\cdot9^{3}3810$	$\cdot9^{3}4024$	$\cdot9^{3}4230$	$\cdot9^{3}4429$	$\cdot9^{3}4623$	$\cdot9^{3}4810$	$\cdot9^{3}4991$
3·3	$\cdot9^{3}5166$	$\cdot9^{3}5335$	$\cdot9^{3}5499$	$\cdot9^{3}5658$	$\cdot9^{3}5811$	$\cdot9^{3}5959$	$\cdot9^{3}6103$	$\cdot9^{3}6242$	$\cdot9^{3}6376$	$\cdot9^{3}6505$
3·4	$\cdot9^{3}6631$	$\cdot9^{3}6752$	$\cdot9^{3}6869$	$\cdot9^{3}6982$	$\cdot9^{3}7091$	$\cdot9^{3}7197$	$\cdot9^{3}7299$	$\cdot9^{3}7398$	$\cdot9^{3}7493$	$\cdot9^{3}7585$
3·5	$\cdot9^{3}7674$	$\cdot9^{3}7759$	$\cdot9^{3}7842$	$\cdot9^{3}7922$	$\cdot9^{3}7999$	$\cdot9^{3}8074$	$\cdot9^{3}8146$	$\cdot9^{3}8215$	$\cdot9^{3}8282$	$\cdot9^{3}8347$
3·6	$\cdot9^{3}8409$	$\cdot9^{3}8469$	$\cdot9^{3}8527$	$\cdot9^{3}8583$	$\cdot9^{3}8637$	$\cdot9^{3}8689$	$\cdot9^{3}8739$	$\cdot9^{3}8787$	$\cdot9^{3}8834$	$\cdot9^{3}8879$
3·7	$\cdot9^{3}8922$	$\cdot9^{3}8964$	$\cdot9^{4}0039$	$\cdot9^{4}0426$	$\cdot9^{4}0799$	$\cdot9^{4}1158$	$\cdot9^{4}1504$	$\cdot9^{4}1838$	$\cdot9^{4}2159$	$\cdot9^{4}2468$
3·8	$\cdot9^{4}2765$	$\cdot9^{4}3052$	$\cdot9^{4}3327$	$\cdot9^{4}3593$	$\cdot9^{4}3848$	$\cdot9^{4}4094$	$\cdot9^{4}4331$	$\cdot9^{4}4558$	$\cdot9^{4}4777$	$\cdot9^{4}4988$
3·9	$\cdot9^{4}5190$	$\cdot9^{4}5385$	$\cdot9^{4}5573$	$\cdot9^{4}5753$	$\cdot9^{4}5926$	$\cdot9^{4}6092$	$\cdot9^{4}6253$	$\cdot9^{4}6406$	$\cdot9^{4}6554$	$\cdot9^{4}6696$
4·0	$\cdot9^{4}6833$	$\cdot9^{4}6964$	$\cdot9^{4}7090$	$\cdot9^{4}7211$	$\cdot9^{4}7327$	$\cdot9^{4}7439$	$\cdot9^{4}7546$	$\cdot9^{4}7649$	$\cdot9^{4}7748$	$\cdot9^{4}7843$
4·1	$\cdot9^{4}7934$	$\cdot9^{4}8022$	$\cdot9^{4}8106$	$\cdot9^{4}8186$	$\cdot9^{4}8263$	$\cdot9^{4}8338$	$\cdot9^{4}8409$	$\cdot9^{4}8477$	$\cdot9^{4}8542$	$\cdot9^{4}8605$
4·2	$\cdot9^{4}8665$	$\cdot9^{4}8723$	$\cdot9^{4}8778$	$\cdot9^{4}8832$	$\cdot9^{4}8882$	$\cdot9^{4}8931$	$\cdot9^{4}8978$	$\cdot9^{5}0226$	$\cdot9^{5}0655$	$\cdot9^{5}1066$
4·3	$\cdot9^{5}1460$	$\cdot9^{5}1837$	$\cdot9^{5}2199$	$\cdot9^{5}2545$	$\cdot9^{5}2876$	$\cdot9^{5}3193$	$\cdot9^{5}3497$	$\cdot9^{5}3788$	$\cdot9^{5}4066$	$\cdot9^{5}4332$
4·4	$\cdot9^{5}4587$	$\cdot9^{5}4831$	$\cdot9^{5}5065$	$\cdot9^{5}5288$	$\cdot9^{5}5502$	$\cdot9^{5}5706$	$\cdot9^{5}5902$	$\cdot9^{5}6089$	$\cdot9^{5}6268$	$\cdot9^{5}6439$
4·5	$\cdot9^{5}6602$	$\cdot9^{5}6759$	$\cdot9^{5}6908$	$\cdot9^{5}7051$	$\cdot9^{5}7187$	$\cdot9^{5}7318$	$\cdot9^{5}7442$	$\cdot9^{5}7561$	$\cdot9^{5}7675$	$\cdot9^{5}7784$
4·6	$\cdot9^{5}7888$	$\cdot9^{5}7987$	$\cdot9^{5}8081$	$\cdot9^{5}8172$	$\cdot9^{5}8258$	$\cdot9^{5}8340$	$\cdot9^{5}8419$	$\cdot9^{5}8494$	$\cdot9^{5}8566$	$\cdot9^{5}8634$
4·7	$\cdot9^{5}8699$	$\cdot9^{5}8761$	$\cdot9^{5}8821$	$\cdot9^{5}8877$	$\cdot9^{5}8931$	$\cdot9^{5}8983$	$\cdot9^{6}0320$	$\cdot9^{6}0789$	$\cdot9^{6}1235$	$\cdot9^{6}1661$
4·8	$\cdot9^{6}2067$	$\cdot9^{6}2453$	$\cdot9^{6}2822$	$\cdot9^{6}3173$	$\cdot9^{6}3508$	$\cdot9^{6}3827$	$\cdot9^{6}4131$	$\cdot9^{6}4420$	$\cdot9^{6}4696$	$\cdot9^{6}4958$
4·9	$\cdot9^{6}5208$	$\cdot9^{6}5446$	$\cdot9^{6}5673$	$\cdot9^{6}5889$	$\cdot9^{6}6094$	$\cdot9^{6}6289$	$\cdot9^{6}6475$	$\cdot9^{6}6652$	$\cdot9^{6}6821$	$\cdot9^{6}6981$

Example: $\Phi(3{\cdot}57) = \cdot9^{3}8215 = 0{\cdot}9998215.$