Module 4D7
Paper 2024

Version JJO/6
Date 10·1·24
Page

1(a) $f_{ck} = f_{cm} - 1.64\sigma$

$= 55 - 1.64(4) = 48\text{ MPa}$

(b)
- only 8 cores
- ⌀50mm is small
- assumes one mix design for whole frame

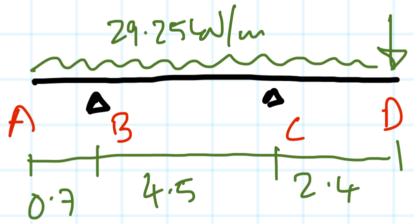
(c) Load take-down

$$\text{Roof} = 1.35(1.5) + 1.5(1) = 3.5\text{ kN/m}$$
$$PL_1 = 1.35(10) = 13.5\text{ kN}$$
$$\text{Floor 1} = 1.35(2.5) + 1.5(10) = 18.4\text{ kN/m}$$
$$PL_2 = 1.35(36) = 48.6\text{ kN}$$

$L = 8.3\text{m}$ ∴ total point load $= (3.5 + 18.4)\left(\frac{8.3}{2}\right)$

$+ 13.5 + 48.6 = 153\text{ kN}$

$$w = 1.35(5) + 1.5(15) = 29.25\text{ kN/m}$$



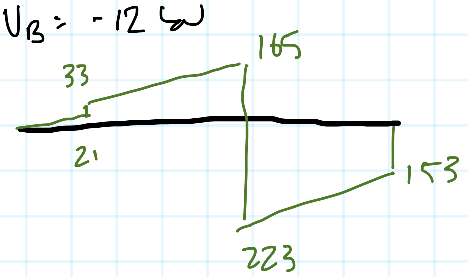
(assume cantilever slab carried by wall)

$$V_B: 29.25\left(\frac{6.9^2}{2}\right) + 153(6.9) - 29.25\left(\frac{0.7^2}{2}\right) = 4.5V_C$$

$V_C = 388\text{ kN}$

$V_B = -12\text{ kN}$

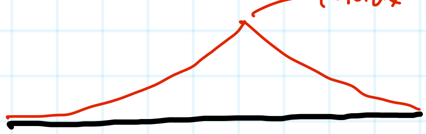
SFD:



(rounded)

BMD

$M_{max} = \frac{1}{2}(153 + 223)2.4 = 451\text{ kNm}$



(1)(d) Section 1, 100mm from tip

$V_{Ed} = 153 + 29.25(0.1) = 156kN$
$M_{Ed} = \frac{1}{2}(153+156)0.1 = 15.5kNm$

$h = 250 + (250/2400)0.1 = 260mm$
$d = 260 - 25 - 10 - 8 = 217mm$
cover - link - guess bar ϕ

$z = 0.9d = 195mm$
$V_{ccd} = M/z = 15.5\times10^6 / 195 = 80kN$
angle = $\tan^{-1}(250/2400) = 5.95°$

$V_{ccd} = 80\times10^3 \sin 5.95 = 8.3kN$
$V_{Ed} = \underline{148kN}$

$V_{Rd,s} = \frac{50\pi}{150}(195)\left(\frac{250}{1.15}\right)2.5 = 111kN$ (not ok)

(could also check $V_{Rd,max} = \frac{(1)(200)(195)(0.6)}{2.5 + 1/2.5}\left(48/1.5\right)$
$= \underline{258kN}$)

Section 2 at 2400mm
$V_{Ed} = 223kN$
$M_{Ed} = 451 kNm$

$h = 500$
$d = 500 - 25 - 10 - 8 = 457mm$; $z = 0.9d = 411mm$

$F_c = 451\times10^6/411 = 1097kN$
$V_{ccd} = 1097\times10^3 \sin 5.95 = 113kN$
$\therefore V_{Ed} = 223 - 113 = \underline{110kN}$

$V_{Rd,s} = \frac{50\pi}{150}(411)\left(\frac{250}{1.15}\right)2.5 = 234kN > V_{Ed}$ $\therefore$ ok

$V_{Rd,max} = \frac{(1)(200)(411)(0.6)}{2.5 + 1/2.5}\left(48/1.5\right) = 544kN > V_{Rd,s}$ $\therefore$ ok

Module 4D7
Paper 2024

Version 5J0/b
Date
Page

1(d)... Section 2 works, but section 1 does not,

to make it viable, we could use full fywd

$\Rightarrow V_{Ed,s} = 1.05(195)(250)(2.5) = 127kN$ ($< V_{ED}$ ∴ not ok)

- we could reduce the load from extension - use other materials, reduce q_k, use other load paths to try and make it work

Module 4D7
Paper 2024

Version J10/B
Date
Page

2(a) Carbonation: we have a range of data - analyse best and worst case:

Core 1 $\quad 20\,mm \propto \sqrt{t}$ $\quad$ (2024 - 1956 = 68 years)

$25\,mm \propto \sqrt{t}$

$\frac{25}{20} = \sqrt{\frac{t}{68}}$ $\quad \therefore t = 106$ years

Core 5 $\quad 2\,mm \propto \sqrt{68}$

$25\,mm \propto \sqrt{t}$

$\frac{25}{2} = \sqrt{\frac{t}{68}}$ $\quad \therefore t > 10{,}000$ years!

Chlorides.

$$C_x = C_0\left(1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right)$$

$$0.006 = C_0\left(1 - \text{erf}\left(5/2\sqrt{Dt}\right)\right)$$
$$0.003 = C_0\left(1 - \text{erf}\left(15/2\sqrt{Dt}\right)\right)$$

$$\frac{0.006}{0.003} = \frac{1 - \text{erf}\left(5/2\sqrt{Dt}\right)}{1 - \text{erf}\left(15/2\sqrt{Dt}\right)} = 2$$

$$\frac{5/2\sqrt{Dt}}{15/2\sqrt{Dt}} = 3 = \frac{x_1}{x_2}$$ guess $x_1 = 0.2$ $\quad \therefore x_2 = 0.6$

erf x_1 = 0.22 (from tables) $\quad \therefore 1 - \text{erf}\, x_1 = 0.78$ $\quad \approx 2\times$ ✓ok

erf x_2 = 0.6 $\quad \therefore 1 - \text{erf}\, x_2 = 0.4$

$$\frac{15}{2\sqrt{Dt_1}} = 0.2 \quad \& \quad \frac{5}{2\sqrt{Dt_1}} = 0.6$$

t_1 = 68 years = 2145916800 seconds

$\therefore D = 72.8 \times 10^{-9}\,m^2/s$

$$C_0 = \frac{0.006}{0.78} = 0.00769$$ (or $C_0 = \frac{0.003}{0.4} = 0.0075$)

Module 4D7
Paper 2024

Version J10'b
Date
Page

(a)... then, find where is 0.4%

$0.004 = 0.00769\left(1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right)$

$1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right) = 0.520$

$\text{erf}\left(\frac{x}{2\sqrt{Dt}}\right) = 0.480 \rightarrow$ from tables $z \simeq 0.45$

$\Rightarrow 0.45 = \frac{x}{2\sqrt{Dt}}$ & $\underline{\underline{x = 11mm}}$

finally, 0.4% at 11mm at 68 years
0.4% at 25mm at X years

$\frac{25}{11} = \sqrt{\frac{X}{68}} \Rightarrow X = \underline{350\,\text{years}}$

Carbonation is critical.

(b) exposure, cracking, quality, surface finish, compaction, permeability, etc...

(c) change environmental exposure; paint or seal concrete surface, etc..

(d) vital. $EC_{A13} = 0$ for reused components
- retention to be prioritised over demolition
- can be more complex, more unknowns, need to understand the structure clearly.

Question #3

(a) Stress limitations:

$$f_{tt} = -1{,}0 \text{ MPa}$$
$$f_{tw} = 0{,}0 \text{ MPa}$$
$$f_{ct} = 0{,}4 \cdot f_{ck}(t_t) = 0{,}4 \cdot 10 = 4{,}0 \text{ MPa}$$
$$f_{cw} = 0{,}33\, f_{ck}(28) = 0{,}33 \cdot 15 = 4{,}95 \text{ MPa}.$$

Effective depth can be calculated for Z_1 and Z_2. Using $d/b = 2$:

$$Z_i = \pm \frac{bd^2}{6} \implies d = \sqrt[3]{12 \cdot Z_i}$$

Inequalities for Z_i:

$$Z_1 \leq \frac{M_a - R \cdot M_t}{R \cdot f_{ct} - f_{tw}} \quad (1) \qquad Z_2 \geq \frac{R \cdot M_t - M_a}{f_{cw} - R \cdot f_{tt}} \quad (4)$$

$$Z_1 \leq -\frac{M_b - R \cdot M_t}{f_{cw} - R f_{tt}} \quad (2) \qquad Z_2 \geq \frac{M_b - R \cdot M_t}{R f_{ct} - f_{tw}} \quad (5)$$

$$Z_1 \leq -\frac{M_b - M_a}{f_{cw} - f_{tw}} \quad (3) \qquad Z_2 \geq \frac{M_b - M_a}{f_{cw} - f_{tw}} \quad (6)$$

with $M_t = 5 \text{ kNm}$; $M_a = 25 \text{ kNm}$; $M_b = 100 \text{ kNm}$ and $R = 0{,}7$

Inequalities (2) and (5) determining:

$$Z_1 \leq \frac{100 \text{ kNm} - 0{,}7 \cdot 5 \text{ kNm}}{4{,}95 - 0{,}7 \cdot (-1{,}0)} \leq -25{,}9 \cdot 10^6 \text{ mm}^3$$

$$Z_2 \geq \frac{100 \text{ kNm} - 0{,}7 \cdot 5 \text{ kNm}}{0{,}7 \cdot 4{,}0 - 0{,}0} \geq 34{,}5 \text{ mm}^3$$

$$\implies \boxed{d \geq 745{,}05 \text{ mm}}$$

(b) $d \geq 745{,}05\,mm$ from (a) rounding to multiple of 50mm:

$$d = 750\,mm$$

so: $$h = d + 50 = 800\,mm$$
$$b = d/2 = 375\,mm.$$

Hence: $$A = 800 \cdot 375 = 300 \cdot 10^3\,mm^2$$
$$I = bh^3/12 = 16 \cdot 10^9\,mm^4$$
$$\bar{y} = h/2 = 400\,mm$$
$$Z_1 = I/-\bar{y} = -40 \cdot 10^6\,mm^3$$
$$Z_2 = I/\bar{y} = 40 \cdot 10^6\,mm^3$$

To draw magnel diagram, choose P. E.g. $P = \frac{f_c}{2} \cdot A$

$$P = 2250\,kN.$$

At top, worst tension when min moment is applied, worst compression when max. moment is applied:

$$\frac{f_{cw} Z_1}{P} - \frac{Z_1}{A} + \frac{M_b}{P} < e < \frac{f_{tw} Z_1}{P} - \frac{Z_1}{A} + \frac{M_a}{P}$$

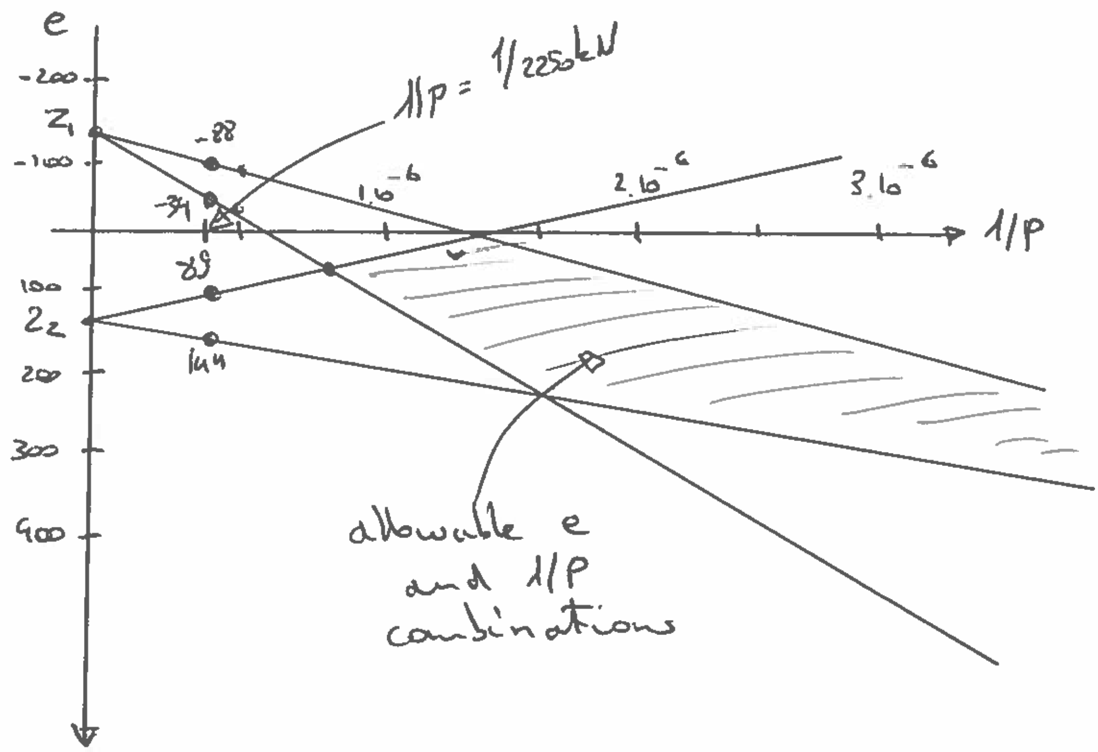
so $$e > 89{,}78\,mm \quad \text{(max. compr.)}$$
$$e < 144{,}44\,mm. \quad \text{(min tension)}$$

Again at bottom fibre stress limits:

$$\frac{f_{cw} Z_2}{P} - \frac{Z_2}{A} + \frac{M}{P} < e < \frac{f_{tw} Z_2}{P} - \frac{Z_2}{A} + \frac{M}{P}$$

$$\Rightarrow \quad e > -88{,}89\,mm$$
$$e < -34{,}22\,mm$$

Drawing Magnel diagrams



→ P_{max} reading from diagram or calculating using eq. above:

$$P_{max} = 1204 \text{ kN}$$

(d) Zero stress at top when prestress is in lower kern point.
For rectangular sections kern points can be found with 'middle third rule'

⇒ P in lower third point.