Version JJO/3

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 29 April 2025 09:30 – 11:10

Module 4D7

CONCRETE AND PRESTRESSED CONCRETE

*Answer **all** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4D7 Data Sheet (5 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 A reinforced concrete beam has a rectangular cross-section with a width of 400 mm and a height of 800 mm. The bottom reinforcement of the beam consists of three ribbed reinforcement bars with a diameter of 20 mm. Two-legged closed rectangular loops with a bar diameter of 10 mm and a cover of 35 mm are provided as shear reinforcement. All reinforcement steel has a design yield strength f_{yd} of 435 MPa. The design compressive strength of the concrete f_{cd} is 25 MPa.

The beam has a simply supported span of 10 m and cantilevers out at one end by a futher 5 m. The design load consists of a uniformly distributed load of 22.5 kNm^{-1} acting along any part of the beam. The self-weight of the beam can be ignored.

(a) Determine and plot the shear force and bending moment envelopes of the beam considering the most unfavourable positions of the load. [15%]

(b) Determine the required spacing s of the stirrups for the most critical section along the beam. Use the variable truss angle method and assume the angle θ of the compression struts such that $cot\theta = 2.5$. [25%]

(c) Determine the positions at which one out of the three bottom reinforcement bars can theoretically be curtailed as it is no longer required to resist the bending moment in the beam (assume the beam to be acting as singly reinforced). [40%]

(d) What would the actual locations of these points be, taking into account the bond length of the reinforcement? (No detailed calculations are required; rule of thumb approximations can be used). [10%]

(e) How could the total length of the reinforcement be reduced further? Explain your answer and give two examples. [10%]

2 A 6 m span prestressed concrete beam has a rectangular cross-section with external dimensions of 250 mm width and 350 mm depth. An initial prestressing force of 350 kN is applied at an eccentricity of 70 mm by steel tendons with total area of 400 mm^2.

The modulus of elasticity of the steel and concrete are 200 GPa and 33 GPa respectively. Assuming slip in the anchorage is 1.5 mm, the creep coefficient in the concrete is 1, the shrinkage strain of concrete is 0.0002, and the relaxation loss in the steel is 3%, find the following:

(a) The initial stress in the tendon. [5%]

(b) The loss of stress due to elastic shortening of the concrete. [25%]

(c) The loss of prestress due to anchorage slip. [20%]

(d) The loss of stress due to creep and shrinkage. [20%]

(e) The loss of stress due to relaxation of the steel. [20%]

(f) The total loss of stress in the tendon. Comment on your result. [10%]

3 You are designing a reinforced concrete tower in a seismically active region. The proposed design has a total height of 400 m and is square on plan with a side length of 33 m. The tower is to have a central reinforced concrete core.

(a) Discuss how upfront embodied carbon could be minimised, considering strategies for element design, material selection, and construction techniques. [30%]

(b) In a seismic event, the design value of the applied torsional moment, T_{Ed}, at the base of the tower is 200 MNm. Design the core to carry this torsion, assuming it to be a hollow square on plan with an outer width of 21 m. You should propose a thickness for the core walls, and calculate the longitudinal and transverse reinforcement area required. [60%]

(c) Without further calculation, discuss other critical design considerations that should be accounted for in the design of the core. [10%]

END OF PAPER

Module 4D7: Data Sheet

Ultimate limit states

It shall be verified that:

$$E_d \leq R_d \tag{1}$$

The design value of the effect of actions, E_d, for a specific combination of actions should be calculated by:

$$E_d = E\left\{\sum_{j\geq 1}\gamma_{G,j}G_{k,j}\text{“}+\text{”}\gamma_p P\text{“}+\text{”}\gamma_{Q,1}Q_{k,1}\text{“}+\text{”}\sum_{i>1}\gamma_{Q,i}\psi_{0,i}Q_{k,i}\right\} \tag{2}$$

Material partial factors are normally $\gamma_s = 1.15$ for steel and $\gamma_c = 1.5$ for concrete;
Partial factors on actions are normally $\gamma_{G,j} = 1.35$ and $\gamma_{Q,1} = 1.5$.

Serviceability limit states

It shall be verified that:

$$E_d \leq C_d \tag{3}$$

The characteristic combination is:

$$E_d = E\left\{\sum_{j\geq 1}G_{k,j}\text{“}+\text{”}P\text{“}+\text{”}Q_{k,1}\text{“}+\text{”}\sum_{i>1}\psi_{0,i}Q_{k,i}\right\} \tag{4}$$

The frequent combination is:

$$E_d = E\left\{\sum_{j\geq 1}G_{k,j}\text{“}+\text{”}P\text{“}+\text{”}\psi_{1,1}Q_{k,1}\text{“}+\text{”}\sum_{i>1}\psi_{2,i}Q_{k,i}\right\} \tag{5}$$

Probability of failure

Design values of actions:

$$F_d = F_k\gamma_f \tag{6}$$

$$F_k = \mu_s + 1.645\sigma_s \tag{7}$$

$$\sigma_s = CoV \times \mu_s \tag{8}$$

Where F_d is the design value; γ_f is the partial safety factor; F_k is the characteristic value, μ_s is the mean value, σ_s is the standard deviation, and CoV is the coefficient of variation.

Design values of product properties:

$$X_d = \frac{X_k}{\gamma_m} \tag{9}$$

$$X_k = \mu_R - 1.645\sigma_R \tag{10}$$

$$\sigma_R = CoV \times \mu_R \tag{11}$$

Where X_d is the design value; γ_m the partial safety factor; X_k the characteristic value, μ_R the mean value, σ_R the standard deviation, and CoV the coefficient of variation.

Reliability index, β

$$\beta = \frac{\mu_R - \mu_s}{\sqrt{\sigma_R^2 + \sigma_s^2}} \tag{12}$$

Probability of failure:

$$P_f = \Phi(-\beta) \tag{13}$$

Where Φ is the standard normal cumulative distribution function.

The difference between two normally distributed variables is itself normally distributed, with mean equal to the difference of the means, and variance the sum of the squares of the standard deviations.

Durability considerations

Uniaxial diffusion into a homogenous material:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} \tag{14}$$

Solution:

$$C_x = C_0\left[1 - \operatorname{erf}(z)\right] \tag{15}$$

$$z = \frac{x}{2(Dt)^{0.5}} \tag{16}$$

Table of *erf(z)*

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	∞
erf(z)	0	0.11	0.22	0.33	0.43	0.52	0.60	0.68	0.74	0.80	0.84	0.88	0.91	0.93	0.95	0.97	1.00

Deflections

Interpolated curvature:

$$\alpha = \zeta\alpha_{||} + (1-\zeta)\,\alpha_{|} \tag{17}$$

Where α is a deflection, $\alpha_{/}$ and $\alpha_{//}$ are the values for the uncracked and fully cracked conditions, ζ is a distribution coefficient:

$$\zeta = 1 - \beta\left(\frac{\sigma_{sr}}{\sigma_s}\right)^2 \tag{18}$$

Where σ_{sr} is the stress in the tension reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking; σ_s is the stress in the tension reinforcement calculated on the basis of a cracked section; $\beta = 1.0$ for single short term loading and $\beta = 0.5$ for sustained loads or many cycles of repeated loading.

ULS Flexure

A doubly reinforced concrete section when flexural strength is reached is shown in Figure 1. It is usual to assume that failure occurs when the extreme fibre compressive strain in the concrete reaches a limiting value of 0.0035. Forces are found by equilibrium of the section.

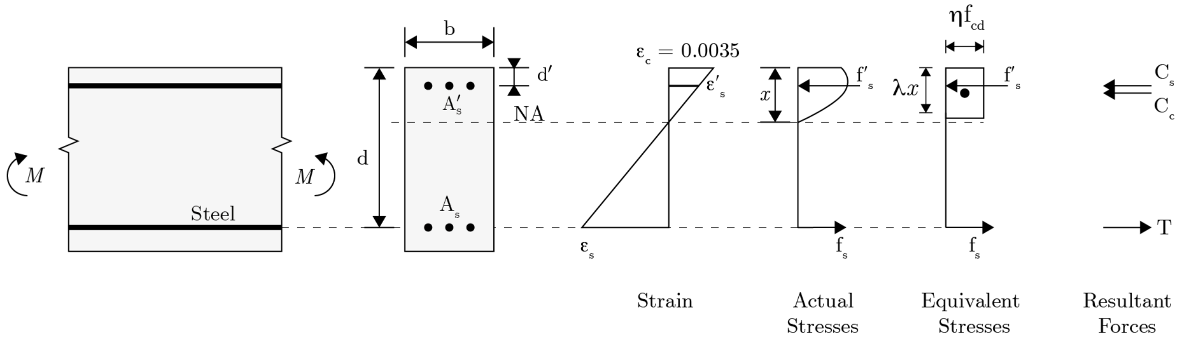

Figure 1

	$f_{ck} \leqslant 50$ MPa	50 MPa $< f_{ck} \leqslant 90$ MPa
λ	0.8	$0.8 - (f_{ck} - 50)/400$
η	1.0	$1.0 - (f_{ck} - 50)/200$

ULS Shear and Torsion

For unreinforced webs at ULS:

$$V_{Rd,c} = \left[\frac{0.18}{\gamma_c} k\,(100\rho_1 f_{ck})^{\frac{1}{3}} + 0.15\sigma_{cp}\right] b_w d \qquad (19)$$

$$\geq (v_{min} + 0.15\sigma_{cp})\, b_w d$$

$k = 1 + (200/d)^{0.5} \leq 2.0$

$\gamma_c = 1.5$

$\rho_1 = A_s/b_w d \ (\rho_1 \leq 0.02)$

$v_{min} = 0.035 k^{3/2} f_{ck}{}^{1/2}$

For reinforced webs at ULS:

$$V_{Rd,s} = \frac{A_{sw}}{s} z f_{ywd} cot\theta \qquad (20)$$

$$V_{Rd,max} = \alpha_{cw} b_w z \nu_1 f_{cd} / \left(cot\theta + tan\theta\right) \qquad (21)$$

$\alpha_{cw} = 1$ for non-prestressed structures

$\nu_1 = 0.6$ *for* $f_{ck} \leq 60$ *MPa and* $\nu_1 = 0.9 - f_{ck}/200 > 0.5$ *for* $f_{ck} \geq 60 MPa$

The shear stress in a wall of a section subject to pure torsion:

$$\tau_{t,i} t_{ef,i} = \frac{T_{Ed}}{2A_k} \qquad (22)$$

$\tau_{t,i}$ = torsional stress in wall i; $t_{ef,i}$ = effective wall thickness (= total area of cross section / outer circumference), A_k = area enclosed by centrelines of the walls including inner hollow areas.

The shear force $V_{Ed,i}$ in wall i due to torsion is given by:

$$V_{Ed,i} = \tau_{t,i} t_{ef,i} z_i \tag{23}$$

z_i = side length of wall i

The required area of longitudinal reinforcement for torsion is given by:

$$\frac{\sum A_{sl} f_{yd}}{u_k} = \frac{T_{Ed}}{2A_k} cot\theta \tag{24}$$

A_{sl} = reinforcement area; u_k = perimeter of area A_k; f_{yd} = design yield stress of longitudinal reinforcement A_{sl}; $cot\theta$ = angle of the compression struts.

Prestressed concrete

Elastic analysis: compression is positive. Eq.(25) applies for both top and bottom fibres since Z_i has sign:

$$\sigma = \frac{P}{A} + \frac{Pe}{Z_i} - \frac{M}{Z_i} \tag{25}$$

To design prestress, stress inequalities take the form:

$$f_c \geq \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} \geq f_t \tag{26}$$

For fibre 1 (top):

$$-\frac{Z_1}{A} + \frac{f_c Z_1}{P} + \frac{M}{P} \leq e \leq -\frac{Z_1}{A} + \frac{f_t Z_1}{P} + \frac{M}{P} \tag{27}$$

For fibre 2 (bottom):

$$-\frac{Z_2}{A} + \frac{f_c Z_2}{P} + \frac{M}{P} \geq e \geq -\frac{Z_2}{A} + \frac{f_t Z_2}{P} + \frac{M}{P} \tag{28}$$

Cumulative normal distribution function

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} e^{-\frac{x^2}{2}}\,dx \quad \text{FOR} \quad 0{\cdot}00 \le u \le 4{\cdot}99.$$

u	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7703	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
1·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
1·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·90147
1·3	·90320	·90490	·90658	·90824	·90988	·91149	·91309	·91466	·91621	·91774
1·4	·91924	·92073	·92220	·92364	·92507	·92647	·92785	·92922	·93056	·93189
1·5	·93319	·93448	·93574	·93699	·93822	·93943	·94062	·94179	·94295	·94408
1·6	·94520	·94630	·94738	·94845	·94950	·95053	·95154	·95254	·95352	·95449
1·7	·95543	·95637	·95728	·95818	·95907	·95994	·96080	·96164	·96246	·96327
1·8	·96407	·96485	·96562	·96638	·96712	·96784	·96856	·96926	·96995	·97062
1·9	·97128	·97193	·97257	·97320	·97381	·97441	·97500	·97558	·97615	·97670
2·0	·97725	·97778	·97831	·97882	·97932	·97982	·98030	·98077	·98124	·98169
2·1	·98214	·98257	·98300	·98341	·98382	·98422	·98461	·98500	·98537	·98574
2·2	·98610	·98645	·98679	·98713	·98745	·98778	·98809	·98840	·98870	·98899
2·3	·98928	·98956	·98983	·9^{2} 0097	·9^{2} 0358	·9^{2} 0613	·9^{2} 0863	·9^{2} 1106	·9^{2} 1344	·9^{2} 1576
2·4	·9^{2} 1802	·9^{2} 2024	·9^{2} 2240	·9^{2} 2451	·9^{2} 2656	·9^{2} 2857	·9^{2} 3053	·9^{2} 3244	·9^{2} 3431	·9^{2} 3613
2·5	·9^{2} 3790	·9^{2} 3963	·9^{2} 4132	·9^{2} 4297	·9^{2} 4457	·9^{2} 4614	·9^{2} 4766	·9^{2} 4915	·9^{2} 5060	·9^{2} 5201
2·6	·9^{2} 5339	·9^{2} 5473	·9^{2} 5604	·9^{2} 5731	·9^{2} 5855	·9^{2} 5975	·9^{2} 6093	·9^{2} 6207	·9^{2} 6319	·9^{2} 6427
2·7	·9^{2} 6533	·9^{2} 6636	·9^{2} 6736	·9^{2} 6833	·9^{2} 6928	·9^{2} 7020	·9^{2} 7110	·9^{2} 7197	·9^{2} 7282	·9^{2} 7365
2·8	·9^{2} 7445	·9^{2} 7523	·9^{2} 7599	·9^{2} 7673	·9^{2} 7744	·9^{2} 7814	·9^{2} 7882	·9^{2} 7948	·9^{2} 8012	·9^{2} 8074
2·9	·9^{2} 8134	·9^{2} 8193	·9^{2} 8250	·9^{2} 8305	·9^{2} 8359	·9^{2} 8411	·9^{2} 8462	·9^{2} 8511	·9^{2} 8559	·9^{2} 8605
3·0	·9^{2} 8650	·9^{2} 8694	·9^{2} 8736	·9^{2} 8777	·9^{2} 8817	·9^{2} 8856	·9^{2} 8893	·9^{2} 8930	·9^{2} 8965	·9^{2} 8999
3·1	·9^{3} 0324	·9^{3} 0646	·9^{3} 0957	·9^{3} 1260	·9^{3} 1553	·9^{3} 1836	·9^{3} 2112	·9^{3} 2378	·9^{3} 2636	·9^{3} 2886
3·2	·9^{3} 3129	·9^{3} 3363	·9^{3} 3590	·9^{3} 3810	·9^{3} 4024	·9^{3} 4230	·9^{3} 4429	·9^{3} 4623	·9^{3} 4810	·9^{3} 4991
3·3	·9^{3} 5166	·9^{3} 5335	·9^{3} 5499	·9^{3} 5658	·9^{3} 5811	·9^{3} 5959	·9^{3} 6103	·9^{3} 6242	·9^{3} 6376	·9^{3} 6505
3·4	·9^{3} 6631	·9^{3} 6752	·9^{3} 6869	·9^{3} 6982	·9^{3} 7091	·9^{3} 7197	·9^{3} 7299	·9^{3} 7398	·9^{3} 7493	·9^{3} 7585
3·5	·9^{3} 7674	·9^{3} 7759	·9^{3} 7842	·9^{3} 7922	·9^{3} 7999	·9^{3} 8074	·9^{3} 8146	·9^{3} 8215	·9^{3} 8282	·9^{3} 8347
3·6	·9^{3} 8409	·9^{3} 8469	·9^{3} 8527	·9^{3} 8583	·9^{3} 8637	·9^{3} 8689	·9^{3} 8739	·9^{3} 8787	·9^{3} 8834	·9^{3} 8879
3·7	·9^{3} 8922	·9^{3} 8964	·9^{4} 0039	·9^{4} 0426	·9^{4} 0799	·9^{4} 1158	·9^{4} 1504	·9^{4} 1838	·9^{4} 2159	·9^{4} 2468
3·8	·9^{4} 2765	·9^{4} 3052	·9^{4} 3327	·9^{4} 3593	·9^{4} 3848	·9^{4} 4094	·9^{4} 4331	·9^{4} 4558	·9^{4} 4777	·9^{4} 4988
3·9	·9^{4} 5190	·9^{4} 5385	·9^{4} 5573	·9^{4} 5753	·9^{4} 5926	·9^{4} 6092	·9^{4} 6253	·9^{4} 6406	·9^{4} 6554	·9^{4} 6696
4·0	·9^{4} 6833	·9^{4} 6964	·9^{4} 7090	·9^{4} 7211	·9^{4} 7327	·9^{4} 7439	·9^{4} 7546	·9^{4} 7649	·9^{4} 7748	·9^{4} 7843
4·1	·9^{4} 7934	·9^{4} 8022	·9^{4} 8106	·9^{4} 8186	·9^{4} 8263	·9^{4} 8338	·9^{4} 8409	·9^{4} 8477	·9^{4} 8542	·9^{4} 8605
4·2	·9^{4} 8665	·9^{4} 8723	·9^{4} 8778	·9^{4} 8832	·9^{4} 8882	·9^{4} 8931	·9^{4} 8978	·9^{5} 0226	·9^{5} 0655	·9^{5} 1066
4·3	·9^{5} 1460	·9^{5} 1837	·9^{5} 2199	·9^{5} 2545	·9^{5} 2876	·9^{5} 3193	·9^{5} 3497	·9^{5} 3788	·9^{5} 4066	·9^{5} 4332
4·4	·9^{5} 4587	·9^{5} 4831	·9^{5} 5065	·9^{5} 5288	·9^{5} 5502	·9^{5} 5706	·9^{5} 5902	·9^{5} 6089	·9^{5} 6268	·9^{5} 6439
4·5	·9^{5} 6602	·9^{5} 6759	·9^{5} 6908	·9^{5} 7051	·9^{5} 7187	·9^{5} 7318	·9^{5} 7442	·9^{5} 7561	·9^{5} 7675	·9^{5} 7784
4·6	·9^{5} 7888	·9^{5} 7987	·9^{5} 8081	·9^{5} 8172	·9^{5} 8258	·9^{5} 8340	·9^{5} 8419	·9^{5} 8494	·9^{5} 8566	·9^{5} 8634
4·7	·9^{5} 8699	·9^{5} 8761	·9^{5} 8821	·9^{5} 8877	·9^{5} 8931	·9^{5} 8983	·9^{6} 0320	·9^{6} 0789	·9^{6} 1235	·9^{6} 1661
4·8	·9^{6} 2067	·9^{6} 2453	·9^{6} 2822	·9^{6} 3173	·9^{6} 3508	·9^{6} 3827	·9^{6} 4131	·9^{6} 4420	·9^{6} 4696	·9^{6} 4958
4·9	·9^{6} 5208	·9^{6} 5446	·9^{6} 5673	·9^{6} 5889	·9^{6} 6094	·9^{6} 6289	·9^{6} 6475	·9^{6} 6652	·9^{6} 6821	·9^{6} 6981

Example: $\Phi(3{\cdot}57) = {\cdot}9^{3}\,8215 = 0{\cdot}9998215.$