4D8 2014

SOLUTIONS.

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29/1/14.

1. By inspection of the figure, only two bound lines come from the lower Kern point, and the tick marks on the line shown indicate that they must relate to upper bounds on the eccentricity for stresses in the top fibre of the beam.

These will be governed by tensile stresses and minimum moment conditions.

∴ The limiting equations will be:—

$$\frac{P_t}{A} + \frac{P_t e}{Z_1} - \frac{M_t}{Z_1} \geqslant f_{tt}$$

and $$\frac{P_w}{A} + \frac{P_w e}{Z_1} - \frac{M_a}{Z_1} \geqslant f_{tw}$$

Rearrange into standard form

$$e \leqslant -\frac{Z_1}{A} + \frac{f_{tt} Z_1}{P_t} + \frac{M_t}{P_t} \quad (1)$$

and $$e \leqslant \frac{-Z_1}{A} + \frac{f_{tw} Z_1}{R P_t} + \frac{M_a}{R P_t} \quad (2)$$

Need to calculate $Z_1 = -\frac{1.152 . 10^{12}}{1200} = -960 . 10^6 \text{ mm}^3$

$$-\frac{Z_1}{A} = +\frac{960 . 10^6}{24 . 10^6} = +400 \text{ mm}$$

Need to choose a value of P to construct Magnel lines. Choose $\frac{f_c . A}{2} = \frac{20 . 2.4 . 10^6}{2} = 24 \text{ MN}$

Say 25 MN as $1/P = 0.04$ so finding point on a gridline

(This value is arbitrary - any reasonable value will do.

Plug values into ① and ②

$$① \Rightarrow \quad e \leq +400 + \frac{(-1)(-960.10^6)}{\cancel{0.8}.25.10^6} + \frac{8000.10^6}{\cancel{0.8}.25.10^6}$$

$$\leq 400 + \frac{\cancel{48}}{38.4} + \frac{\cancel{400}}{320}$$

$$\leq \cancel{848\,mm}\ 758.4\ mm$$

$$② \quad e \leq +400 + 0 + \frac{8000}{0.8.25.10^6}\ (400)$$

$$\leq 800\ mm$$

$\therefore$ ① will govern.

Plot lines as shown on Figure.

(b) (i) Minimum prestress will be at Point A on diagram. Candidates should scale from the diagram. $P = 10435$ KN ($\equiv 1/P = 0.096\,MN^{-1}$)

$e = 1277$ mm (This value obtained by calculation)

This point lies outside the section.

(ii) If cover limited to 200 mm

$\therefore e = 1000$ mm — Point B on figure

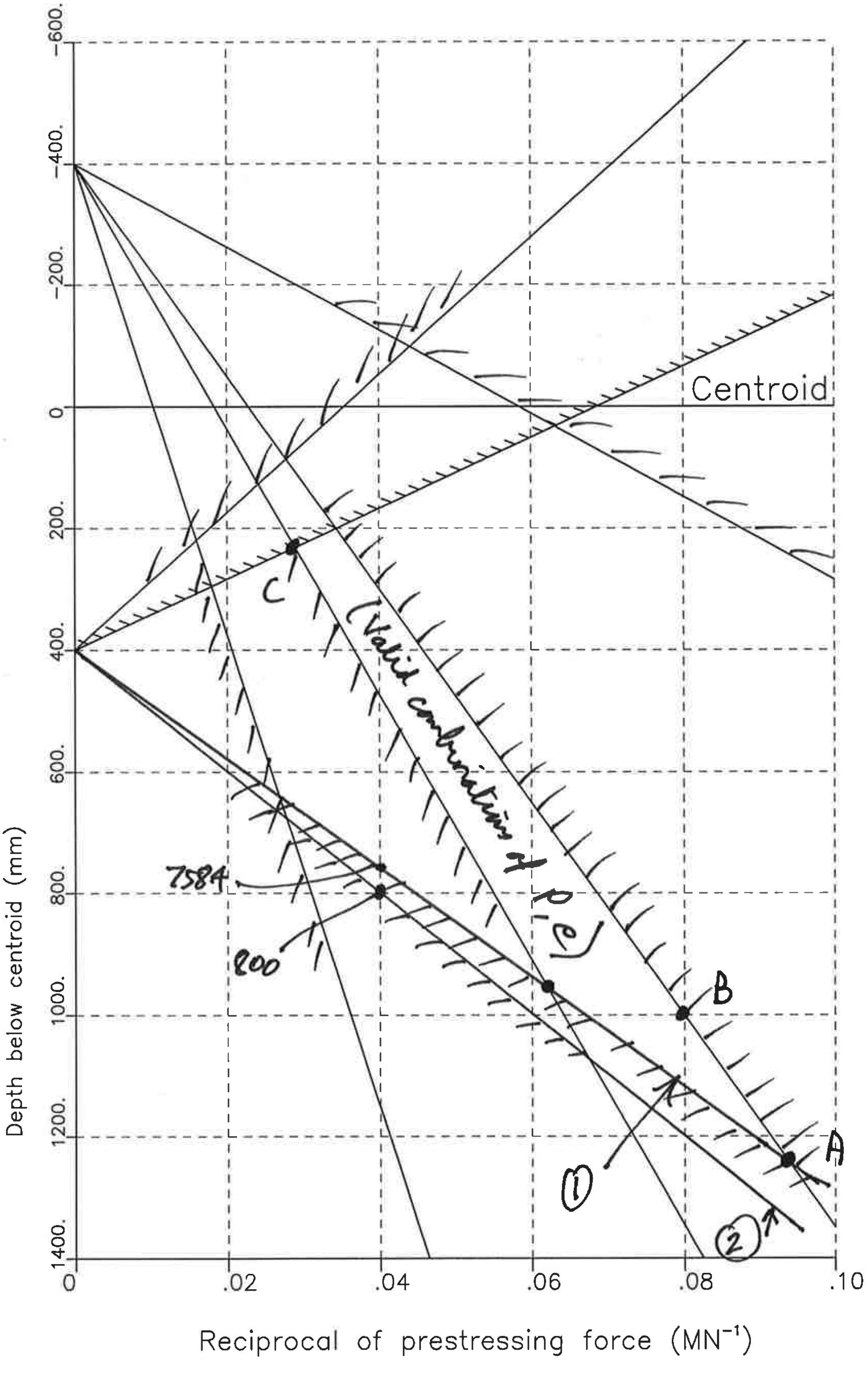
$1/P = .08 \Rightarrow P = 12500$ KN, $e = 1000$ mm (This is exact solution)

(c) Maximum prestress at point C

$P = \cancel{41405}\ 34560$ KN $\Rightarrow 1/P = 0.028$

@ $e = 232$ mm

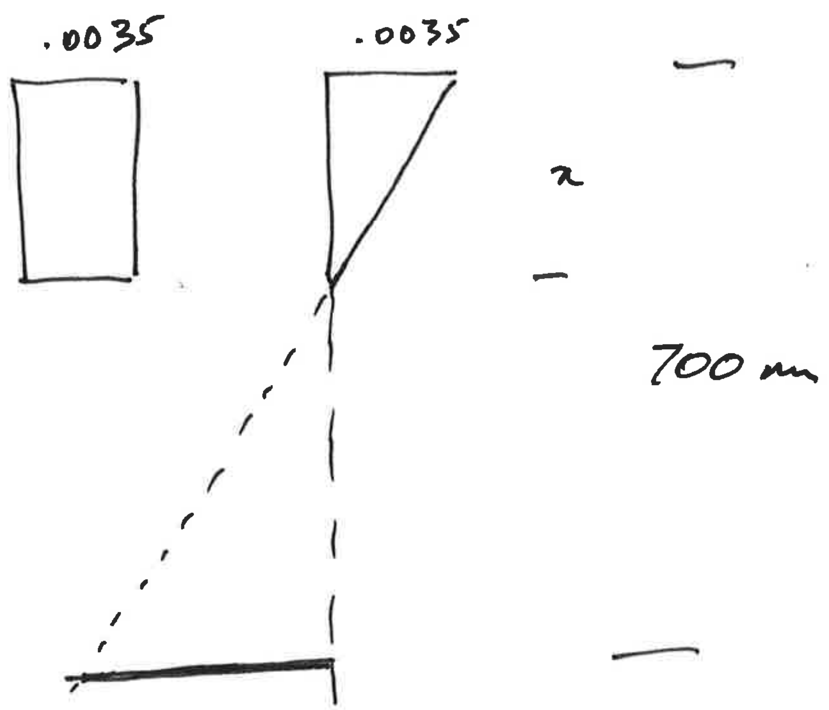
4D8 Prestressed Concrete 2014 Qu 2
Partial Magnel diagram for exam



(d) The calculation here relates to the line of thrust of the tendon, not its actual position. In determinate structures these coincide but in statically indeterminate structures they do not need to.

In order to have an actual cable profile (e_s) inside the section, but a line of thrust (e_p) outside the section, there would need to be significant sagging secondary moments (M_2).

[This question is not particularly long but it does require detailed understanding of the logic behind the Magnel diagram, so significant thinking time is allowed!].



Prestrain in CFRP $= \frac{1000}{200.10^3} = 0.005$

$E = \frac{\sigma}{\varepsilon}$

$\therefore \varepsilon = \frac{\sigma}{E} =$ $\therefore$ Strain in CFRP $= \underbrace{\frac{(700-x)}{x} . .0035}_{\text{bending}} \quad +0.005$ Prestrain

Stress in CFRP $= \left(\frac{(700-x)}{x} . .0035 + 0.005\right) 200.10^3$

$= 700 \left(\frac{700-x}{x}\right) + 1000$

$\therefore$ Force in CFRP $= 1.75.10^6 \left(\frac{700-x}{x}\right) + 2.5.10^6$

Force in Concrete $= 0.4 . 60 \times x . 500$

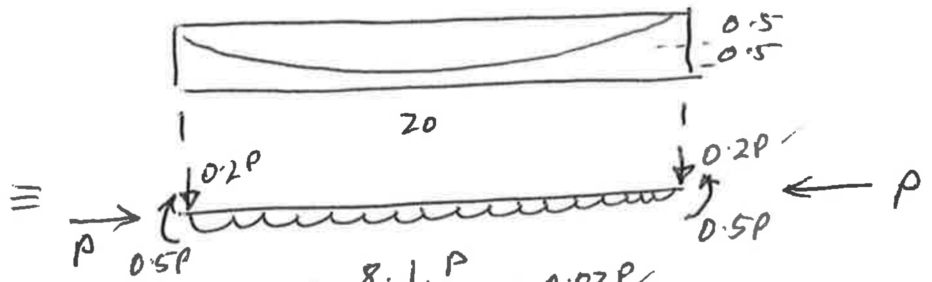
$= 12000x$

These must be equal

$1.75.10^6 \left(\frac{700-x}{\cancel{x}}\right) + 2.5.10^6 x = 12000x^2$

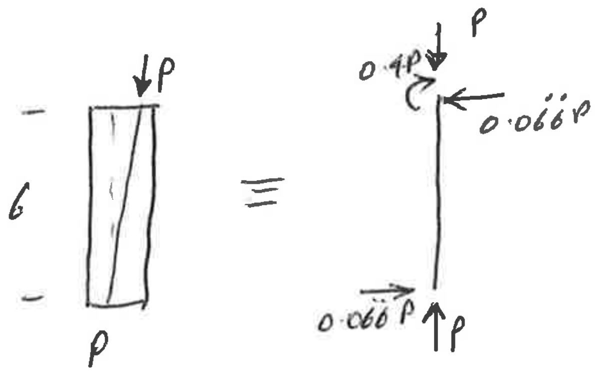
2

(a) Equivalent loads – in beam

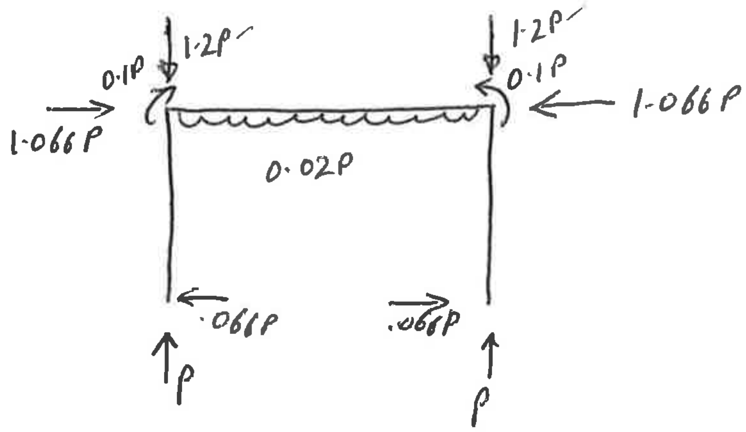


$$q = \frac{8.1.P}{20^2} = 0.02P$$

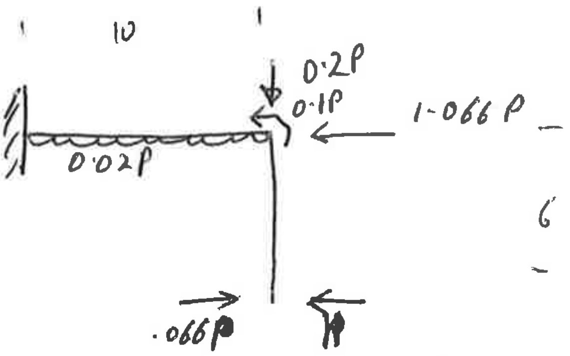
Equivalent loads in column



∴ In total

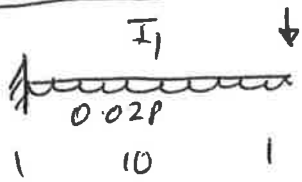


(b) Analyse using deflection coefficients to find H such that deflection at base = 0. Consider half of structure.



Only need horizontal deflection at base – vertical deflection won't cause secondary effects.

Beam will see



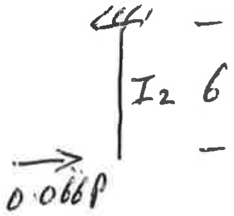
$\circlearrowleft 0.5P \quad (= 0.1P + 0.0\dot{6}\dot{6}P.6)$

∴ End rotation will be

$$\theta = \frac{0.02P.1000}{EI_1} + \frac{0.5P.10}{EI_1} - \frac{0.2P.100}{2EI_1} = -1.\dot{6}\dot{6}\frac{P}{EI_1} \quad +ve \circlearrowright$$

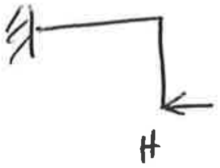
This will cause a displacement at base of column of $6\theta = \frac{10P}{EI_1}$ $\leftarrow$ +ve

Column will see



$$\therefore \delta = \frac{0.066P.6^3}{3EI_2} = \frac{4.8P}{EI_2} \quad \underset{+ve}{\rightarrow}$$

Due to indeterminacy



$$\delta = \frac{6H.10.6}{EI_1} + \frac{H.6^3}{3EI_2} = \frac{H}{E}\left(\frac{360}{I_1} + \frac{72}{I_2}\right)$$

$\leftarrow$ ve

Equating these

$$\frac{10P}{EI_1} - \frac{4.8P}{EI_2} = \frac{H}{E}\left(\frac{360}{I_1} + \frac{72}{I_2}\right)$$

But $I_1 = \frac{1.2^3}{12}.1 = 0.144\,m^4$ $\qquad I_2 = \frac{1^3}{12} = 0.0833\ m^4$

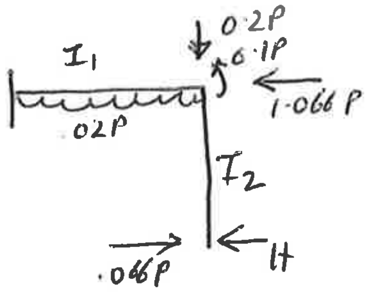
$$P\underbrace{\left(\frac{10}{.144} - \frac{4.8}{.0833}\right)}_{11.88} = -H\underbrace{\left(\frac{360}{.144} + \frac{72}{.0833}\right)}_{3364} \Rightarrow \underline{\underline{H = -0.0035P}}$$

Extra shortening due to axial force $1.066P = \frac{1.066P.10}{E.1.2.1} = 8.88\frac{P}{E}$

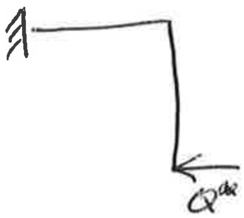
Effect of H will be shortening of $8.33\frac{H}{E}$

$$(11.88 + 8.88)P = -H(3364 + 8.33) \Rightarrow \underline{\underline{H = -0.0061P}}$$

Alternatively   Use V.W



Real

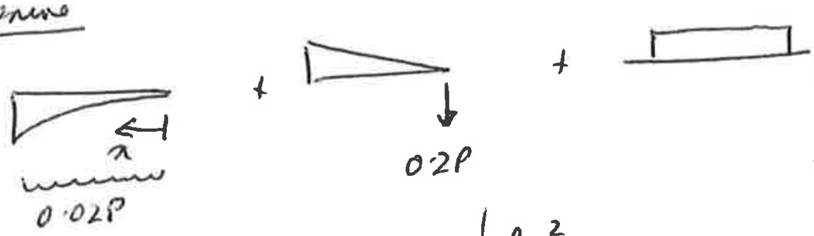


Fictitious

$$\Sigma W^* \Delta = \int M^* \hat{\kappa} + P^* e$$

In Beam - flexure

Real M =



$H.6 - .0\dot{6}\dot{6}.6 - 0.1P = 6H - 0.5P$

$$M = 6H - 0.5P + 0.2Px - 0.0\overset{1}{\cancel{2}}\frac{Px^2}{\underset{1}{\cancel{2}}}$$

Real $\hat{\kappa} = M/EI_1$

$$M^* = 6Q^*$$

$$\int_0^{10} M^* \hat{\kappa} = 6Q^* \left[60H + P\underbrace{\left(-5 + \frac{0.2}{2}.100 - \frac{0.01}{3}.1000\right)}_{1.\dot{6}\dot{6}}\right] / EI_2$$

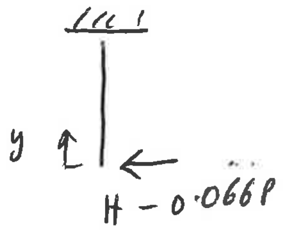
In beam - axial

Axial shortening will be $\frac{(P+H).10}{EA_1}$

Fict. axial force Q^*

$$\therefore P^*e = \frac{Q^*}{EA_1}(10H + 10P)$$

In column - flexure



$\therefore M = (H - 0.0\dot{6}\dot{6}P)y$ $M^* = Q^* y$

$$\therefore \int M^* \hat{\kappa} = Q^* \frac{(H - 0.0\dot{6}\dot{6}P)}{EI_2} \cdot \frac{6^3}{3}$$
$$(72)$$

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$$\therefore\ 6\cancel{Q}\frac{[60H + 1.66P]}{\cancel{E}I_2} + 72\cancel{Q}\frac{[H - 0.066P]}{\cancel{E}I_2} + \frac{\cancel{Q}}{\cancel{E}A_1}(10H+10P) = 0$$

Omit if ignoring axial effects

$$\Rightarrow\quad 3372H + 20.18P = 0$$

$$\Rightarrow -0.006P \quad \text{as before}$$

if axial terms are ignored

$$3364H + 11.84H = 0 \quad\Rightarrow\quad \underline{H = -.0035P}.$$

So, either way: If axial shortening of the beam is ignored the horizontal reaction at the base will be

0.0035 P outwards.

If they are included, the outward reaction goes up to

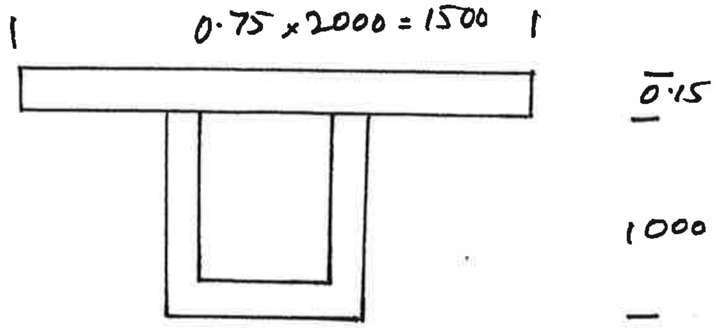
0.0061 P

(b)

This will cause a sagging secondary moment in the beam of $6 \times 0.0061P = 0.036P$

Thus, the prestress will behave as if the eccentricity in the beam has been reduced by 36 mm (i.e. it appears to be higher in the beam than the actual tendon). This effect will be constant all along the beam.

3 (a)



Note reduction in width of flange to allow for modular ratio

Find new $\bar{y}$ (above soffit)

$$\bar{y} = \frac{0.405 \times 0.39 + 1.5 \times 0.15 \times 1.075}{0.405 + 1.5 \times 0.15} = \frac{0.4}{0.63} = 0.635\,\text{m}$$

New area = $0.63\,\text{m}^2$

$$\text{New } I = 0.0392 + 0.405\,(0.635 - 0.39)^2 + \frac{1.5 \cdot 0.15^3}{12} + 1.5 \cdot 0.15 \cdot (1.075 - 0.635)^2$$

$$= \underline{\underline{0.1075\ \text{m}^4}}$$

In the precast beam

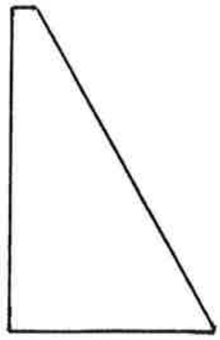
$$\left.\begin{aligned} Z_1 &= -0.064\ \text{m}^3 \\ Z_2 &= +0.100\ \text{m}^3 \end{aligned}\right\} \text{precast only}$$

$$\left.\begin{aligned} Z_1' &= -0.295\ \text{m}^3 \\ Z_2' &= +0.169\ \text{m}^3 \end{aligned}\right\} \text{Composite Section}$$

[6 marks]

(b) Stress due to prestress (never acts alone

$e = 0.39 - 0.24 = 0.15\,m$



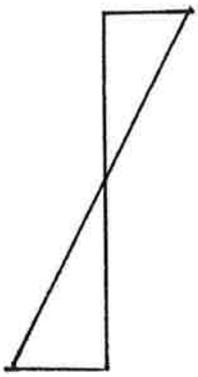
$\leftarrow \quad 9.9 - \dfrac{4000.10^3 \cdot 0.15.10^3}{0.064.10^9} = 0.5\ N/mm^2$

$\leftarrow \quad \dfrac{4000.10^3}{0.405.10^6} + \dfrac{4000.10^3\,0.15.10^3}{0.1.10^9} = 15.9\,N/mm^2$

[3 marks]

Effect of beams own dead weight

$\text{Moment} = 0.405 \times 24 \times \dfrac{20^2}{8} = 486\ kNm$



$\leftarrow \quad \dfrac{486.10^6}{0.064.10^9} = 7.6\ N/mm^2$

$\leftarrow \quad -\dfrac{486.10^6}{0.1.10^9} = -4.86\ N/mm^2$

$\therefore$ Total stress $= 8.1\ N/mm^2$ in top
$11.0\ N/mm^2$ in bottom

[3 marks]

Loss of prestress $= -0.1\,N/mm^2$ in top $\Rightarrow 8.0\,N/mm^2$

$= -2.4\,N/mm^2$ in bottom $\Rightarrow 8.6\,N/mm^2$

[2 marks]

Effect of in situ dead weight

$= 0.15 \times 2 \times 24 = 7.2$ kN/m

↑ N.B not 1.5 since all concrete contributes to weight

Additional moments $= \dfrac{7.2 \times 20^2}{8} = 360$ kNm

$\therefore$ Stresses are $+5.6\,N/mm^2$ in top

$-3.6\,N/mm^2$ in bottom

$\therefore$ Total $= 13.6\,N/mm^2$ in top

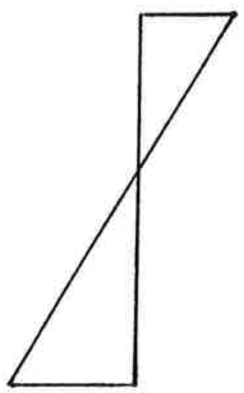
$5.0\,N/mm^2$ in bottom

[3 marks]

Live load $10\,kN/m^2$

$\therefore$ Moment $= 10 \times 2 \times \dfrac{20^2}{8} = 1000$ kNm

Total



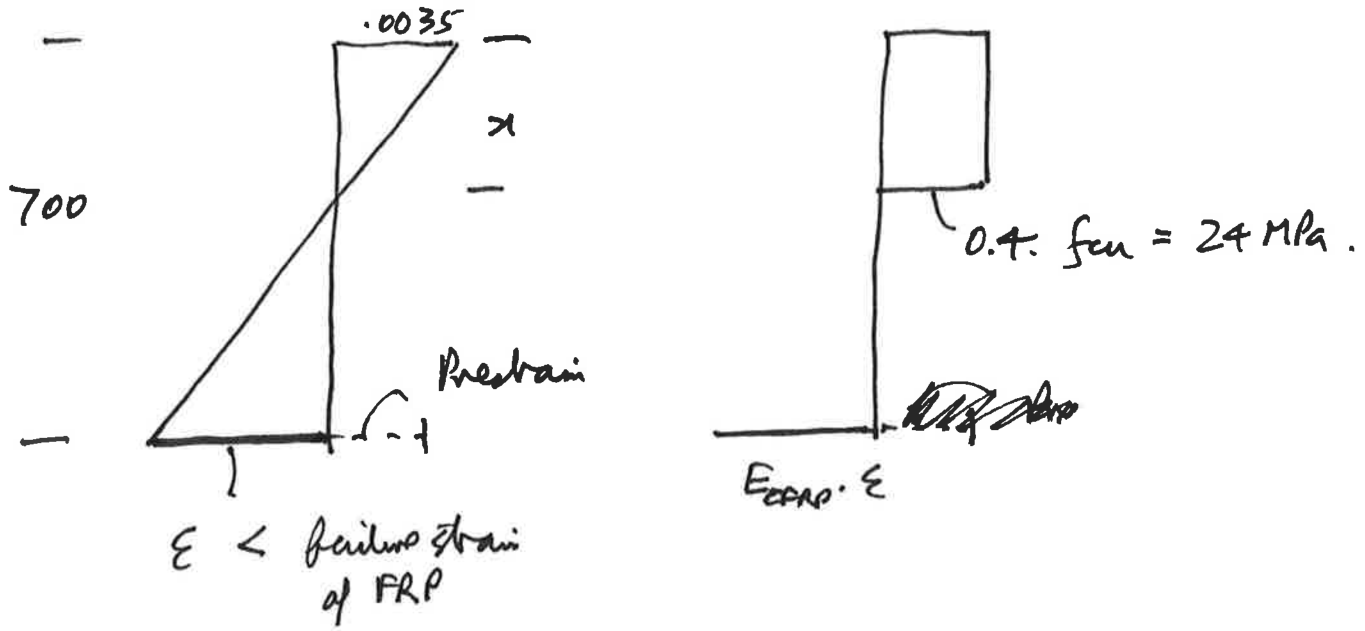
$\leftarrow \dfrac{1000.10^6}{0.295.10^9} = 3.4\,N/mm^2$ $\quad$ $17.0\,N/mm^2$

$\leftarrow \dfrac{1000.10^6}{0.169.10^9} = 5.9\,N/mm^2$ $\quad$ $-0.9\,N/m$

[3 marks]

(a) Beams with CFRP tendons will not be ~~expected~~ allowed to reach the failure strain of the tendon. So the normal calculation, where the steel tendon has been yielding for some lower loads and the concrete just reaches the failure strain cannot apply to CFRP.

(b) Strain at failure   Stress at failure



Prestrain in FRP = $\dfrac{1000}{200.10^3} = 0.005$

Strain in FRP from linearity of strain diagram

$$= \frac{(700-x)}{x} \cdot .0035 + 0.005$$

$\therefore$ Force in CFRP = $\varepsilon.E.A. = 1.75.10^6\left(\dfrac{700-x}{x}\right) + 2.5.10^6$

Force in concrete $= 0.4.60.x.500 = 12000x$

These must be equal. Gives a quadratic in x

$$\Rightarrow x = 352 \text{ mm}$$

$\therefore$ Compression force $= 12000x = 4227$ kN

Lever arm $= 700 - \dfrac{352.3}{2} = 524$ mm

$\therefore$ Moment will be $\underline{2214 \text{ kNm}}$

(c)

Check stress in CFRP

$$\text{Strain} = \frac{(700-352.3)}{352.3} \cdot .0035 + .005 = 0.00845$$

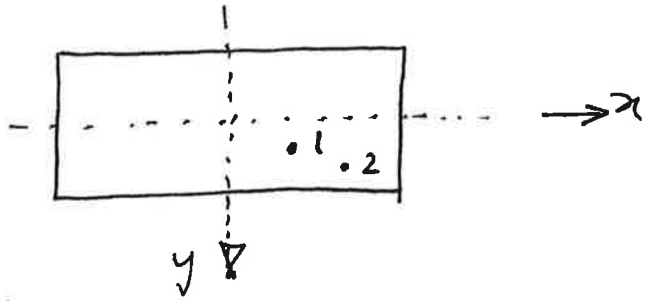
$$\therefore \text{Stress} = .00845 \; 200.10^3 = 1690 \text{ MPa}$$

$\therefore$ CFRP does not fail.

Failure mode will be by crushing of the concrete but without significant cracking of the concrete in tension before failure.

$\therefore$ Regarded as very undesirable.

5.



Effect of stressing tendon 2 on tendon 1 which has already been stressed is:

(40%)

$$\delta\sigma_1 = \frac{E_s}{E_c} P_2 \left(\frac{1}{A} + \frac{e_{y2}\, e_{y1}}{I_x} + \frac{e_{x2}\, e_{x1}}{I_y} \right)$$

For this beam $P = 1250 \sim 800 = 1000\,\text{kN}$ for all tendons

$$\therefore \frac{P E_s}{E_c} = 10^6 \cdot \frac{200}{16} = 12.5 \cdot 10^6 \text{ N}$$

$A = 3\text{m}^2$

$I_x = \frac{3.1^3}{12} = \frac{1}{4} \text{ m}^4$

$I_y = \frac{1.3^3}{12} = \frac{9}{4} \text{ m}^4$

Order of stressing tendons is important (10%)

make up table of products $e_{x1}\, e_{x2}$ etc.

$\sum e_{yi} e_{yj}$

This tendon stressed

			1	2	3	4	$\sum$
		e_y	0.4	0.4	−0.4	0.4	
effect on this tendon	1	0.4	•	0.16	−0.16	0.16	0.16
	2	0.4	•	•	−0.16	0.16	0
	3	−0.4	•	•	•	−0.16	−0.16
	4	0.4	•	•	•	•	0

10%

(• indicates no effect because of order of stressing)

$\sum e_{xi} e_{xj}$

		1	2	3	4	$\sum$
		−0.8	0	0	0.8	
1	−0.8	•	0	0	−0.64	−0.64
2	0	•	•	0	0	0
3	0	•	•	•	0	0
4	0.8	•	•	•	•	0

10%

Loss of ~~force~~ stress in tendon 1

30%

$$= 12.5 \cdot 10^6 \left(\frac{3}{3} + \frac{0.16}{1/4} - \frac{0.64}{9/4} \right) \cdot \frac{1}{10^6} = \underline{\underline{12.9 \text{ N/mm}^2}}$$

Loss in tendon 2

$$= 12.5 \cdot 10^6 \left(\frac{2}{3} + 0 + 0 \right) \frac{1}{10^6} = \underline{\underline{8.3 \text{ N/mm}^2}}$$

Loss in tendon 3 $= 12.5 \cdot 10^6 \left(\frac{1}{3} - \frac{0.16}{1/4} + 0 \right) = \underline{\underline{-3.8 \text{ N/mm}^2}}$