EGT3/EGT2
ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Thursday 24 April 20142 to 3.30

## Module 4D8

## PRESTRESSED CONCRETE

Answer not more than one question from Section $A$ and two questions from Section B.

Questions from Section A carry twice as many marks as questions from Section B.

The approximate percentage of marks allocated to each part of a question is indicated in the right margin.

Write your candidate number not your name on the cover sheet.

## STATIONERY REQUIREMENTS

Single-sided script paper
Graph Paper

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed
Engineering Data Book
Supplementary page: one extra copy of Fig. 1 (Section A, Question 1)

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

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## SECTION A

## Answer one question

1 A beam has the dimensions and loading as set out in Table 1. A partly completed Magnel diagram is shown in Fig. 1, a copy of which is attached at the back of this paper and which should be completed and handed in with your answers. Two of the bound lines are missing. You may assume that the section is large enough for a feasible solution to exist.
(a) Complete the diagram by adding the remaining bound lines and adding tick marks to all bound lines.
(b) By scaling from the diagram, determine the minimum allowable prestressing force and its eccentricity:
(i) ignoring any limits on the maximum eccentricity of the tendon;
(ii) if the centre-line of the tendon must have a minimum cover of 200 mm .
(c) Determine the maximum prestressing force and its eccentricity.
(d) Under what circumstances is it possible for the eccentricity of the tendon, as used in these calculations, to lie outside the cross-section?

| Depth of beam <br> $(\mathrm{mm})$ | 2,400 | $f_{t t}(\mathrm{MPa})$ | -1 | Moment at <br> transfer <br> $(\mathrm{kNm})$ | 8,000 |
| :--- | :---: | :---: | :---: | :--- | :---: |
| Height of centroid <br> $(\mathrm{mm})$ | 1,200 | $f_{c t}(\mathrm{MPa})$ | 12 | Minimum <br> moment <br> $(\mathrm{kNm})$ | 8,000 |
| Cross sectional area <br> $\left(\mathrm{mm}^{2}\right)$ | $2.4 \times 10^{6}$ | $f_{\text {tw }}(\mathrm{MPa})$ | 0 | Maximum <br> moment <br> $(\mathrm{kNm})$ | 14,000 |
| Second mom. of <br> area $\left(\mathrm{mm}^{4}\right)$ | $1.152 \times 10^{12}$ | $f_{c w}(\mathrm{MPa})$ | 20 | Loss Ratio | 0.8 |

Table 1

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Fig. 1 (reduced scale)

A full size supplementary Page of Fig. 1 is attached to the back of this paper. It should be detached and handed in with your paper after completion.

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2 A concrete portal frame has columns 1 m square and a beam that is 1 m wide and 1.2 m deep. The concrete has a Young's modulus of 30 GPa . The columns are pinned at the base. The columns are prestressed with straight but inclined tendons, and the beam is prestressed with a parabolic tendon. All tendons carry the same force, $P$. The dimensions and eccentricities are shown in Fig 2.
(a) By using deflection coefficients, virtual work or otherwise, calculate the horizontal reaction at the base of each column assuming:
(i) the axial shortening of the beam can be ignored;
(ii) the axial shortening of the beam has to be taken into account.
(b) What is the effect on the apparent eccentricity of the prestressing tendon at the centre of the beam in these two cases?


Fig. 2

## SECTION B

## Answer two questions

3 The precast U-beam shown in Fig. 3 has a prestressing force at transfer of 4000 kN , acting at a height of 240 mm above the soffit. The section properties of the precast unit are: area $0.405 \mathrm{~m}^{2}$, second moment of area $0.0392 \mathrm{~m}^{4}$, and height of centroid above soffit 0.390 m . It is simply supported over a span of 20 m without propping, and an in-situ slab of thickness 150 mm and width 2000 mm is then added on the top. $15 \%$ of the prestress is lost due to creep and shrinkage between applying the prestress and the addition of the in-situ slab, after which no significant creep occurs. After the in-situ concrete has hardened it has a Young's modulus that is $75 \%$ of that of the precast concrete. Concrete weighs $24 \mathrm{kNm}^{-3}$. A uniform live-load of $10 \mathrm{kNm}^{-2}$ is then added to the beam.
(a) Determine the section properties of the composite section.
(b) Plot the stress distribution in the beam at mid-span at each stage of the beam's construction from transfer of prestress to final service.


Fig. 3

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4 A rectangular beam 1 m deep and 0.5 m wide is prestressed with bonded CFRP tendons of area $2500 \mathrm{~mm}^{2}$ located 300 mm above the bottom surface. The tendons carry a prestress of 1000 MPa . The CFRP has a Young's modulus of 200 GPa and a strength of 2500 MPa . The concrete has a cube strength of 60 MPa and a Young's modulus of 30 GPa ; assume its ultimate strain in compression is 0.0035 .
(a) Explain why beams with CFRP need to be analysed in a different way from beams with steel tendons.
(b) Calculate the ultimate strength of the beam in sagging bending.
(c) Calculate the strain in the CFRP at failure and describe the expected failure mode.

5 A post-tensioned concrete beam has breadth of 3 m and depth of 1 m . It is prestressed by four tendons whose eccentricities relative to the horizontal and vertical principal axes are shown in Table 2. The order of stressing of the tendons is also given in the table. Each tendon has a cross-sectional area of $1250 \mathrm{~mm}^{2}$ and is stressed to 800 MPa. The Young's modulus of concrete is 16 GPa and that of steel is 200 GPa .

Determine the loss of force in each cable due to elastic shortening of the concrete.

| Order of prestressing | Eccentricity from the <br> horizontal principal axis <br> $(\mathrm{m})$ | Eccentricity from the <br> vertical principal axis (m) |
| :---: | :---: | :---: |
| 1 | +0.4 | -0.8 |
| 2 | +0.4 | 0 |
| 3 | -0.4 | 0 |
| 4 | +0.4 | +0.8 |

Table 2

## END OF PAPER

## 4D8 2014. Numerical answers.

1. (b)(i) $10.43 \mathrm{MN}, 1277 \mathrm{~mm}$ (ii) $12.5 \mathrm{MN}, 1000 \mathrm{~mm}$ (c) $34.56 \mathrm{MN}, 232 \mathrm{~mm}$
2. (a)(i) 0.0035 P (ii) 0.0061 P (b) 36 mm higher
3. (a) $0.1075 \mathrm{~m}^{4}, 0.635 \mathrm{~m}$ (b) $8.1 \& 11.0 \mathrm{MPa} ; 13.6 \& 5.0 \mathrm{MPa} ; 17.0 \&-0.9 \mathrm{MPa}$
4. (b) 2214 kNm (c) 0.00845
5. $12.9,8.3 \&-3.8 \mathrm{MPa}^{\prime}$

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