EGT3
ENGINEERING TRIPOS PART IIB

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Friday May 3 2024 14.00 to 15.40

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**Module 4D9**

**OFFSHORE GEOTECHNICAL ENGINEERING**

*Answer not more than* ***three*** *questions.*

*All questions carry the same number of marks.*

*The* ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number* ***not*** *your name on the cover sheet and at the top of each answer sheet.*

**STATIONERY REQUIREMENTS**
Write on single-sided paper.

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
Engineering Data Book
CUED approved calculator allowed
4D9: Offshore Geotechnical Engineering Data Book (20 pages)

**10 minutes reading time is allowed for this paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

**You may not remove any stationery from the Examination Room.**

1 A site on the edge of the continental shelf is proposed as a development location for a floating offshore wind farm. The location is in deep water (>1000 m), at the base of an escarpment, and several tens of kilometres from shore. An offshore site investigation has specified that a number of cone penetrometer tests be performed at the proposed wind farm location. The cone outer diameter $D$ = 35.7 mm and the shaft diameter $D_s$ = 30.0 mm. Measured data from one of the cone penetrometer tests is given in Table 1.

(a) Describe the most likely soil conditions that will be found at the potential development location and explain why they are likely to be prevalent. [15%]

(b) Describe the most likely geohazards that would be found along a potential export cable route back to shore. [15%]

(c) Estimate the cone area ratio from the geometry. [10%]

(d) Describe an alternative method to obtain the cone area ratio for the cone. [10%]

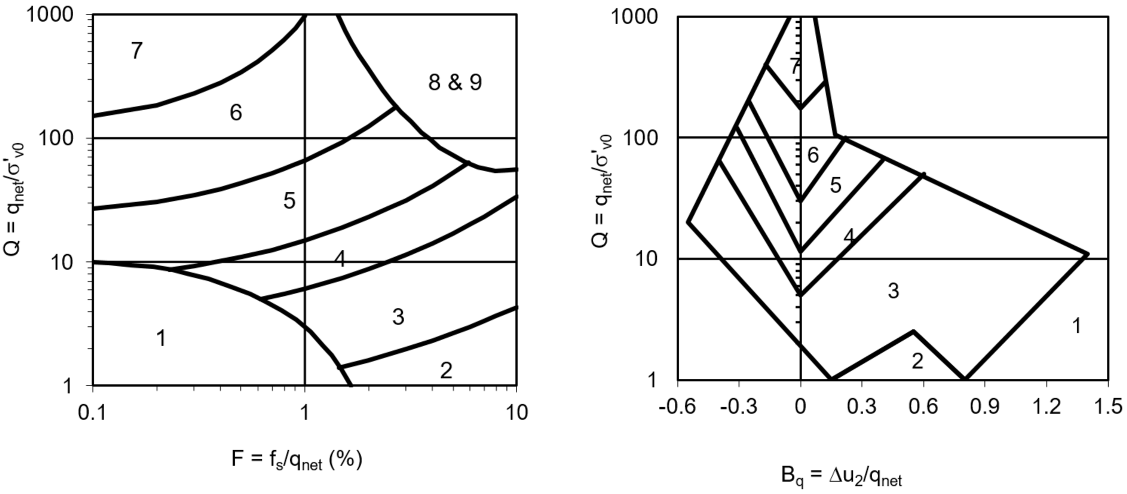
(e) Use the Robertson charts given in Fig. 1 to indicate the type of sediment at the cone penetrometer test location. [30%]

(f) Describe any additional tests required to estimate the appropriate cone factor $N_{kt}$ to use to analyse the cone penetration data and derive an undrained shear strength. [10%]

(g) Summarise the shortcoming of using the cone for measuring undrained shear strength. [10%]

Table 1

| $z$ [m] | $q_c$ $\left[\text{kN m}^{-2}\right]$ | $f_s$ $\left[\text{kN m}^{-2}\right]$ | $u_2$ $\left[\text{kN m}^{-2}\right]$ | $\gamma$ $\left[\text{kN m}^{-3}\right]$ |
|---|---|---|---|---|
| 20 | 600 | 1.0 | 1020 | 16 |




| Zone | Soil behaviour type | Zone | Soil behaviour type | Zone | Soil behaviour type |
|---|---|---|---|---|---|
| 1. | Sensitive, fine grained | 4. | Silt mixtures, clayey silt to silty clay | 7. | Gravelly sand to sand |
| 2. | Organic soils, peats | 5. | Sand mixtures, silty sand to sandy silt | 8. | Very stiff sand to clayey sand |
| 3. | Clayey clays to silty clay | 6. | Sands, clean sand to silty sands | 9. | Very stiff fine grained |

Fig. 1

2 Suction caisson anchors are to be used to moor an offshore facility at a site comprising soft clay, with a uniform shear strength $s_u$ = 16 kN m$^{-2}$ and sensitivity $S_t$ = 4. The proposed caisson has a diameter $D$ = 4 m, length $L$ = 20 m and wall thickness $t_w$ = 0.05 m. The effective unit weight of the soil $\gamma'$ = 6 kN m$^{-3}$. Take vertical bearing capacity factors on tip bearing $N_c$ = 7.5 and on plug bearing $N_c$ = 9, and take an interface roughness coefficient $\alpha$ = 0.3 for installation. The submerged weight of the caisson $W'$ = 500 kN. The caisson is provided with ring stiffeners at 1 m intervals along the internal shaft, starting 0.5 m above the toe of the caisson and protruding 0.05 m inward from the caisson wall.

(a) Calculate the installation resistance corresponding to the final depth assuming that the stiffeners degrade the soil strength on the inner wall of the caisson during penetration. [40%]

(b) Calculate the factor of safety on plug stability. [20%]

(c) Calculate the depth of self-weight penetration. [20%]

(d) Discuss any simplifications implicitly applied in the above calculations compared to reality. [10%]

(e) Comment on how to optimise the "padeye" position in order to maximise the anchoring capacity assuming the offshore floating facility is anchored by a catenary mooring system. [10%]

3 A subsea cable with diameter $D = 0.25$ m and specific gravity $SG = 3$, exits a J-tube on a monopile in a sandy area of the North Sea and is buried to a cover depth $C = 2D$. Engineers responsible for the cable are concerned about uplift breakout of the cable close to the termination of the J-tube due to hydrodynamic loading. The sand has a critical state friction angle $\phi_{cs} = 33°$, relative density $I_D = 0.6$, natural logarithm of the grain crushing stress $Q = 10$ and effective unit weight $\gamma' = 10$ kN m$^{-3}$.

(a) Assuming no energy is dissipated in shear within the sand on uplift and accounting for the buoyant weight of the cable, show that a conservative estimate for the uplift force is:

$$F_{up} = \left[ CD + \frac{D^2}{2}\left(1 - \frac{\pi}{4}\right) + \left(C + \frac{D}{2}\right)^2 \tan\psi \right] \gamma' + (SG - 1)\,\frac{\pi D^2}{4}\gamma_w$$

where $\psi$ is a dilation angle. Provide diagrams to illustrate your approach. [30%]

(b) Using a suitable method to estimate the dilation angle $\psi$, calculate an estimate for the uplift force $F_{up}$. [30%]

(c) Comment on any assumptions made in (b) and the sensitivity of the calculation to those assumptions. [30%]

(d) Suggest two methods of stabilising the buried cable in order to increase the uplift breakout force $F_{up}$. [10%]

 (TURN OVER

4 A suction embedded plate anchor with width $B$ = 5 m and height $H$ = 5 m is installed to a depth $z_a$ = 20 m in a soft clay seabed. The plate anchor is attached to a mooring system comprising of a mooring chain with link diameter $d$ = 0.1 m and submerged weight $w$ = 2 kN m$^{-1}$. For simplicity assume that there is no loss of embedment on keying. The strength profile of the seabed is:

$$s_u = s_{um} + kz$$

where the undrained strength at the mudline $s_{um}$ = 2 kN m$^{-2}$ and the gradient of undrained shear strength with depth $k$ = 1 kN m$^{-3}$.

(a) Calculate the ultimate holding capacity of the plate anchor. [20%]

(b) Determine the angle of the mooring chain at the anchor padeye and the capacity of the mooring system at the mudline. [30%]

(c) Estimate the distance from the anchor padeye that the mooring chain first breaks through the surface of the seabed and the total length of the embedded mooring chain. [20%]

(d) Describe the process of anchor keying using a sequence of supporting diagrams and identify any implications this process will have on the calculations performed in parts (a), (b) and (c). [30%]

**END OF PAPER**

# Module 4D9: Offshore Geotechnical Engineering — Supplementary Data Book —

This supplementary data books contains relationships and associated data that you are not expected to remember, but you will be expected to understand what the parameters and relationships represent and how to apply them in analysis.

## 1 Offshore environment

### 1.1 Slope failure

Factor of safety against infinite slope failure:

$$F = \frac{\tau_{ult}}{\tau_{mob}}$$

Undrained strength ratio:

$$k = s_u/\sigma'_v$$

Factor of safety for undrained infinite slope failure:

$$F = \frac{2k}{\sin 2\alpha}$$

Critical slope angle for undrained conditions:

$$\alpha_{ult} = 0.5\arcsin(2k)$$

Factor of safety for drained conditions:

$$F = \frac{\tan\phi_{cr}}{\tan\alpha}$$

Critical slope angle for drained conditions:

$$\alpha_{ult} = \phi_{cr}$$

Factor of safety for partially drained conditions and an undrained failure criteria:

$$F = k\frac{(\cos^2\alpha - r_u)}{\sin\alpha\cos\alpha}$$

Factor of safety for partially drained conditions and an drained failure criteria:

$$F = \frac{(\cos^2\alpha - r_u)\tan\phi_{cr}}{\sin\alpha\cos\alpha}$$

# 2 Site investigation

## 2.1 CPT interpretation

Cone total resistance:

$$q_t = q_c + (1-\alpha)u_2$$

Cone area ratio from laboratory pressure chamber:

$$\alpha = \frac{A_{shoulder}}{A_{shaft}}$$

Cone area ratio from geometry:

$$\alpha = \frac{q_c}{q_{chamber}}$$

Net cone resistance:

$$q_{net} = q_t - \sigma_{v0} = q_t - \gamma z$$

Undrained strength from cone net resistance:

$$s_u = \frac{q_{net}}{N_{kt}}$$

Normalised cone tip resistance:

$$Q = \frac{q_{net}}{\sigma'_{v0}} = \frac{q_{net}}{\gamma' z}$$

Friction ratio:

$$R_f = \frac{f_s}{q_{net}} \cdot 100 \text{ (i.e. expressed as percentage)}$$

Excess pore pressure ratio:

$$B_q = \frac{u_2 - u_0}{q_{net}} = \frac{\Delta u_2}{q_{net}}$$

Cone relative density:

$$I_D = 0.34 \ln\left(0.04 \frac{q_c}{p'_0}\left(\frac{p'_0}{p_a}\right)^{0.54}\right)$$

## 2.2 Shear vane interpretation

Undrained strength from vane shear:

$$T = \frac{\pi d^3}{6}\left(1 + 3\frac{h}{d}\right) s_u$$

## 2.3 "Full flow" penetrometer interpretation

Full flow penetrometer undrained shear strength:

$$s_u = \frac{q}{N}$$

# 3 Pipelines / cables

## 3.1 Dynamic lay effects

Pipeline / cable tension during laying process:

$$\frac{T_0}{z_w W'} = \left(\frac{\cos\phi}{1-\cos\phi}\right)$$

Stress concentration factor in touchdown zone:

$$f \approx 0.6 + 0.4\left(\frac{\lambda^2 k}{T_0}\right)^{0.25}$$

Seabed secant stiffness:

$$k = \frac{V}{w}$$

Characteristic length:

$$\lambda = \sqrt{\frac{EI}{T_0}}$$

Pipe second moment of area:

$$I_{pipe} \approx \frac{\pi}{8} D^3 t$$

## 3.2 Pipeline / cable geometry

Effective weight:

$$W' = (SG-1)\left(\frac{\pi D^2}{4}\right)\gamma_w$$

Semi-angle of the embedded pipeline / cable segment:

$$\theta = \cos^{-1}\left(1 - \frac{2w}{D}\right)$$

Effective contact diameter:

$$D' = D\sin\theta$$

Embedded area:

$$A' = \left[\frac{\pi D^2}{4}\cdot\frac{\theta}{\pi}\right] - \left[\left(\frac{D}{2}\right)^2 \sin\theta\cos\theta\right] = \frac{D^2}{4}(\theta - \sin\theta\cos\theta)$$

## 3.3 Undrained bearing capacity

Undrained bearing capacity on effective contact diameter:

$$V_{max} = N_c D' s_u + f_b A'\gamma \text{ (where } N_c = 5 \text{ and } f_b \approx 1.5)$$

Undrained bearing capacity on diameter:

$$V_{max} = N_c D s_u + f_b A'\gamma \text{ (where } N_c \approx 6\,(w/D)^{0.25} \text{ and } f_b \approx 1.5)$$

## 3.4 Drained bearing capacity

Drained bearing capacity can be estimated from the following system of equations after Tom and White (2019):

$$V_{\max} = A\left(\frac{w}{D}\right)^{B}\gamma' D^2$$

$$A = C_1\left(e^{\phi_{peak}C_2}\right)^{C_3\phi_{peak}}$$

$$B = 1.3067 - 0.0123\phi_{\text{peak}}$$

$$C_i = I_{c,i} + \phi_{cs}S_{c,i}$$

Constants for $C$ parameter determination, after Tom and White (2019):

| Coefficient | | Value |
|---|---|---|
| $C_1$ | $S_{C,1}$ | 0.07 |
| | $I_{C,1}$ | 1.75 |
| $C_2$ | $S_{C,2}$ | 0.0163 |
| | $I_{C,2}$ | 0.6467 |
| $C_3$ | $S_{C,3}$ | -5.97e-5 |
| | $I_{C,3}$ | 0.0030 |

Bolton's (1986) equations for calculating peak friction and dilation angles relevant to partially buried pipelines and cables:

$$\phi_{peak} = \phi_{cs} + 0.8\psi$$

$$\psi = \frac{5I_R}{0.8}$$

$$I_R = min\left(I_D\left(Q - \ln p'\right), 4\right)$$

$$p' \approx \frac{(1+K_0)}{2}\gamma' w$$

Jaky's in-situ stress approximation:

$$K_0 = 1 - sin(\phi_{cs})$$

## 3.5 Axial friction

Axial force:

$$F = \mu\zeta W$$

Wedging factor:

$$\zeta = \frac{2\sin\theta}{\theta + \sin\theta\cos\theta} \leq 1.27$$

Undrained friction coefficient:

$$\mu \approx \left(\frac{s_{u-int}}{\sigma''_{vc}}\right)_{NC} OCR^{0.6}$$

Drained friction coefficient:

$$\mu = \tan\delta$$

## 3.6 Undrained lateral breakout

Undrained lateral breakout can be calculated using the following system of equations:

$$\frac{H}{H_{max}} = \beta\left(\frac{V}{V_{max}}\right)^{\beta_1}\left(1-\frac{V}{V_{max}}\right)^{\beta_2}$$

$$\beta = \frac{(\beta_1+\beta_2)^{\beta_1+\beta_2}}{\beta_1^{\beta_1}\beta_2^{\beta_2}}$$

$$\beta_1 = (0.8-0.15\alpha)(1.2-w/D)$$

$$\beta_2 = 0.35(2.5-w/D)$$

$$\frac{H_{\max}}{V_{\max}} = \left(0.48-\frac{\alpha}{25}\right)\left(\frac{w}{D}\right)^{\left(0.46-\frac{\alpha}{25}\right)}$$

## 3.7 Drained lateral breakout

Drained lateral breakout can be calculated using the following system of equations:

$$\frac{\bar{H}}{\bar{V}_{\max}} = \mu\left(\frac{\bar{V}}{\bar{V}_{\max}}+\beta\right)^{n}\left(1-\frac{\bar{V}}{\bar{V}_{\max}}\right)^{m}$$

$$\bar{V} = \frac{V}{\gamma' D^2}; \quad \bar{V}_{max} = \frac{V_{max}}{\gamma' D^2}; \quad \bar{H} = \frac{H}{\gamma' D^2}$$

$$\mu = 0.2w/D + \mu_0$$

$$\mu_0 = -0.00437\phi_{peak} + 0.42$$

$$m = 0.013\phi_{peak} + 0.4$$

## 3.8 Pipeline thermal expansion

Free end expansion:

$$S_{FE} = 0.5\alpha\Delta T L$$

Fully constrained axial force:

$$P_{FC} = AE\alpha\Delta T$$

Temperature variation due to thermal losses:

$$\Delta T = \Delta T_{\max} - KP\Delta T_{loss}$$

Partially constrained free end expansion:

$$S_{PC} \approx 0.5\left(\alpha\Delta T L - \frac{P_{ave}L}{EA}\right)$$

Axial force:

$$P = \mu\zeta W' L_{FE}$$

Hobb's critical buckling force:

$$P_{buckle} = 3.86\sqrt{\frac{EIH}{D}}$$

# 4 Piles

Structure natural frequency of a wind turbine on pile:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{1}{m_T\left(\frac{h_T^3}{3EI}+\frac{h_T^2}{k_s}\right)}}$$

Dynamic Amplification Factor:

$$DAF = \frac{1}{\sqrt{(1-\beta^2)^2+(2\xi\beta)^2}}$$

## 4.1 Axial Response

### 4.1.1 Axial capacity: DNV-OS-J101 (DNV 2014) / API (2000) method

**Clay**

*α-Method*:

Unit shaft resistance: $\alpha = \frac{\tau_s}{s_u} = 0.5max\left[\left(\frac{\sigma'_{v0}}{s_u}\right)^{0.5}, \left(\frac{\sigma'_{v0}}{s_u}\right)^{0.25}\right]$

Note: it is assumed that equal shaft resistance acts inside and outside open-ended piles.

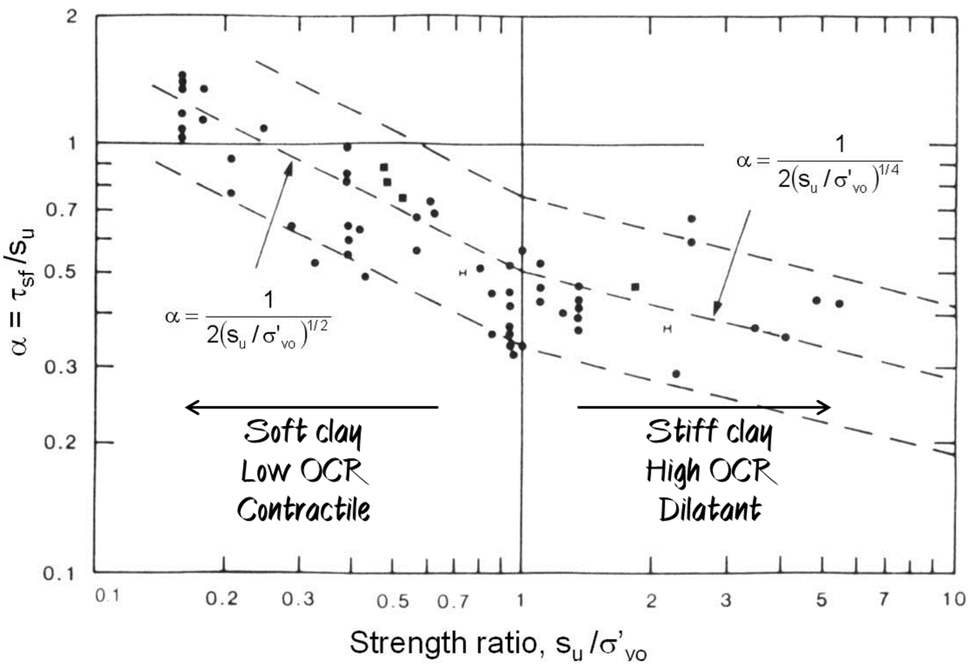



Figure 1: API (2000) $\alpha$-correlations for ultimate unit shaft resistance in clay (Randolph & Murphy 1985)

*$\beta$-Method*:

$$\tau_{sf} = \beta\sigma'_{v0} = K\sigma'_{v0}tan\delta$$

*$\lambda$-Method*:

$$\tau_{sf} = \lambda\left(\sigma'_{0m} + 2s_{um}\right)$$

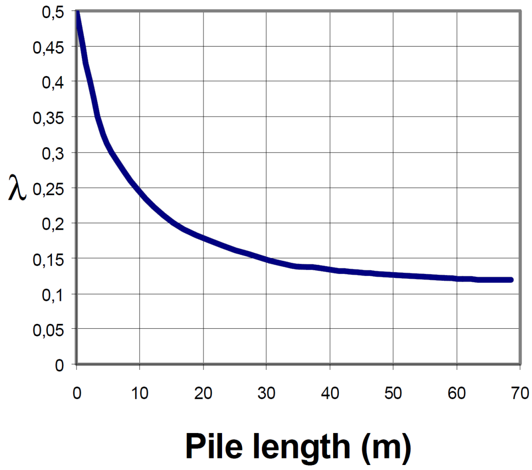



Figure 2: Coefficient $\lambda$ vs. pile length

*Unit base resistance*:

$$q_b = N_c s_u; \quad N_c = 9$$

**Sand**

Unit shaft resistance: $\tau_{sf} = \sigma'_{hf}tan\delta = K\sigma'_{v0}tan\delta \leq \tau_{s,lim}$

Closed-ended piles: $K = 1$
Open-ended piles: $K = 0.8$

Unit base resistance: $q_b = N_q\sigma'_{v0} < q_{b,limit}$

| Soil category | Soil density | Soil type | Soil-pile friction angle, $\delta$ (°) | Limiting value $\tau_{s,lim}$ (kPa) | Bearing capacity factor, $N_q$ | Limiting value, $q_{b,lim}$ (MPa) |
|---|---|---|---|---|---|---|
| 1 | Very loose<br>Loose<br>Medium | Sand<br>Sand-silt<br>Silt | 15 | 48 | 8 | 1.9 |
| 2 | Loose<br>Medium<br>Dense | Sand<br>Sand-silt<br>Silt | 20 | 67 | 12 | 2.9 |
| 3 | Medium<br>Dense | Sand<br>Sand-silt | 25 | 81 | 20 | 4.8 |
| 4 | Dense<br>Very dense | Sand<br>Sand-silt | 30 | 96 | 40 | 9.6 |
| 5 | Dense<br>Very dense | Gravel<br>Sand | 35 | 115 | 50 | 12 |

Figure 3: DNV-OS-J101 (DNV 2014) recommendations for driven pile capacity in sand (following API, 2000).

### *t-z* curves: DNV-OS-J101 (DNV 2014)

Governing equation:

$$\frac{d^2w}{dz^2} = \frac{\pi D}{(EA)_p}\tau_s$$

The $t$-$z$ curves can be generated with a nonlinear relation between the origin and the point where the maximum skin resistance $t_{max}$ is reached:

$$z = t\frac{R}{G_0}ln\left(\frac{z_{IF} - r_f\frac{t}{t_{max}}}{1 - r_f\frac{t}{t_{max}}}\right) \quad for\ 0 \leq t \leq t_{max}$$

in which:

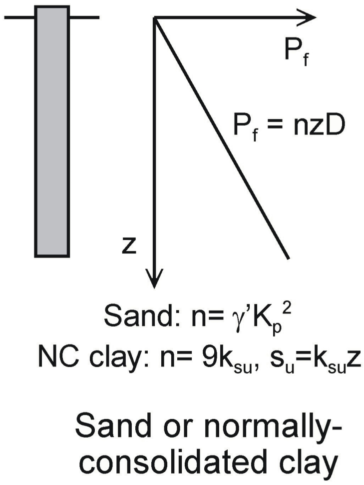
| | |
|---|---|
| $R$ | Radius of the pile |
| $G_0$ | Initial shear modulus of the soil |
| $z_{IF}$ | Dimensionless zone of influence<br>(defined as the radius of the zone of influence around the pile divided by $R$) |
| $r_f$ | curve fitting factor |

## 4.2 Lateral Response

### 4.2.1 Lateral capacity: linearly increasing lateral resistance with depth

Lateral soil resistance (force per unit length): $P_f = nzD$

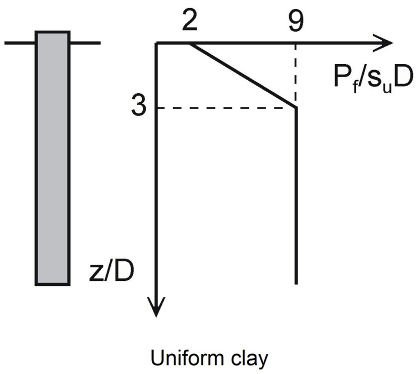
In sand: $n = \gamma' K_p^2$
In normally consolidated clay with strength gradient $k$: $s_u = kz; n = 9k$




| | |
|---|---|
| $H_{ult}$ | Ultimate horizontal load on pile |
| $D$ | Pile diameter |
| $L$ | Pile length |
| $\gamma'$ | Effective unit weight |
| $K_p$ | Passive earth pressure coefficient, $\approx (1 + sin(\phi))/(1 - sin(\phi))$ |

### 4.2.2 Lateral capacity: uniform clay

Lateral soil resistance (force per unit length), $P_f$, increases from $2s_uD$ at surface to $9s_uD$ at $3D$ depth then remains constant.




| | |
|---|---|
| $H_{ult}$ | Ultimate horizontal load on pile |
| $D$ | Pile diameter |
| $L$ | Pile length |
| $s_u$ | Undrained shear strength |

### 4.2.3 DNV-OS-J101 (DNV 2014) / API (2000) $p$-$y$ curves method

Governing equation:

$$E_p I_p \frac{\mathrm{d}^4 y}{\mathrm{d}z^4} + V \frac{\mathrm{d}^2 y}{\mathrm{d}z^2} + k_{py} y = 0$$

**Clay**

The static ultimate lateral resistance is recommended to be calculated as:

$$P_f = \begin{cases} (3s_u + \gamma' z)D + J s_u z & for\ 0 < z < z_R \\ 9 s_u D & for\ z > z_R \end{cases}$$

where:

| | |
|---|---|
| $z$ | Depth below soil surface |
| $z_R$ | Transition depth |
| | below which the value of $(3s_u + \gamma' z)D + J s_u z$ exceeds $9 s_u D$ |
| $D$ | Pile diameter |
| | defined as the radius of the zone of influence around the pile divided by $R$ |
| $s_u$ | Undrained shear strength of the soil |
| $\gamma'$ | Effective unit weight of soil |
| $J$ | Dimensionless empirical constant |
| | with values in the range of 0.25 to 0.50 |
| | with 0.50 recommended for soft normally consolidated clay. |

For static loading, the $p - y$ curve can be generated according to:

$$p = \begin{cases} \frac{p_u}{2} \left( \frac{y}{y_c} \right)^{1/3} & for\ y < 8y_c \\ p_u & for\ y > 8y_c \end{cases}$$

For cyclic loading and $z > z_R$, the $p$-$y$ curves can be generated according to:

$$p = \begin{cases} \frac{p_u}{2} \left( \frac{y}{y_c} \right)^{1/3} & for\ y < 3y_c \\ 0.72 p_u & for\ y > 3y_c \end{cases}$$

For cyclic loading and $z \leq z_R$, the $p$-$y$ curves can be generated according to:

$$p = \begin{cases} \frac{p_u}{2} \left( \frac{y}{y_c} \right)^{1/3} & for\ y < 3y_c \\ 0.72 p_u \left( 1 - \left( 1 - \frac{z}{z_R} \right) \frac{y - 3y_c}{12 y_c} \right) & for\ 3y_c < y < 15y_c \\ 0.72 p_u \frac{z}{z_R} & for\ y > 15y_c \end{cases}$$

Here, $y_c = 2.5 \epsilon_c D$, in which $D$ is the pile diameter and $\epsilon_c$ is the strain which occurs at one-half the maximum stress in laboratory undrained compression tests of undisturbed soil samples.

**Sand**

For piles in cohesionless soils, the static ultimate lateral resistance is recommended to be calculated as:

$$p_u = min\left((C_1 z + C_2 D)\gamma' z; C_3 D\gamma' z\right)$$

where the coefficients $C_1$, $C_2$ and $C_3$ depend on the friction angle $\phi$ as shown in Figure 2 (RHS), and where:

| | |
|---|---|
| $z$ | Depth below soil surface |
| $z_R$ | Transition depth<br>below which the value of $(C_1 z + C_2 D)\gamma' z$ exceeds $C_3 D\gamma' z$ |
| $D$ | Pile diameter<br>defined as the radius of the zone of influence around the pile divided by $R$ |
| $\gamma'$ | Effective unit weight of soil |

The $p$-$y$ curve can be generated according to:

$$p(z, y) = Ap_u \cdot tanh\left(\frac{k.z}{Ap_u} y\right)$$

in which $k$ is the initial modulus of subgrade reaction and depends on the friction angle $\phi$ as given in the Figure below (LHS), and $A$ is a factor to account for static or cyclic loading conditions as follows:

$$\begin{cases} A_{static} = max\left(0.9, \left(3 - 0.8\frac{z}{D}\right)\right) \\ A_{cyclic} = 0.9 \end{cases}$$

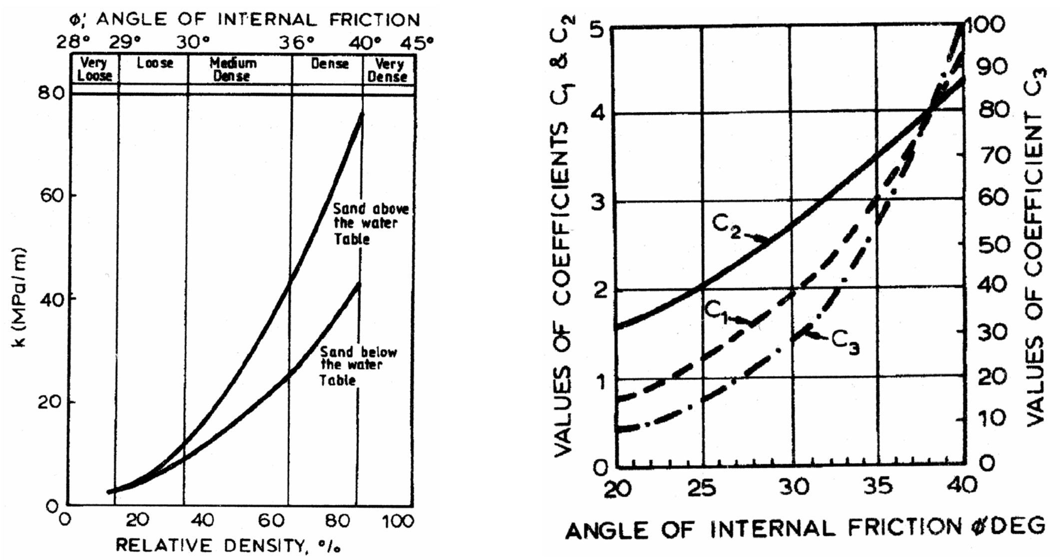



Figure 4: Modulus of subgrade reaction and Empirical coefficients in DNV as a function of friction angle.

# 5 Anchors

## 5.1 Equilibrium equations for an embedded anchor line

Change in line tension:

$$\frac{dT}{ds} = F + wsin\theta$$

Change in mooring line angle:

$$\frac{d\theta}{ds} = \frac{-Q + wcos\theta}{T}$$

Unit soil friction acting on the anchor line (parallel to the line):

$$F = A_s \alpha s_u$$

Limiting normal force transmitted to the anchor line from the soil:

$$Q = A_b N_c s_u$$

## 5.2 Analytical solution for embedded anchor line

Line tension at padeye:

$$\frac{T_a}{2}\left(\theta_a^2 - \theta_m^2\right) = z_a Q_{av}$$

Bearing resistance:

$$z_a Q_{av} = bN_c \int_0^{z_a} s_u dz$$

Angle at padeye:

$$\theta_a = \sqrt{\frac{2}{T^*}}$$

where:

$$T^* = \sqrt{\frac{T_a}{z_a Q_{av}}}$$

Relationship between the anchor line tension at the mudline $T_m$ and that at the padeye $T_a$:

$$\frac{T_m}{T_a} = e^{\mu(\theta_a - \theta_m)}$$

Normalised profile of the anchor line:

$$z^* = e^{x^* \theta_a}$$

Effective bearing resistance:

$$Q_{eff} = Q - w$$

## 5.3 Analytical solution for drag anchors

Holding capacity:

$$T_a = A_f N_c s_u$$

Force acting on the anchor parallel to the direction of travel:

$$T_p = (fA_p)N_c s_u$$

Anchor capacity at any embedment:

$$T_w = \frac{T_p}{cos\theta} = \frac{fA_pN_cs_u}{cos\theta_w}$$

Resultant force in the anchor chain at the anchor attachment point:

$$T_a = \frac{T_p}{cos\theta'_w} = \frac{fA_pN_cs_u}{cos\theta'_w}$$

Angle of the resultant anchor line tension $T_a$ to the fluke for a weighty anchor:

$$\theta'_w = tan^{-1}\left(\frac{W + T_p tan\theta_w}{T_p}\right)$$

Anchor holding capacity:

$$T_a = \frac{2z_aQ_{av}}{(\theta_a^2 - \theta_m^2)}$$

Bearing resistance:

$$z_aQ_{av} = bN_c\int_0^{z_a} s_u dz$$

Relationship between the anchor line tension at the mudline $T_m$ and that at the padeye $T_a$:

$$\frac{T_m}{T_a} = e^{\mu(\theta_a - \theta_m)}$$

Performance ratio:

$$\eta = \frac{T_m}{W}$$

## 5.4 Drop anchors

Impact velocity:

$$V_{terminal} = \sqrt{\frac{2mg}{C_{drag}A_{end}\rho_{water}}} \quad where \;\; C_{drag} \approx 0.035 + 0.01\frac{L}{D}$$

Equation of motion:

$$m\frac{d^2z}{dt^2} = W_s - F_{bear} - F_{fric} - F_d$$

where:

| | |
|---|---|
| $m$ | anchor mass |
| $z$ | depth |
| $t$ | time |

Shaft resistance :

$$F_{fric} = Q_{sf} = \alpha s_u A_{shaft} = \alpha k_{su} z A_{shaft}$$

Front and rear "base resistance":

$$F_{bear} \approx 2N_c s_u A_{tip}$$

Inertial drag:

$$F_d = \frac{1}{2}C_d \rho_s A_{tip} v^2$$

where:

| | |
|---|---|
| $C_d$ | drag coefficient, estimated as 0.24 |
| $\rho_s$ | soil density |
| $A_{tip}$ | projected anchor area |
| $v$ | current anchor velocity |

Final penetration depth:

$$z_{final} = \frac{W_s + \sqrt{(W_s)^2 + mv_{impact}^2 k_{su}(\alpha A_{shaft} + 2N_c A_{tip})}}{k_{su}\left(\alpha A_{shaft} + 2N_c A_{tip}\right)}$$

## 5.5 Suction caissons

### 5.5.1 Installation resistance

Clay:

$$Q = A_s \alpha \overline{s}_u + A_{tip} \left( N_c s_u + \gamma' z \right)$$

Required under-pressure:

$$\Delta u_{req} = \frac{Q - W'}{A_i}$$

Allowable under-pressure:

$$\Delta u_a = \frac{A_i N_c s_u + A_{si} \alpha \overline{s}_u + W'_{plug} - \gamma' d A_{plug}}{A_i} = \frac{A_i N_c s_u + A_s \alpha \overline{s}_u}{A_i}$$

Factor of safety:

$$F = \frac{\Delta u_a}{\Delta u_{req}}$$

Soil plug stability criterion:

$$\left( \frac{L}{D} \right)_{limit} \simeq \frac{1}{4\alpha_e} \left[ N_c + \left( N_c^2 + \frac{32 W \alpha_e}{\pi k D^3} \right) \right]$$

Sand:

$$W' + 0.25 \pi D_i^2 p = F_o + (F + Q_{tip}) \left( 1 - \frac{p}{P_{crit}} \right) \text{ for } \quad p \leq p_{crit}$$

### 5.5.2 Vertical capacity

Without suction:

$$V_{ult} = W' + A_{se} \alpha_e \overline{s}_{u(t)} + A_{si} \alpha_i \overline{s}_{u(t)}$$

Or:

$$V_{ult} = W' + A_{se} \alpha_e \overline{s}_{u(t)} + W'_{plug}$$

With suction:

$$V_{ult} = W' + A_{se} \alpha_e \overline{s}_{u(t)} + N_c s_u A_e$$

### 5.5.3 Maximum horizontal resistance

$$H_{max} = L D_e N_p \overline{s}_u$$

### 5.5.4 Inclined loading

$$\left( \frac{H}{H_{ult}} \right)^a + \left( \frac{V}{V_{ult}} \right)^b = 1$$

# 6 Shallow foundations

## 6.1 Clay

Ultimate capacity:

$$V_{ult} = A'\left(s_{u0}\left(N_c + kB'/4\right)\frac{FK_c}{\gamma_m} + p'_0\right)$$

Dimensionless undrained strength gradient:

$$\kappa = \frac{kB'}{s_{u0}}$$

Modification factor:

$$K_c = 1 - i_c + s_c + d_c$$

where:

$$i_c = 0.5(1 - \sqrt{1 - H/A's_{u0}})$$
$$s_c = s_{cv}\left(1 - 2i_c\right)B'/L$$
$$d_c = 0.3e^{-0.5kB'/s_{u0}}\arctan\left(d/B'\right)$$

Shape factor $s_{cv}$ after Salençon and Matar (1982):

| $\kappa = kB/s_{u0}$ | $s_{cv}$ |
|---|---|
| 0 | 0.20 |
| 2 | 0.00 |
| 4 | -0.05 |
| 6 | -0.07 |
| 8 | -0.09 |
| 10 | -0.10 |

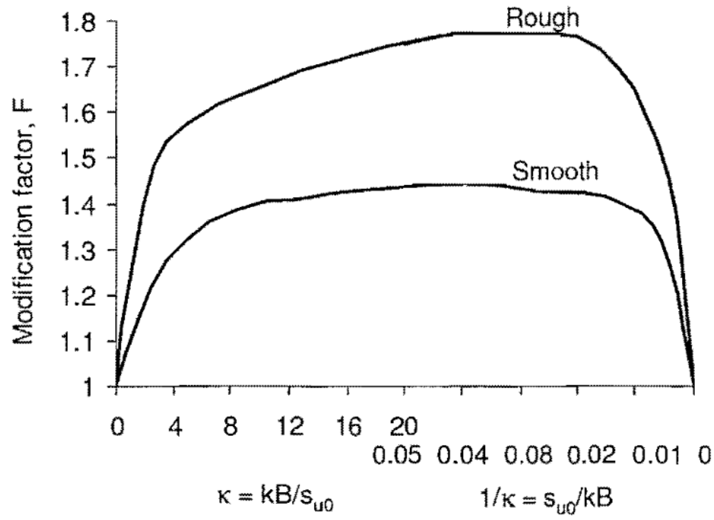



Figure 5: Modification factor $F$, after Davis and Booker (1973).

Horizontal failure criterion gives:

$$\frac{H_{ult}}{A's_{u0}} = 1$$

Ultimate moment:

$M_{ult} = 0.64A'Bs_{u0}$ for a strip foundation
$M_{ult} = 0.61A'Ds_{u0}$ for a circular foundation

## 6.2 Sand

$$V_{ult} = A'\left(\frac{1}{2}\gamma' B' N_\gamma K_y + (p_0 + a)N_q K_q - a\right)$$

Where:

| | |
|---|---|
| $V_{ult}$ | Ultimate vertical load |
| $A'$ | Effective bearing area of the foundation |
| $\gamma'$ | Effective unit weight of the soil |
| $B'$ | Effective width of the foundation |
| $N_\gamma,\ N_q$ | Bearing capacity factors for self-weight and surcharge |
| $K_\gamma, K_q$ | Modification factors to account for foundation shape, embedment and load inclination |
| $p_0'$ | Effective overburden acting to either side of the foundation |
| $a$ | Soil attraction factor which accounts for cementation equal to the point of interception of the tangent to the Mohr Circle and the normal stress axis. |
| $\phi$ | Effective internal friction angle of the soil |
| $\gamma_m$ | material factor on shear strength |

$$N_q = tan^2\left(\frac{\pi}{4} + \frac{1}{2}tan^{-1}\left(\frac{tan\phi}{\gamma_m}\right)\right)e^{\frac{\pi tan\phi}{\gamma_m}}$$

$$N_\gamma = 1.5(N_q - 1)tan\left(\frac{tan\phi}{\gamma_m}\right)$$

Modification factors:

$$K_q = s_q d_q i_q$$

and:

$$K_\gamma = s_\gamma d_\gamma i_\gamma$$

$$s_q = 1 + i_q\frac{B'}{L}sin\left(tan^{-1}\left(\frac{tan\phi}{\gamma_m}\right)\right)$$

$$d_q = 1 + 2\frac{d}{B'}\left(\frac{tan\phi'}{\gamma_m}\right)\left\{1 - sin\left(tan^{-1}\left(\frac{tan\phi}{\gamma_m}\right)\right)\right\}^2$$

$$i_q = \left\{1 - 0.5\left(\frac{H}{V + A'a}\right)\right\}^5$$

$$s_\gamma = 1 - 0.4 i_\gamma\frac{B'}{L}$$

$$d_\gamma = 1$$

$$i_\gamma = \{1 - 0.7(\frac{H}{V + A'a})\}^5$$

## 6.3 Spudcan Foundations

### 6.3.1 Clay

Bearing capacity:

$$V = (N_c s_u + \sigma'_{v0})A$$

Table 1: Bearing capacity factors for rough circular plate in homogeneous soil (Houlsby & Martin 2003)

| $Embedment\ depth/D$ | Bearing factor, $N_c$ |
|---|---|
| 0 | 6 |
| 0.1 | 6.3 |
| 0.25 | 6.6 |
| 0.5 | 7.1 |
| 1.0 | 7.7 |
| $\leq 2.5$ | 9.0 |

Conditions for backflow:

$$Flow\ failure\ occurs\ if\ : \quad \frac{D}{B} > \left(\frac{s_{uD}}{\gamma' B}\right)^{0.55} - \frac{1}{4}\left(\frac{s_{uD}}{\gamma' B}\right)$$

### 6.3.2 Sand

$$V = \gamma' N_\gamma \frac{\pi D^3}{8} \tag{1}$$

Table 2: Bearing capacity factors for a flat, rough circular footing (from use of ABC software of Martin (2003))

| Friction angle $\phi$ (degrees) | Bearing factor, $N_\gamma$ |
|---|---|
| 20 | 2.42 |
| 25 | 6.07 |
| 30 | 15.5 |
| 35 | 41.9 |
| 40 | 124 |
| 45 | 418 |

# 7 Hydrodynamics / scour

Surface elevation:

$$\eta(x,t) = \frac{H}{2}\sin\left[2\pi\left(\frac{t}{T} - \frac{x}{L}\right)\right]$$

Dispersion equation:

$$\omega^2 = gk\tanh(kh)$$

Particle velocity:

$$v_x = \omega\frac{H}{2}\frac{\cosh[k(h-z)]}{\sinh(kh)}\sin(\omega t - kx)$$
$$v_z = \omega\frac{H}{2}\frac{\sinh[k(h-z)]}{\sinh(kh)}\cos(\omega t - kx)$$

Drag force due to fluid flow:

$$F_D = \frac{1}{2}\rho v^2 C_D D$$

Lift force due to fluid flow:

$$F_L = \frac{1}{2}\rho v^2 C_L D$$