EGT3
ENGINEERING TRIPOS PART IIB

Friday 25 April 2014 9.30 to 11

Module 4F1

CONTROL SYSTEM DESIGN

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4F1 Formulae sheet (3 pages).
Supplementary pages: one extra copy of Fig. 1 (Question 1) and two extra copies of Fig. 2 (Question 2)
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) (i) Explain how the Nyquist diagram and root-locus diagram for a system can be defined in terms of a mapping between complex planes. [15%]

(ii) State the definition and property of a conformal mapping. Explain the use of the property in the construction of Nyquist and root-locus diagrams. [15%]

(b) A transfer function $G(s)$ has a pole of multiplicity one at the origin and exactly one pole satisfying $\text{Re}(s) > 0$. Fig. 1 shows part of the frequency response $G(j\omega)$ for a range of positive frequencies with specific frequencies indicated: $\omega = 4, 5, \ldots, 9$ rad s^{-1}. Constant gain feedback k is applied to the system with the negative feedback convention.

(i) Sketch the complete Nyquist diagram for $G(s)$ and use it to determine the number of poles of the closed-loop system in the right half-plane for all values of k both positive and negative. [20%]

(ii) By use of a sketch on the extra copy of Fig. 1 estimate the position of any closed-loop poles close to the imaginary axis for $k = 1/8$ and $k = 1/10$. [25%]

(iii) Provide a sketch of any part of the root-locus diagram which may be deduced from (b)(i) and (b)(ii). What can be said about the location of any zeros of $G(s)$? What can be said about the total number of zeros in relation to the total number of poles of $G(s)$? [25%]

An extra copy of Fig. 1 is provided on a separate sheet. This should be handed in with your answers.

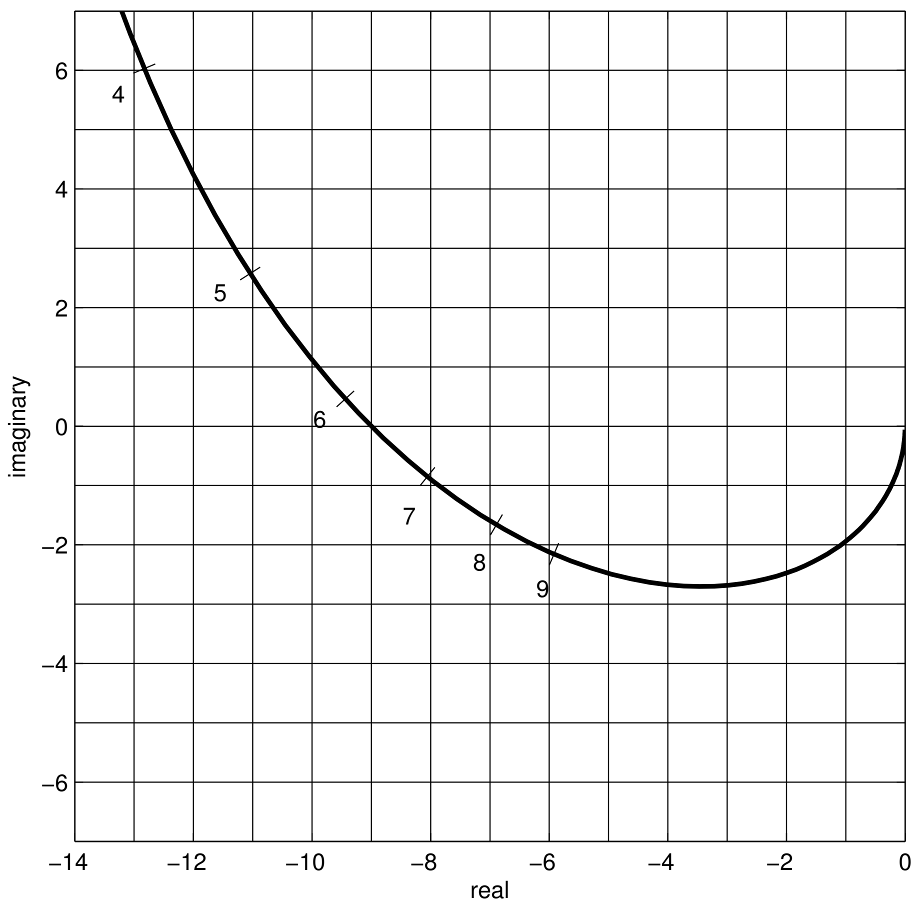

Fig. 1

(TURN OVER

2 Figure 2 is the Bode diagram of a system with transfer function $G(s)$ for which a compensator $K(s)$ is to be designed. It is known that the system has no poles satisfying $\mathrm{Re}(s) > 0$.

(a) (i) Sketch on a copy of Fig. 2 the expected phase of $G(j\omega)$ if $G(s)$ were stable and minimum phase. [15%]

(ii) Determine the form of the all-pass part of the transfer function and estimate any associated parameters. [10%]

(iii) Comment briefly on any limitations that may be experienced in the design of $K(s)$. [10%]

(b) Suppose it is desired to achieve the following specifications for the return ratio $L(s) = G(s)K(s)$:

A: Velocity error constant $K_v = 10$ where $K_v = \lim_{s\to 0}(sL(s))$;

B: Gain cross-over frequency ω_c (i.e. $|L(j\omega_c)| = 1$) satisfying $\omega_c \geq 1\ \mathrm{rad\,s^{-1}}$;

C: Phase margin of at least 45°;

D: $|L(j\omega)| \leq 0.1$ for $\omega \geq 20\ \mathrm{rad\,s^{-1}}$.

(i) By considering the cases of a lead and lag compensator separately, or otherwise, explain why it is not possible to achieve the specifications using a compensator with one pole and one zero. [25%]

(ii) Choose constants $\omega_1 > 0$, $\omega_2 > 0$ and $\alpha > 1$ to show that the specifications A–D can be met with $\omega_c = 1$ using [40%]

$$K(s) = \left(\frac{s+\omega_1\alpha}{s+\omega_1/\alpha}\right)\left(\frac{s+\omega_2/\alpha}{s+\omega_2\alpha}\right).$$

Sketch the Bode diagram of $K(s)$ and $G(s)K(s)$ for your design on a copy of Fig. 2.

Two copies of Fig. 2 are provided on separate sheets. These should be handed in with your answers.

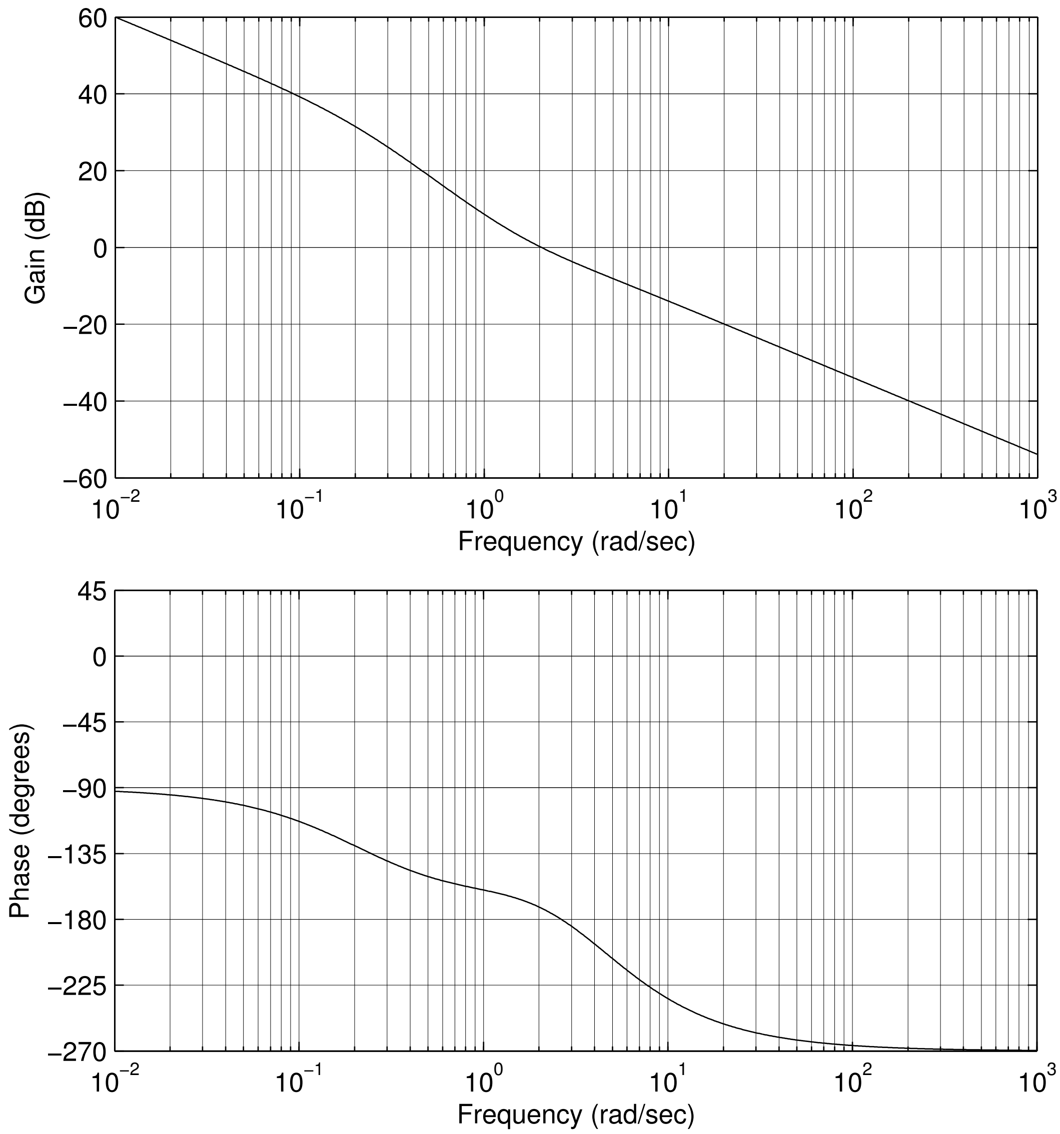
60
40
20
0
−20
−40
−60
Gain (dB)
10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3}
Frequency (rad/sec)
45
0
−45
−90
−135
−180
−225
−270
Phase (degrees)
10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3}
Frequency (rad/sec)

Fig. 2

3 (a) For a plant with transfer function $G(s)$ sketch the block diagram for a two-degree-of-freedom control system. Write down (but do not prove) the conditions which must apply to the transfer function from reference input $r(t)$ to plant output $y(t)$ in any design. [15%]

(b) Let $R(s)$ be the transfer function of a stable system. Suppose that $R(s)$ has a right half-plane zero at $s = \alpha$. By considering the definition of the Laplace transform, or otherwise, show that the step response $y(t)$ of the system satisfies [15%]

$$0 = \int_0^\infty y(t)e^{-\alpha t}\,dt.$$

(c) A control system is to be designed for the superconducting magnet in a magnetic levitation train (maglev). The transfer function from coil voltage $u(t)$ to vertical displacement $y(t)$ after linearisation and normalisation takes the form:

$$G(s) = \frac{\omega_1^2}{s^2 - \omega_1^2}$$

where $\omega_1 > 0$.

(i) Show that this plant can be stabilised by a lead compensator of the form: [15%]

$$K(s) = \frac{k(s+\omega_1)}{s+b}.$$

(ii) Hence, or otherwise, design a two-degree-of-freedom control system for which the transfer function from reference input $r(t)$ to plant output $y(t)$ is given by: [20%]

$$T_{r\to y} = \frac{\omega_1^3}{(s+\omega_1)^3}.$$

(iii) Find the transfer function $T_{r\to u}$ from reference input $r(t)$ to plant input $u(t)$ for any design satisfying part (c)(ii). Find the initial slope and final value of the step response of $T_{r\to u}$. Sketch the form of the step response without further calculation. [20%]

(iv) Prove that $T_{r\to u}$ has a zero at $s = \omega_1$ for *any* two-degree-of-freedom design and explain the consequences of this fact making use of part (b) or otherwise. [15%]

END OF PAPER

Version MCS/4 Candidate Number:

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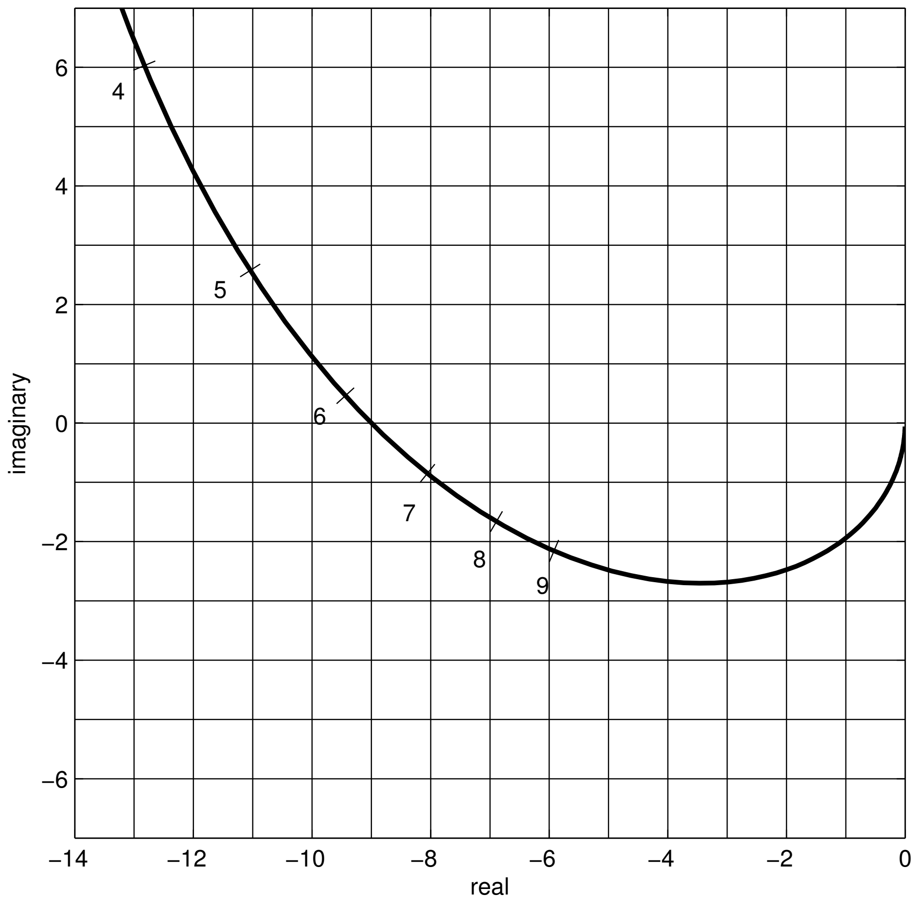
Friday 25 April 2014, Module 4F1, Question 1.



Extra copy of Fig. 1: Nyquist diagram for Question 1.

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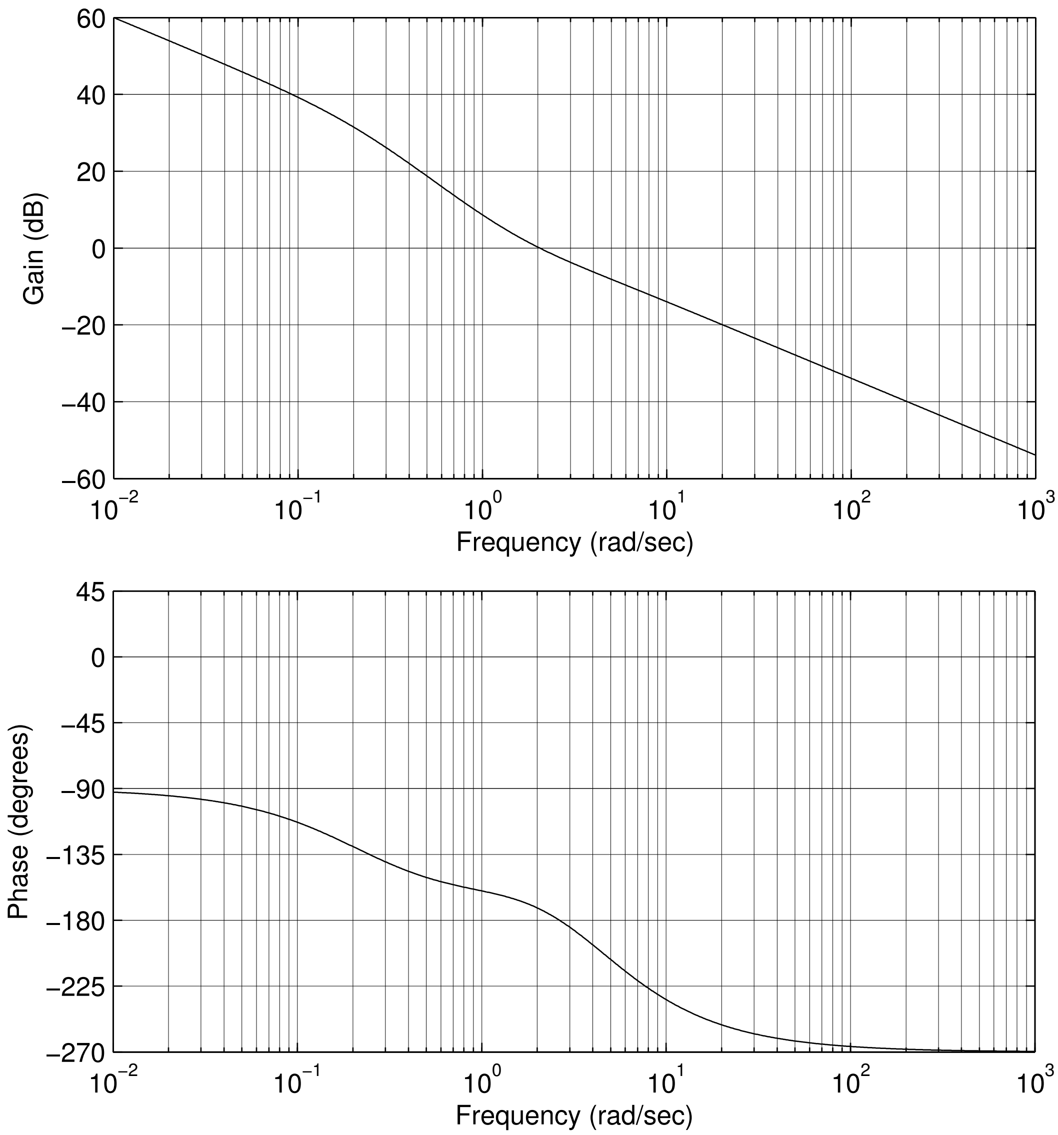
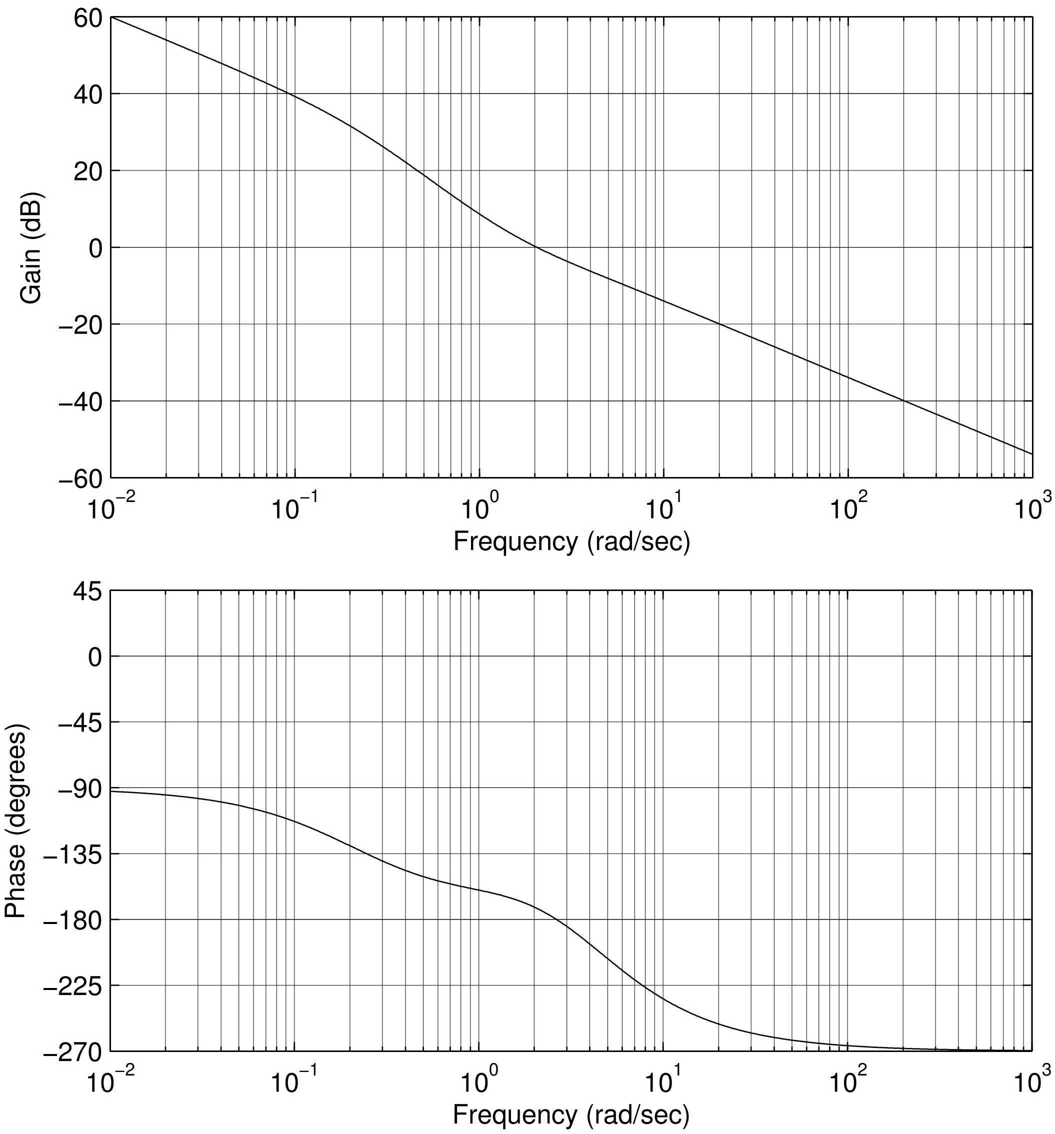
Friday 25 April 2014, Module 4F1, Question 2.



Extra copy of Fig. 2: Bode diagram for Question 2.

Version MCS/4 Candidate Number:



Extra copy of Fig. 2: Bode diagram for Question 2.