4F3 2014: Numerical answers

Question 1.

(a)(i) $F = \begin{bmatrix} A & 0 \\ CA & I \end{bmatrix}$, $G = \begin{bmatrix} B \\ CB \end{bmatrix}$

(ii) $P \geq 0$, $Q_1 \geq 0$, $Q_2 \geq 0$, $R \geq 0$. ($R > 0$ for uniqueness)

(iii) Local = global optimum. Efficient algorithms.

(b)(i) $\begin{bmatrix} (A-I) & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x_\infty \\ u_\infty \end{bmatrix} = \begin{bmatrix} 0 \\ r \end{bmatrix}$ or $\begin{bmatrix} I-A & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} x_\infty \\ u_\infty \end{bmatrix} = \begin{bmatrix} 0 \\ r \end{bmatrix}$

Existence: Full row rank

Uniqueness: Invertible

(ii) $\Delta x_\infty = 0$, $\Delta u_\infty = 0$, $z_\infty = r$.

(iii) $z_i \leftarrow z_i - r$, $z_N \leftarrow z_N - r$.

(iv) –

Question 2.

(a)(i) –

(ii) –

(b)(i) $\Phi = \begin{bmatrix} I \\ A \\ A^2 \\ A^3 \end{bmatrix}$, $\Gamma = \begin{bmatrix} 0 & & \\ B & & \\ AB & B & \\ A^2B & AB & B \end{bmatrix}$ (ii) $F = \begin{bmatrix} C & & & \\ & C & & \\ & & C & \\ & & & T \end{bmatrix}$, $G = \begin{bmatrix} D & & \\ 0 & D & \\ 0 & 0 & D \\ 0 & 0 & 0 \end{bmatrix}$, $g = \begin{bmatrix} e \\ e \\ e \\ t \end{bmatrix}$

(iii) $(F\Gamma + G)\underline{\mathbf{u}} \leq g - F\Phi x(k)$

(c)(i)

$$\mathbb{T} = \left\{ x : (C+DK)(A+BK)^i x \leq e, \quad \forall i = 0, \ldots, \infty \right\}.$$

(ii) Recursive feasibility of RHC in closed loop

Question 3.

(a) –

(b)(i) –

(ii) $-\dot{p}(t) = 1 - p(t)^2 + 4p(t)$, $P(T) = 10$.

Question 4.

(a) $\|y\|_\infty \leq \frac{1}{2\pi}\|G(s)\|_2 \|u\|_2$., where $\|y\|_\infty = \sup_t \sqrt{y(t)^T y(t)}$

(b)(i) –

(ii) –

(iii) $g(t) = Ce^{At}B$

(iv) $\|G(s)\|_2 = \sqrt{2\pi}\sqrt{\text{trace}(B^T Q B)}$

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