

## 4F5 2012 Exam Answers

- (1) (a)  $\|x_1(\cdot)\| = \frac{\sqrt{6}}{3}$ ,  $\|x_2(\cdot)\| = \frac{\sqrt{10}}{5}$ ,  $\|x_3(\cdot)\| = \frac{\sqrt{14}}{7}$   
 (b)  $\langle x_1(\cdot), x_2(\cdot) \rangle = 0$ ,  $\langle x_1(\cdot), x_3(\cdot) \rangle = \frac{2}{5}$ ,  $\langle x_2(\cdot), x_3(\cdot) \rangle = 0$ .  
 (c) For  $t \in [-1, 1]$ ,  $f_1(t) = \frac{\sqrt{6}}{2}t$ ,  $f_2(t) = \frac{\sqrt{10}}{2}t^2$ , and  $f_3(t) = \frac{5\sqrt{14}}{4}(t^3 - \frac{3}{5}t)$ .  
 (d)  $\mathbf{x}_1 = (\frac{\sqrt{6}}{3}, 0, 0)$ ,  $\mathbf{x}_2 = (0, \frac{\sqrt{10}}{5}, 0)$ ,  $\mathbf{x}_3 = (\frac{\sqrt{6}}{5}, 0, \frac{2\sqrt{14}}{35})$ ,  
 (e)  $\|\mathbf{x}_1 - \mathbf{x}_2\| = \frac{4\sqrt{15}}{15}$ ,  $\|\mathbf{x}_2 - \mathbf{x}_3\| = \frac{2\sqrt{210}}{35}$ ,  $\|\mathbf{x}_3 - \mathbf{x}_1\| = \frac{4\sqrt{105}}{105}$ .  
 (f)  $P_e \leq \frac{2}{3} \left[ Q\left(\frac{4\sqrt{15}}{15}\right) + Q\left(\frac{2\sqrt{210}}{35}\right) + Q\left(\frac{4\sqrt{105}}{105}\right) \right]$
- (2) (a) ...  
 (b)  $P_Z(1) = \delta$ ,  $H(X|Y, Z=0) = 0$ ,  $H(X|Y, Z=1) = H_2(p)$ ,  
 (c)  $I(X; Y) = H_2(p)(1 - \delta)$ ,  $\max_p I(X; Y) = 1 - \delta$ .  
 (d)  $P_e \geq \frac{1}{5} - \frac{2}{n}$ .  
 (e)  $I(X; Y_1, Y_2) = H_2(p)(1 - \delta_1\delta_2)$ ,  $C = \max_{P_X} I(X; Y_1, Y_2) = 1 - \delta_1\delta_2$ .  
 (f)  $I(X_2; Y_1, Y_2) = (1 - \delta)^2$ ,  $I(X_1; X_2, Y_1, Y_2) = 1 - \delta^2$ .
- (3) (a)  $R = 1/2$ , generator polynomials  $(6, 7)_8$ .  
 (b) 4 states  
 (c) ...
- (d) 

$x_i \backslash y_j$	A	B	C
0	1	0	0
1	0	0	1
- (e) 1,1,0,0
- (4) (a) (i) channel underspread  
 (ii) flat in time, selective in frequency  
 (b) (i) ...  
 (ii) ...  
 (iii) ...  
 (iv) diversity 3  
 (v) diversity less than 3.