

4F7 Digital filters and spectrum estimation

List of numerical answers to 2010 exam

Question 1

Part (a): –

Part (b): –

Part (c): –

Question 2

Part (a):

$$K(n) = \frac{\sigma(n-1)^2}{\sigma_v^2 + \sigma(n-1)^2}.$$

Part (b):

$$\hat{x}(n) = \frac{\sigma_0^2}{n\sigma_0^2 + \sigma_v^2} \sum_{i=1}^n y(i).$$

Part (c) Sample mean:

$$E \left\{ \left(\frac{1}{n} \sum_{i=1}^n y(i) \right)^2 \right\} = \frac{\sigma_v^2}{n} + \sigma_0^2.$$

Kalman estimate:

$$E \{ \hat{x}(n)^2 \} = \left(\frac{\sigma_0^2}{n\sigma_0^2 + \sigma_v^2} \right)^2 (n\sigma_v^2 + n^2\sigma_0^2).$$

Since $\frac{\sigma_0^2}{n\sigma_0^2 + \sigma_v^2} < \frac{1}{n}$, Kalman is better.

Question 3

Part (a): –

Part (b): –

Part (c): $N \geq 143.4/2 = 71.7$.

Part (d): –

Question 4

Part (a): –

Part (b-i): $b_0 = 1, b_1 = 1$.

Part (b-ii): $a_1 = 1, b_0 = \sqrt{3}, b_1 = -\sqrt{3}$.

Part(c): –