EGT3
ENGINEERING TRIPOS PART IIB

Monday 5 May 2014 2 to 3.30

Module 4F8

IMAGE PROCESSING AND IMAGE CODING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) If an image displays poor use of available grey levels we can often improve its appearance via the process of *histogram equalisation*.

(i) Consider the 6×6 image given in Fig. 1, where the range of greyscale values is 1 to 9.

9	8	7	7	8	9
7	9	8	8	9	7
6	8	9	9	8	6
6	9	8	8	9	6
9	8	7	7	8	9
9	7	6	6	7	9

Fig. 1

Sketch and comment on the histogram of this image. [10%]

(ii) Perform histogram equalisation on this image by finding the set of transformed values $\{y_k\},\ k = 1,..,9$ onto which the original greylevels are mapped. Sketch the new equalised image and its histogram, commenting on how well the process has worked. [25%]

(iii) The spread of greylevels might be improved by a simple interpolation process. Using any reasonable interpolation, show how a resultant histogram might look. (Note histogram values must be integers). [20%]

(b) Assume that an observed image, $y(\mathbf{n})$, can be modelled as a convolution of the true image, $x(\mathbf{n})$, with a point spread function (psf), $h(\mathbf{n})$, plus additive noise, $d(\mathbf{n})$, i.e.

$$y(\mathbf{n}) = \sum_{\mathbf{m} \in \mathbb{Z}^2} h(\mathbf{m})\, x(\mathbf{n} - \mathbf{m}) + d(\mathbf{n})$$

(i) Explain how the process of *generalised inverse filtering* can be used for deconvolving noiseless images when the psf is known, detailing why this method performs poorly for images with significant noise. [20%]

(ii) A deconvolution method that attempts to take account of the noise is the *Wiener filter*, $G(\omega)$. Give an expression for the Wiener filter in terms of the power spectrum of the image, $P_{xx}(\omega)$, the power spectrum of the noise, $P_{dd}(\omega)$, and the frequency response of the psf, $H(\omega)$. [10%]

(iii) In matrix terms we can write our image model as $\mathbf{y} = L\mathbf{x} + \mathbf{d}$. Give the matrix form of the Weiner filter, W, in terms of L and the covariances $N = E[\mathbf{d}\mathbf{d}^T]$ and $C = E[\mathbf{x}\mathbf{x}^T]$. Indicate the relationships between $\{L,\ N,\ C\}$ and the quantities in part (b)(ii). [15%]

2 (a) An ideal filter, $H(\omega_1,\omega_2)$, is shown in Fig. 2, with $H=1$ in the shaded regions and $H=0$ otherwise.

(i) Sampling is performed on a rectangular grid with spacings of Δ_1 and Δ_2 in the u_1 and u_2 directions respectively. Using standard results, or otherwise, find the ideal impulse response, $h(n_1,n_2)$, of this filter. [40%]

(ii) If $\Omega_{L1}\to 0$ and $\Omega_{L2}\to 0$, show that the expression obtained in part (a)(i) reduces to that for a rectangular lowpass filter with dimensions Ω_{U1} and Ω_{U2}. [10%]

(b) (i) Explain the occurrence of *aliasing* effects in images and describe the common distortions that are visible due to such aliasing. [15%]

(ii) If we sample a continuous 2D image $g(u_1,u_2)$ via a uniform grid at spacings Δ_1 in u_1 and Δ_2 in u_2, the sampled image $g_s(u_1,u_2)$ is given by:

$$g_s(u_1,u_2)=s(u_1,u_2)\,g(u_1,u_2)$$

where $s(u_1,u_2)=\sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty}\delta(u_1-n_1\Delta_1,u_2-n_2\Delta_2)$.
Derive the Fourier transform (spectrum) of the sampled signal and hence explain the phenomenon of aliasing discussed in part (b)(i). [35%]

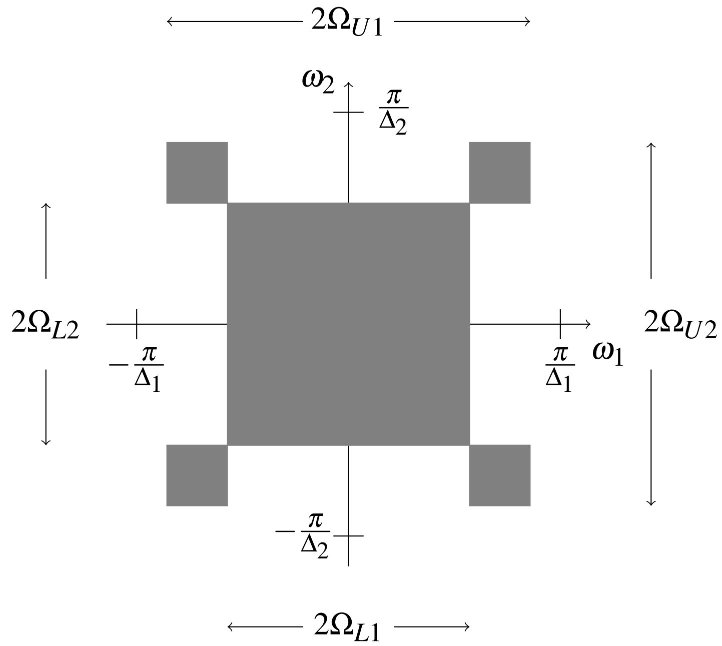

Fig. 2

3 The Haar transform matrix is given by:

$$T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

(a) Demonstrate that the matrix T is orthonormal, and show why it preserves the total energy of any 2-element input vector $\mathbf{x}$, when it is transformed into a vector $\mathbf{y} = T\mathbf{x}$. [15%]

(b) When the N pixels of an image are taken in non-overlapping vertical pairs, $\{\mathbf{x}_i\}$ for $i = 1$ to $N/2$, it is found that the covariance matrix of the pairs of pixels is

$$C_{xx} = E[\mathbf{x}_i \mathbf{x}_i^T] = \sigma^2 \begin{bmatrix} 1 & \alpha \\ \alpha & 1 \end{bmatrix}$$

where σ is a constant, and the expectation $E[\cdot]$ is taken over all $N/2$ non-overlapping pixel pairs in the image. If each vertical pair of pixels $\mathbf{x}_i$ is transformed into a pair of transform coefficients $\mathbf{y}_i$, derive the covariance matrix C_{yy} for the pairs $\{\mathbf{y}_i\}$. Hence estimate the total energy of each of the two subbands formed by grouping all of the $\{\mathbf{y}_i\}_1$ coefficients together, and all of the $\{\mathbf{y}_i\}_2$ coefficients together, respectively. Assume the input pixels come from a pdf with zero mean. [25%]

(c) When the Haar transform is applied to a 2D image, explain how the matrix T is used to generate the Haar transform of a 2×2 block of pixels. Also show how the inverse transform may be calculated to convert a 2×2 block of coefficients back into the original pixels. [25%]

(d) Explain how a simple 2D Haar transform may be applied to an image of size 768×1024 pixels over several levels in a *wavelet-like* way, such that three bandpass subbands are generated at each level together with a lowpass subband at the final level used. For a 3-level transform, give sizes for the subimages of Haar coefficients at each level, and hence show that the transform is non-redundant overall. [35%]

4 (a) Draw a 2-band analysis filter-bank with down-samplers, suitable for one dimensional (1D) signals. Draw the equivalent 2-band reconstruction filter-bank which is normally used to reconstruct the signal from the down-sampled outputs of the analysis system. [15%]

(b) Explain how a multi-level wavelet transform for 1D signals and its inverse may be built from these components. [20%]

(c) If the two analysis filters have z-transforms, $H_0(z)$ and $H_1(z)$, and the equivalent synthesis filters have transforms, $G_0(z)$ and $G_1(z)$, show that perfect-reconstruction (P-R) of the input signal may be achieved at the output of the reconstruction system if the following conditions are satisfied:

$$G_0(z)H_0(z) + G_1(z)H_1(z) = 2$$

and

$$G_0(z)H_0(-z) + G_1(z)H_1(-z) = 0$$

Hint: You may assume that, in the z-domain, the output of a downsample-by-2 and upsample-by-2 system is $\frac{1}{2}[Y(z) + Y(-z)]$, when its input is $Y(z)$. [25%]

(d) Highpass filters, H_1 and G_1, are obtained from the lowpass filters using

$$H_1(z) = z^{-1}\, G_0(-z) \quad \text{and} \quad G_1(z) = zH_0(-z)$$

A designer chooses

$$H_0(z) = (1+Z)(1 - \tfrac{2}{7}Z)$$

where $Z = \frac{1}{2}(z + z^{-1})$ in order to achieve filters that are symmetric about the term in z^0 and such that $H_0(z) = \{\sim\!\sqrt{2},\ 1,\ 0\}$ at $z = \{1,\ \pm j, -1\}$. If G_0 is of the form $(1+Z)(1 + aZ + bZ^2)$, determine rational values for the coefficients a and b that will produce P-R. [40%]

END OF PAPER

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