4G6 crib (22/23)

Q1

(a) $$-v\frac{dn}{dx} = (1-n)f(x) - ng(x)$$

under isometric conditions $v = 0$

$$\Rightarrow \quad (1-n)f = ng$$

$$n = \frac{f(x)}{f(x)+g(x)}$$

(b)

For shortening $n = 0$ $x > h$ as no crossbridges are dragged there & probability of attachment $= 0$ for $x > h$.

$0 < x < h$

$$-v\frac{dn}{dx} = (1-n)f_0 - ng_0$$

$$-v\frac{dn}{dx} = -n(f_0+g_0) + f_0$$

Homogeneous solution is $n = A \exp\left[\frac{(f_0+g_0)x}{V}\right]$

Particular solution is $n = B$

$$\Rightarrow B = \frac{f_0}{f_0+g_0}$$

$$n(x) = A \exp\left[\frac{(f_0+g_0)x}{V}\right] + \frac{f_0}{f_0+g_0}$$

$$n(h) = A \exp\left[\frac{(f_0+g_0)h}{V}\right] + \frac{f_0}{f_0+g_0} = 0$$

$$A = \frac{-f_0}{(f_0+g_0)\exp\left[\frac{(f_0+g_0)h}{V}\right]}$$

$$n(x) = \frac{f_0}{f_0+g_0}\left[1 - \exp\left\{\frac{(f_0+g_0)(x-h)}{V}\right\}\right]$$

$$n(0) = \frac{f_0}{f_0+g_0}\left[1 - \exp\left\{-\frac{(f_0+g_0)h}{V}\right\}\right]$$

$x < 0$

$$-V\frac{dn}{dx} = -ng_1$$

$$n = c\exp\left[\frac{g_1 x}{V}\right]$$

$$n(0) = c = \frac{f_0}{f_0+g_0}\left[1-\exp\left\{-\frac{(f_0+g_0)h}{V}\right\}\right]$$

$$n(x) = \frac{f_0}{f_0+g_0}\left[1-\exp\left\{-\frac{(f_0+g_0)h}{V}\right\}\right]\exp\left(\frac{g_1 x}{V}\right)$$

(c) When the tension reduces from its isometric value of T_0 to T_1, the extension of each crossbridge reduces by ΔL such that n changes from

$$n = \frac{f(x)}{f(x)+g(x)} \quad \text{to} \quad n_s(x+\Delta L)$$

where $$T_1 = \frac{n\,s\,A}{2}\int_{-\infty}^{\infty} b\,x\,n(x+\Delta L)\,dx$$

Now recall $\frac{\partial T_1}{\partial t} = 0 \Rightarrow$

$$0 = \int_{-\infty}^{\infty} \left(x \frac{\partial n}{\partial t} - v n \right) dx$$

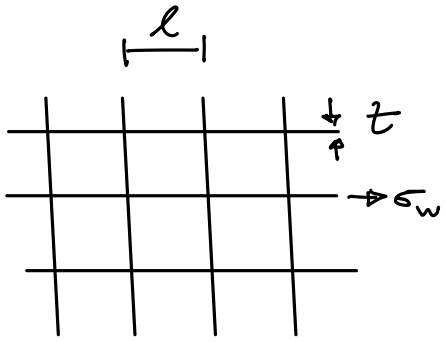
But $\frac{\partial n}{\partial t} - v \frac{\partial n}{\partial x} = (1-n) f - n g$

Thus,

$$\int_{-\infty}^{\infty} \left[x \left\{ (1-n) f - n g + v \frac{\partial n}{\partial x} \right\} - v n \right] dx = 0$$

where $v = -\frac{dx}{dt}$ can be solved with initial conditions $n(x, 0) = n_s(x + \Delta L)$ to obtain $v(t)$.

Q2.



(a)

$$\bar{\rho} = \frac{2tl}{l^2} = 2\left(\frac{t}{l}\right)$$

(b) let wall stress be σ_w & macroscopic stress is

$$\sigma_{11} l = \sigma_w t \quad \Rightarrow \sigma_w = \frac{\sigma_{11} l}{t}$$

wall strain $\varepsilon_w = \frac{\sigma_w}{E_s}$

Macroscopic strain = wall strain

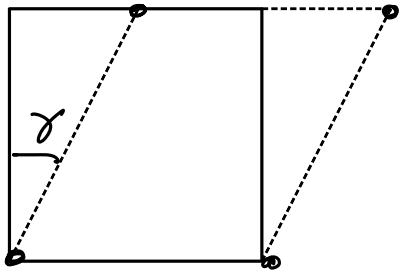
$$\Rightarrow \quad E_{11} = \frac{\sigma_{11}}{\varepsilon_w} = \frac{\sigma_w}{\varepsilon_w}\,\frac{t}{l}$$

$$= E_s\left(\frac{t}{l}\right)$$

$$E_{11} = \frac{E_s \bar{\rho}}{2}$$

(c) $\sigma_{11}^{Y} = \frac{t}{\ell} Y$

$\Rightarrow \sigma_{11}^{Y} = \frac{Y\bar{\rho}}{2}$

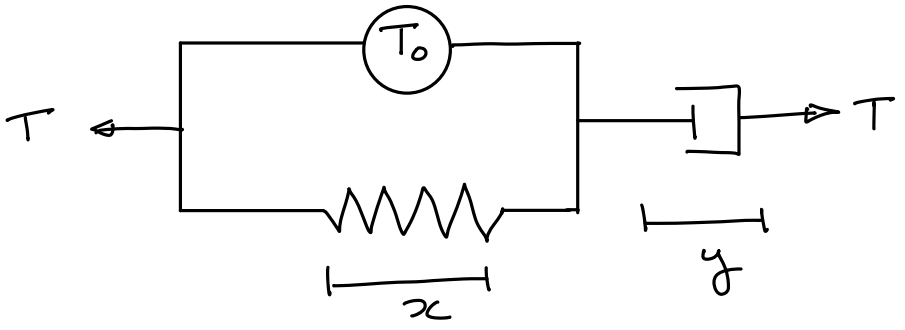


$\sigma_{12}^{Y} (t\ell)(\gamma\ell) = M_p \gamma \quad ; \quad M_p = \frac{1}{4} Y t^3$

$\sigma_{12}^{Y} = \frac{1}{4} Y \left(\frac{t}{\ell}\right)^2 = \frac{1}{16} \bar{\rho}^2 Y$

(d) The fact that the joints are not rigid but tied together by elastin reduces their rotational stiffness/strength $\Rightarrow$ drop in σ_{12}^{Y} but little effect on E_{11}.

Q3



(a)

$$T_0 + kx = -\eta \dot{x} \qquad \text{since } \dot{y} = -\dot{x} \text{ under isometric conditions}$$

$$-\eta \frac{\dot{x}}{T_0 + kx} = 1$$

(b) $$-\frac{\eta}{k} \ln\left(\frac{T_0 + kx}{C}\right) = t$$

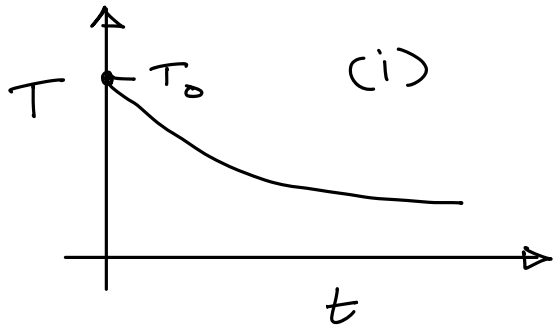
$$kx = C \exp\left(-\frac{kt}{\eta}\right) - T_0$$

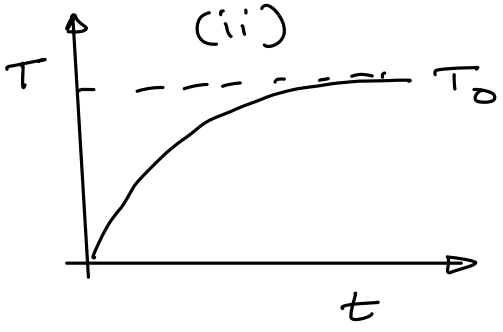
$$kx = 0 \text{ @ } t = 0 \Rightarrow C = T_0$$

$$kx = T_0\left[\exp\left(-\frac{kt}{\eta}\right) - 1\right]$$

$$T = T_0 + kx = T_0 \exp\left(-\frac{kt}{\eta}\right).$$

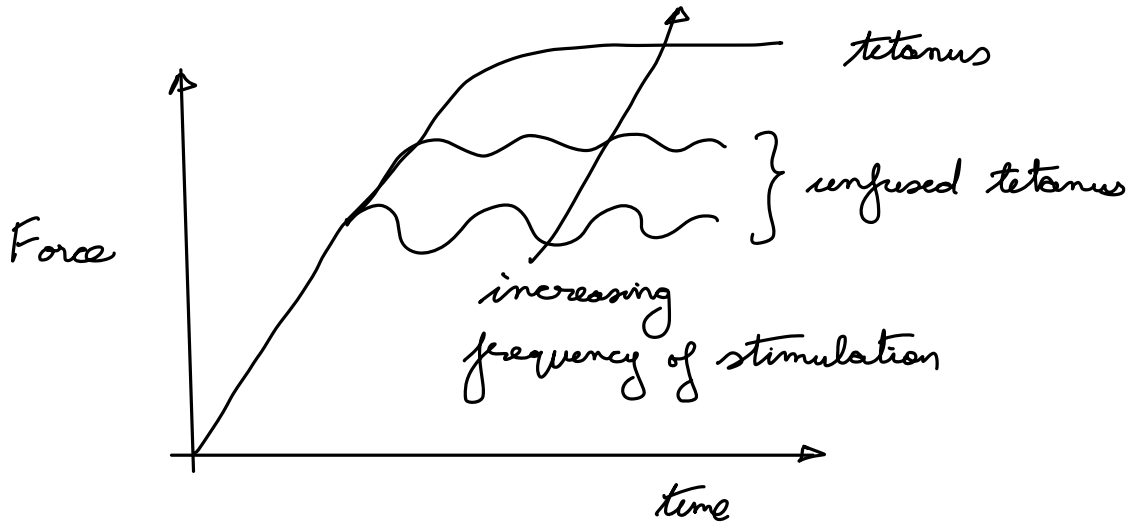
(c)





The model predicts (i) where the tension rises to T_0 instantaneously & then decays while under isometric conditions we would expect a response as sketched in (ii).

(d)



With increasing stimulation frequency, the twitches fuse so that both ripples in the force reduce & the force increases due to merging of the twitches. This is unfused tetanus. At tetanus the force is constant & the maximum achievable.

The Hexley model is <u>only</u> applicable at tetanus.

Q4.

(a) Animals only have a semi-permeable cell membrane encompassing cells. Thus, when animal cells are placed in a concentrated solution osmosis drives up the pressure inside the cells due to the influx of water. This pressure can build up to a level that bursts the cell membrane thereby killing the cell.

Plant cells are surrounded by a strong cellulose cell wall that can sustain these high osmotic pressure & ∴ plant cells can survive in a concentrated solution.

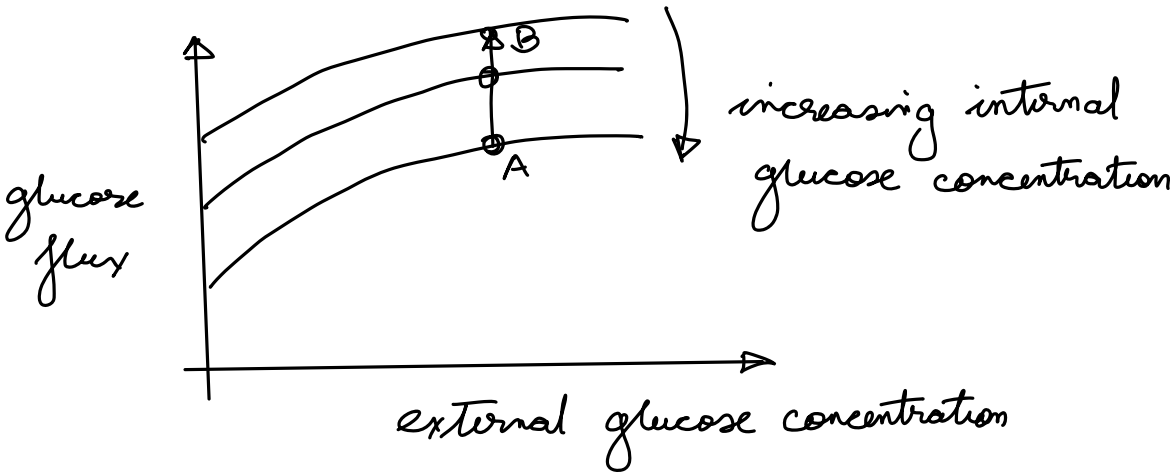
(b) The collagen fibres within skin are in a wavy network

This means the modulus of skin is not governed by the stretching of collagen fibres but rather by their bending as they straighten out under tension. Hence the modulus of skin is much less than the collagen fibres.

(c)

Myoglobin in a large molecule & relatively immobile. However, it stores oxygen & releases it when the enviromental oxygen concentration is low. This gives rise to an high effective diffusion rate as myoglobin acts as a source of oxygen. Thus, it increases the effective diffusion rate without actually transporting oxygen.

(d)



The phosphorylation of glucose decreases the internal glucose concentration & thereby results in the operating point moving from A to B on the above graph resulting in an increase in the glucose flux.