EGT3
ENGINEERING TRIPOS PART IIB

Wednesday 30 April 2014 9.30 to 11.00

Module 4G6

CELLULAR AND MOLECULAR BIOMECHANICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3C7 datasheet (2 pages).
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1. Wood cells are prismatic in shape, with the cross-section of a regular hexagonal honeycomb, as shown in Fig. 1(a). The cell walls comprise cellulose fibres and these are idealised by a prismatic, regular triangular lattice, as sketched in Fig. 1(b).

(a) Obtain an expression for the relative density $\bar{\rho}_1$ of the hexagonal lattice in terms of its strut thickness t_1 and strut length l_1. Likewise, obtain an expression for the relative density $\bar{\rho}_2$ of the triangular lattice in terms of its strut thickness t_2 and strut length l_2. [20%]

(b) Determine the uniaxial strength of the hexagonal lattice σ_{Y1} in terms of the yield strength σ_{YS} of the cell wall material. [40%]

(c) Determine the uniaxial strength of the triangular lattice σ_{Y2} in terms of the yield strength σ_F of the cellulose fibres. [30%]

(d) Hence obtain an expression for σ_{Y1} in terms of σ_F. [10%]

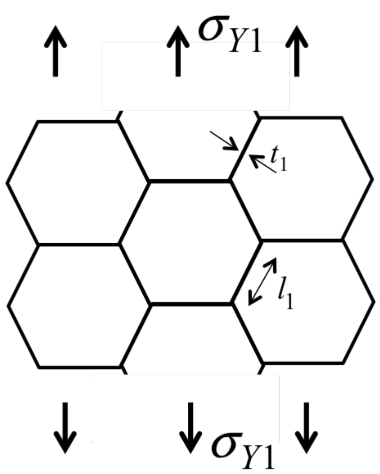

Fig. 1(a)

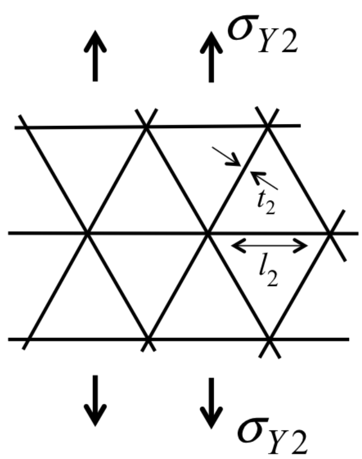

Fig. 1(b)

2 In the Huxley crossbridge model for a muscle, $n(x)$ is the fraction of attached crossbridges, where x is the position of an actin binding site from the equilibrium position of a myosin head. Assume that the attachment and detachment of the crossbridges is governed by a first order kinetic scheme with attachment and detachment rate constants $f(x)$ and $g(x)$, respectively.

(a) Determine the steady-state $n(x)$ in terms of $f(x)$ and $g(x)$ for a muscle in isometric tension. [20%]

(b) Given that:

$f(x)=0$; and $g(x)=g_1$; for $x<0$;
$f(x)=f_0$; and $g(x)=g_0$; for $0 \le x \le h$;
$f(x)=0$; and $g(x)=g_0$; for $x>h$;

determine $n(x)$ for shortening at a constant velocity $V=-dx/dt$. Here g_0, f_0, and h are constants. [50%]

(c) Qualitatively discuss how one might use the Huxley crossbridge dynamics model to calculate the response of a muscle in Hill's quick-release experiments (step change in tension). [30%]

3 (a) Describe the physical basis for the Young's modulus of biological tissues by explaining how the following concepts dictate the modulus:

(i) persistence length [25%]

(ii) nodal connectivity [25%]

(b) Qualitatively describe the myosin crossbridge cycle with reference to conversions of ATP to ADP. [25%]

(c) Describe the tension versus length curve of a single muscle fibre. Suppose that the tension decreased nonlinearly with increasing length for striation spacings greater than 2.5 μm. Would this invalidate the theory that the crossbridges working independently generate the tension? [25%]

4 (a) Summarize the main mechanical functions of the cytoskeleton within a cell by reference to the microtubules, actin cortex and intermediate filaments. [25%]

(b) (i) Briefly describe the working of the sodium-potassium pump in animal cells. [25%]

(ii) How does the concentration of ATP affect the rate of the sodium-potassium pump? [25%]

(iiii) The drug Ouabain competes with K^+ for external potassium binding sites of the Na^+-K^+ ATPase. Discuss the effects of Ouabain on animal cells with reference to the sodium-potassium pump. [25%]

END OF PAPER

Module 3C7: Mechanics of Solids

ELASTICITY and PLASTICITY FORMULAE

1. Axi-symmetric deformation : discs, tubes and spheres

	Discs and tubes	Spheres
Equilibrium	$\sigma_{\theta\theta} = \dfrac{d(r\sigma_{rr})}{dr} + \rho\omega^2 r^2$	$\sigma_{\theta\theta} = \dfrac{1}{2r}\dfrac{d(r^2\sigma_{rr})}{dr}$
Lamé's equations (in elasticity)	$\sigma_{rr} = A - \dfrac{B}{r^2} - \dfrac{3+\nu}{8}\rho\omega^2 r^2 - \dfrac{E\alpha}{r^2}\displaystyle\int_c^r rT\,dr$	$\sigma_{rr} = A - \dfrac{B}{r^3}$
	$\sigma_{\theta\theta} = A + \dfrac{B}{r^2} - \dfrac{1+3\nu}{8}\rho\omega^2 r^2 + \dfrac{E\alpha}{r^2}\displaystyle\int_c^r rT\,dr - E\alpha T$	$\sigma_{\theta\theta} = A + \dfrac{B}{2r^3}$

2. Plane stress and plane strain

Plane strain elastic constants $\quad \bar{E} = \dfrac{E}{1-\nu^2}\ ;\ \bar{\nu} = \dfrac{\nu}{1-\nu}\ ;\ \bar{\alpha} = \alpha(1+\nu)$

	Cartesian coordinates	Polar coordinates
Strains	$\varepsilon_{xx} = \dfrac{\partial u}{\partial x}$	$\varepsilon_{rr} = \dfrac{\partial u}{\partial r}$
	$\varepsilon_{yy} = \dfrac{\partial v}{\partial y}$	$\varepsilon_{\theta\theta} = \dfrac{u}{r} + \dfrac{1}{r}\dfrac{\partial v}{\partial\theta}$
	$\gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}$	$\gamma_{r\theta} = \dfrac{\partial v}{\partial r} + \dfrac{1}{r}\dfrac{\partial u}{\partial\theta} - \dfrac{v}{r}$
Compatibility	$\dfrac{\partial^2\gamma_{xy}}{\partial x\partial y} = \dfrac{\partial^2\varepsilon_{xx}}{\partial y^2} + \dfrac{\partial^2\varepsilon_{yy}}{\partial x^2}$	$\dfrac{\partial}{\partial r}\left\{r\dfrac{\partial\gamma_{r\theta}}{\partial\theta}\right\} = \dfrac{\partial}{\partial r}\left\{r^2\dfrac{\partial\varepsilon_{\theta\theta}}{\partial r}\right\} - r\dfrac{\partial\varepsilon_{rr}}{\partial r} + \dfrac{\partial^2\varepsilon_{rr}}{\partial\theta^2}$
or (in elasticity)	$\left\{\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right\}(\sigma_{xx} + \sigma_{yy}) = 0$	$\left\{\dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2}{\partial\theta^2}\right\}(\sigma_{rr} + \sigma_{\theta\theta}) = 0$
Equilibrium	$\dfrac{\partial\sigma_{xx}}{\partial x} + \dfrac{\partial\sigma_{xy}}{\partial y} = 0$	$\dfrac{\partial}{\partial r}(r\sigma_{rr}) + \dfrac{\partial\sigma_{r\theta}}{\partial\theta} - \sigma_{\theta\theta} = 0$
	$\dfrac{\partial\sigma_{yy}}{\partial y} + \dfrac{\partial\sigma_{xy}}{\partial x} = 0$	$\dfrac{\partial\sigma_{\theta\theta}}{\partial\theta} + \dfrac{\partial}{\partial r}(r\sigma_{r\theta}) + \sigma_{r\theta} = 0$
$\nabla^4\phi = 0$ (in elasticity)	$\left\{\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right\}\left\{\dfrac{\partial^2\phi}{\partial x^2} + \dfrac{\partial^2\phi}{\partial y^2}\right\} = 0$	$\left\{\dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2}{\partial\theta^2}\right\} \times \left\{\dfrac{\partial^2\phi}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial\phi}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2\phi}{\partial\theta^2}\right\} = 0$
Airy Stress Function	$\sigma_{xx} = \dfrac{\partial^2\phi}{\partial y^2}$	$\sigma_{rr} = \dfrac{1}{r}\dfrac{\partial\phi}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2\phi}{\partial\theta^2}$
	$\sigma_{yy} = \dfrac{\partial^2\phi}{\partial x^2}$	$\sigma_{\theta\theta} = \dfrac{\partial^2\phi}{\partial r^2}$
	$\sigma_{xy} = -\dfrac{\partial^2\phi}{\partial x\partial y}$	$\sigma_{r\theta} = -\dfrac{\partial}{\partial r}\left\{\dfrac{1}{r}\dfrac{\partial\phi}{\partial\theta}\right\}$

3. Torsion of prismatic bars

Prandtl stress function: $\sigma_{zx}\ (=\tau_x) = \dfrac{\partial\psi}{\partial y}, \quad \sigma_{zy}\ (=\tau_y) = -\dfrac{\partial\psi}{\partial x}$

Equilibrium: $T = 2\int_A \psi\, dA$

Governing equation for elastic torsion: $\nabla^2\psi = -2G\beta$ where β is the angle of twist per unit length.

4. Total potential energy of a body

$$\Pi = U - W$$

where $U = \frac{1}{2}\int_V \underset{\sim}{\varepsilon}^{T} [D]\, \underset{\sim}{\varepsilon}\, dV$, $W = \underset{\sim}{P}^{T}\, \underset{\sim}{u}$ and $[D]$ is the elastic stiffness matrix.

5. Principal stresses and stress invariants

Values of the principal stresses, σ_P, can be obtained from the equation

$$\begin{vmatrix} \sigma_{xx}-\sigma_P & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy}-\sigma_P & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz}-\sigma_P \end{vmatrix} = 0$$

This is equivalent to a cubic equation whose roots are the values of the 3 principal stresses, i.e. the possible values of σ_P.

Expanding: $\sigma_P^3 - I_1\,\sigma_P^2 + I_2\sigma_P - I_3 = 0$ where $I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$,

$$I_2 = \begin{vmatrix} \sigma_{yy} & \sigma_{yz} \\ \sigma_{yz} & \sigma_{zz} \end{vmatrix} + \begin{vmatrix} \sigma_{xx} & \sigma_{xz} \\ \sigma_{xz} & \sigma_{zz} \end{vmatrix} + \begin{vmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{vmatrix} \quad \text{and} \quad I_3 = \begin{vmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{vmatrix}.$$

6. Equivalent stress and strain

Equivalent stress $\bar{\sigma} = \sqrt{\frac{1}{2}}\ \{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\}^{1/2}$

Equivalent strain increment $d\bar{\varepsilon} = \sqrt{\frac{2}{3}}\ \{d\varepsilon_1^2 + d\varepsilon_2^2 + d\varepsilon_3^2\}^{1/2}$

7. Yield criteria and flow rules

<u>Tresca</u>

Material yields when maximum value of $|\sigma_1-\sigma_2|$, $|\sigma_2-\sigma_3|$ or $|\sigma_3-\sigma_1|$ $= Y = 2k$, and then,

if σ_3 is the intermediate stress, $d\varepsilon_1 : d\varepsilon_2 : d\varepsilon_3 = \lambda(1 : -1 : 0)$ where $\lambda \neq 0$.

<u>von Mises</u>

Material yields when, $(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 = 2Y^2 = 6k^2$, and then

$$\frac{d\varepsilon_1}{\sigma'_1} = \frac{d\varepsilon_2}{\sigma'_2} = \frac{d\varepsilon_3}{\sigma'_3} = \frac{d\varepsilon_1-d\varepsilon_2}{\sigma_1-\sigma_2} = \frac{d\varepsilon_2-d\varepsilon_3}{\sigma_2-\sigma_3} = \frac{d\varepsilon_3-d\varepsilon_1}{\sigma_3-\sigma_1} = \lambda = \frac{3}{2}\frac{d\bar{\varepsilon}}{\bar{\sigma}}.$$

Numerical answers to 4G6

1. (a) $\rho_1 = \frac{2t_1}{\sqrt{3}l_1}$; $\rho_2 = 2\sqrt{3}\, t_2/l_2$

   (b) $\sigma_{Y1} = \frac{\rho_1^2}{2}\sigma_{YS}$; $\sigma_{Y2} = \frac{\rho_2}{2}\sigma_{YS}$;

   (c) $\sigma_{Y1} = \frac{\rho_1\, \rho_2}{4}\sigma_f$

2. (a) $n = f/(f+g)$