Version CRIB v2

EGT4
ENGINEERING TRIPOS PART IIB

CRIB 2025

Module 4G7

CONTROL & COMPUTATION IN LIVING SYSTEMS

*Answer **both** questions in section A. Answer **one** question in section B.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment 1: Hurkey et al. (2023) paper for Question 3 (8 pages)
Attachment 2: London et al. (2010) paper for Question 4 (6 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

Version CRIB v2

SECTION A

Answer **both** questions.

1 (a) (i) elementary reactions are:

- $X_i \to \emptyset$
- $X_i + X_j \to X_k$
- $X_i \to X_j$

as well as the reverse reactions [15%]

1a(ii) Such reaction rate forms arise from taking a (quasi) steady state of an enzymatic reaction e.g.

$$S + E \rightleftharpoons SE \to P$$

The exponent n comes from multiple binding steps in $S+E$, e.g. $S + nE \rightleftharpoons SE + (n-1)E \rightleftharpoons \ldots SE_n$

[15%]

1b(i) The positive (autocatalitic) term $\dot{x} = x$ comes from breeding

Negative self/cross interactions $\dot{x} = -x^2$ or $\dot{x} = -xy$ represent competition between or within species

Assumption: animal numbers are well-described by positive real numbers, i.e. population is large enough that ± 1 increments are negligible.

[10%]

[50%]

b(ii) equilibria (x, y) s.t. $\begin{cases} \dot{x} = x(1-x-y) = 0 \\ \dot{y} = y(3-x-\frac{3}{2}y) = 0 \end{cases}$

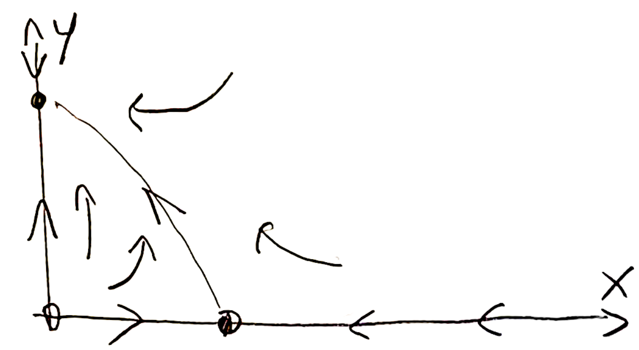
Jacobian, J: $\begin{pmatrix} \delta\dot{x} \\ \delta\dot{y} \end{pmatrix}\Bigg|_{\substack{\text{equil.}\\(x,y)}} = \begin{pmatrix} 1-2x-y & -x \\ -y & 3-3y-x \end{pmatrix}$

@ $(0,0)$ $J = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$ unstable node.

@ $(0,2)$ $J = \begin{pmatrix} -2 & 0 \\ -2 & -3 \end{pmatrix}$ stable node

@ $(1,0)$ $J = \begin{pmatrix} -1 & -1 \\ 0 & 2 \end{pmatrix}$ saddle (unstable)

Sketch



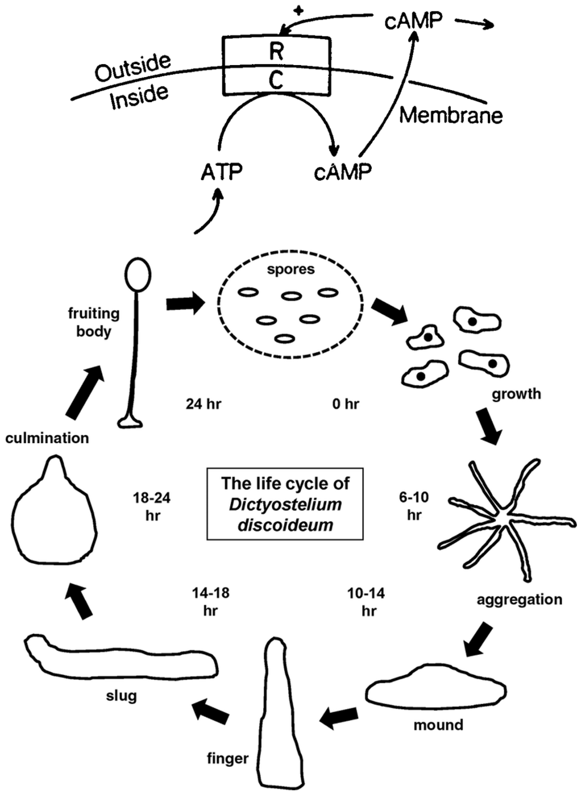
b(iii) For $x(0) \geq 0$, $y(0) \geq 0$ system always approaches $(0,2)$ as $t \to \infty$. So species x dies out in the long run.

[10%]

2 . [30%]

2(a) social amoeba Dictostelium (slime mould) enters different stages in its life cycle via **quorum sensing**

- during the growth phase cells multiply and aggregate
- as food becomes scares, cells release cyclic AMP, a **chemoattractant**
- cells also possess cAMP receptors, that convert ATP to cAMP
- thus there is positive feedback between population-level cAMP and cellular secretion of cAMP which leads to **coherent population oscillations**
- in populations this results in spiral and circular waves, allowing cells to navigate toward each other and form a fruiting body
- the fruiting body generates spores that diffuse to other locations, spawning new colonies

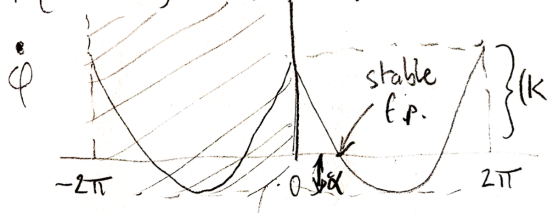


b(i)

$$\dot{\varphi} = \dot{\theta}_1 - \dot{\theta}_2 = \overbrace{\omega_1 - \omega_2}^{\alpha} + k_1(|\theta_1 - \theta_2| - \pi)^2 - k_2(|\theta_1 - \theta_2| - \pi)^2$$

$$\dot{\varphi} = \alpha + (k_1 - k_2)(|\varphi| - \pi)^2 \qquad \text{wlog } k_1 > k_2$$

$\varphi \pmod{2\pi}$. Synchrony requires stable f.p. for φ.



$\}(k_1 - k_2)\pi^2 - \alpha > 0$ (or < 0)

$\dot{\varphi}$ has one stable fixed point $\iff$ two cases

$\alpha < 0$ ($\omega_1 < \omega_2$) $\Rightarrow$ $k_1 - k_2 > \frac{\alpha}{\pi^2}$ or $k_1 > \frac{\alpha}{\pi^2} + k_2$

OR $\alpha > 0$ ($\omega_1 > \omega_2$) $\Rightarrow$ $(k_1 - k_2) < \frac{\alpha}{\pi^2}$ or $k_1 < \frac{\alpha}{\pi^2} + k_2$

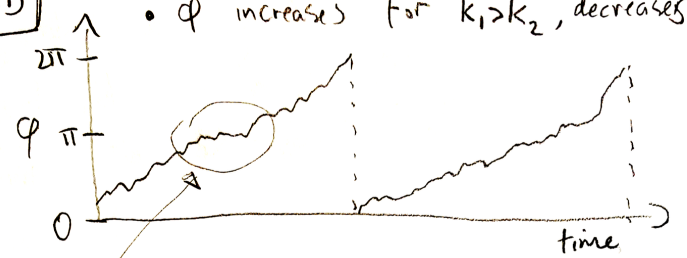
[40%]

b(ii) Dynamics of φ undergo a saddle-node (fold) bifurcation as k_1, k_2 are varied inside/outside the set in (i)

[10%]

b(iii)

- φ is random walk for $k_1 = k_2$
- φ increases for $k_1 > k_2$, decreases for $k_1 < k_2$



Dynamics slow down near $\varphi = \pi$ for $k_1 \neq k_2$ (have longer autocorrelation time).

[20%]

SECTION B

Answer **one** question.

3 (a) Essay key points:

- •background: study focussed on a network of motor neurons in Drosophila that provide rhythmic input to wing muscles during flight; a key property is that the discharge of these units is desynchronized, evenly spacing spikes to provide smooth control of muscle contraction and wingbeat frequency and vigour
- •goal: understand how synchronization/desynchronization of motor units occurs
- •methods: in-vivo electrophysiological recordings and laser-based wingbeat detection, combined with computational and mathematical analysis of a network of motor units with the same connection topology (gap junctions) as in the animal
- •main findings: the dynamical type of the motor units have a determining effect on synchronization, as revealed by the phase resetting/response curve (PRC); a SNIC PRC produces synchrony, a SNL PRC produces a splay state (sketch helps); the PRC is in turn determined by expression level of a potassium channel (Shab) which the investigators manipulate to test the prediction of this model

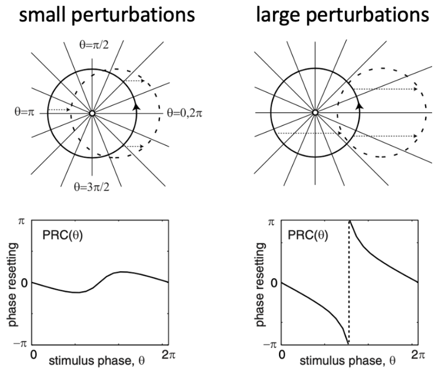
[60%]

(b) . [40%]

the phase resetting/response curve (PRCs) maps a perturbation (or transient input) to a change in oscillator phase, as a function of phase

its shape depends on the **size** and **direction** of the perturbation in state space as well as the underlying dynamics of the oscillator

for large perturbations it can change shape completely (and thus give misleading predictions), as shown below



4 (a) Essay key points:

- background: cortical signalling uses spikes, but there is debate about whether information is encoded in the rate of spiking, or whether coordinated spiking events carry information and enable computation
- goal: determine whether cortex likely uses rate coding or coordinated spiking, by perturbing spiking events in an intact brain
- methods: hypothesis is illustrated by showing spiking sensitivity in a simple model; the input-output sensitivity of single units is characterised using single-cell recordings in which single spikes are injected into the surrounding network
- main findings: state of cortical networks is sensitive to addition/perturbation of individual spikes in a single cell

[60%]

(b) Rate coding means that information is carried in the (time-averaged) rate of spiking events, or can be defined as the instantaneous rate of a suitable point process (e.g. Poisson process). Alternatives would include coding via time differences between spikes. [20%]

(c) The authors demonstrate that cortex can be sensitive to single spiking events, then use this as an argument that the cortex cannot process information using spikes, which is somewhat circular. [20%]

END OF PAPER

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