EGT3
ENGINEERING TRIPOS PART IIB

Monday 2 May 2016 2 to 3.30

Module 4I10 CRIB

REACTOR ENGINEERING AND THERMAL-HYDRAULICS

1

(a)

For a single phase flow, heat transfer coefficient can be approximated using Dittus-Boelter correlation:

$Nu=0.023\ Re^{0.8}\ Pr^{0.4}$

Water properties at 2 bar since the core is 10 m below the pool surface:

$\mu=0.000687$ Pa-s;

k=0.625 W/m/K;

$C_p=4212$ J/kg/K

Number of assemblies Na=25, assembly size = 10 x 10 x 50 cm

Number of plates Np=20, power per plate $Q_p=5{,}000{,}000/25/20=10{,}000$ W

Flow rate per plate $\dot{m}=Q_p/\ C_p/(T_{out} - T_{in})=10{,}000/4212/(50\text{-}25)=0.095$ kg/s

Flow area per plate A=0.10x0.10x0.3/20 = 0.00015 m^2

Heated perimeter $P_h=2\text{x}10=20$ cm = 0.2 m

Equivalent diameter $d=4A/P_h=4\text{x}0.0004/0.2=0.003$m

$Re=Gd/\mu=\dot{m}/A\ d/\mu = 0.095/0.00015 \text{ x } 0.003/0.000687 = 2765$

$Pr= C_p\mu/k = 4212 \text{ x } 0.000687 / 0.625 = 4.63$

$Nu = 0.023\ Re^{0.8}\ Pr^{0.4} = 24$

h= Nu k/d = 5012 $W/m^2/K$

(b)

$\Delta p=\Delta p_{fric}+\Delta p_{acc}+\Delta p_{grav}$

Gravity losses are: $\Delta p_{grav} = \int_{z_{in}}^{z_{out}} \rho(z)gdz \approx \rho_{av}g(z_{out} - z_{in})$

$\rho(25C)=997$ kg/m^3 ; $\rho(50C)=988$ kg/m^3 ; $\rho_{av} = 993$ kg/m^3 ; z=0.5m ; g=9.8 m/s^2

$\Delta p_{grav} = \rho_{av}\ g\ \Delta z = - 4858$ Pa ;

$\Delta p_{acc} \sim 0$ assuming the flow area does not change. It is not exactly zero because water does heat up and expands, so that for a fixed mass flow rate, the velocity has to increase.

$\Delta p_{fric} = \rho_{av}\ v^2/2\ L/d\ f$;

$f = 0.184/Re^{0.2} = 0.184/2765^{0.2} = 0.0377$

$v = \dot{m}/\ A\rho_{av} = 0.19/0.0004/991 = 0.6376$ m/s

$\Delta p_{fric} = 991 \text{ x } 0.6376^2/2 \text{ x } 0.5/0.008\ 0.0328 = 1269$ Pa

(c)

Pumping power to overcome frictional losses is: $W_p= \Delta p_{fric}\ \dot{m}\ /\ \rho_{av}$

It is clear that gravity pressure head is more than sufficient to overcome the frictional pressure losses even without additional head provided by a pump. In a closed circuit however, gravitational pressure losses roughly balance each other with an exception of buoyancy effects due to the liquid density changes as a result of heating or cooling. In this case, buoyancy would introduce an additional pressure loss because the flow through the core is downwards.

(d)

(i) The pipe is needed to thermally isolate the flow of the rising hot water so that the buoyancy driving force will have larger height differential. In the absence of the pipe, the hot water would mix with the surrounding cold water which will increase the water density and reduce the buoyancy. Longer pipe would increase the height differential between the thermal centers but will also introduce additional pressure drop. Large pipe diameter would reduce the additional frictional Δp but might allow cold water entry if the diameter is much larger than the size of the core.

(ii)

Power at nominal value: Q = 5,000,000 W

Assuming the pipe is perfectly insulating, boiling might develop even outside the core because of the drop in pressure as the water is rising. Therefore, limiting T_{out} should be 100 C, i.e. saturation temperature at 1 bar (pool surface) and not 2 bar (core outlet).

Core flow rate to remove the power with $\Delta T=(100-25)=75$ C:

$\dot{m}=Q/C_p/(T_{out}-T_{in}) = 5{,}000{,}000 / 4212 / 75 = 15.8$ kg/s

use upper estimate of C_p (at saturation) to be on conservative side, i.e. underestimation of temperature rise for a given heat.

Velocity in the core: $v_c = \frac{\dot{m}}{\rho A} = \frac{15.8}{977 \times 0.5 \times 0.5 \times 0.3} = 0.216$ m/s

$Re=\dot{m}/A\ d/\mu = \frac{0.95 \times 0.003}{25 \times 20 \times 0.00015 \times 0.000591} = 1071$ thus the flow is laminar and

$f = 64/Re = 0.06$

Neglecting entry-exit Δp, frictional Δp in the core:

$\Delta p_{fric} = \rho \frac{v^2}{2} f \frac{L}{d} = 977\ \frac{(0.216)^2}{2} 0.06 \frac{0.5}{0.003} = 227$ Pa

Same for the pipe:

$Re=\dot{m}/A\ d/\mu = \frac{15.8 \times d}{\frac{\pi d^2}{4} \times 0.000282} = 142{,}926$ – turbulent flow

$f = 0.184/Re^{0.2} = 0.017$

$\Delta p_{fric} = \rho \frac{v^2}{2} f \frac{L}{d} = 977\ \frac{\left(\frac{15.8}{958\ A}\right)^2}{2} 0.017 \times \frac{3}{0.5} = 0.35$ Pa – can be neglected compared to the core frictional Δp.

Buoyancy force: $\Delta p = g\ \Delta\rho\ \Delta z = 9.8\ m/s^2 \times (997\text{–}958)\ kg/m^3 \times 3m = 1147\ Pa > \Delta p_{fric}$

This is sufficient to drive the necessary flow.

2

(a) Potential benefits of annular fuel:

- Larger cooling surface area reduces the heat flux allowing power uprate.
- Shorter heat conduction path would reduce fuel operating temperature resulting in:
    - Less swelling and fission gas release, thus higher achievable burnup
    - Less stored energy ~ $m_{fuel}C_{fuel}$ ($T_{fuel} – T_{heat\ sink}$)
    - Lower Doppler reactivity decrement, thus lower enrichment

(b)

(i) Hot to Cold reactivity decrement is due to two reactivity feedback effects: Fuel temperature and Moderator density. Given that H/HM and enrichment remain the same, MTC should be largely unaffected. Doppler coefficient should be larger because resonance absorption in U238 (the main contributor to DC) is happening mostly near the surface of the fuel – which will roughly be doubled for the internally cooled fuel. The heat up of the fuel will be much smaller because of the better cooling, therefore partially compensating for the larger DC.

(ii) As in (i), the same enrichment and H/HM suggest similar neutron spectrum, therefore similar shape of the reactivity vs burnup curve. The higher resonance absorption in U238 due to large surface area however will reduce the reactivity throughout the cycle. More efficient conversion of U238 into Pu239 on the other hand may at least partially compensate for this reactivity penalty (i.e. by reducing the slope of the reactivity vs burnup curve). Extra cladding will also introduce additional parasitic absorption lowering the achievable burnup.

(iii) The presence of additional cladding and gas gap suggests that the fuel HM loading in the core will be slightly reduced even if H/HM ratio is kept the same. Therefore, even for the same core power density, the core specific power will be higher. As a result, the fuel will arrive at its discharge burnup faster meaning that the fuel cycle length should be reduced.

(c)

Since inner and outer channels share the same upper and lower plena, the flow rate will be distributed such that the inner and outer channel pressure losses are the same. Therefore, assuming friction is the dominant mechanism:

$$(\rho v^2/2\ L/D\ f)_1 = (\rho v^2/2\ L/D\ f)_2 \qquad (1)$$

Where friction factor $f = K\ Re^{-0.2}$

Flow areas as a function of R_1 (inner) and R_2 (outer) radii:

$A_1 = \pi R_1^2$ $\qquad$ $A_2 = P^2 - \pi R_2^2 = (1.1 \times 2R_2)^2 - \pi R_2^2 = 1.698\ R_2^2$

Equivalent channel hydraulic diameters:

$D_1 = 2R_1$ $\qquad$ $D_2 = 4A_2/p_w = (4\times 1.698\ R_2^2)/(2\pi R_2) = 1.081\ R_2$

Neglecting the differences in water density due to potentially un-equal inner and outer heat fluxes.

Expanding the relation (1) and noting that $Re = \rho\ v\ D/\mu$ and the channels have the same length, we obtain:

$$(f\ v^2/D)_1 = (f\ v^2/D)_2$$
$$((vD)^{-0.2}\ v^2/D)_1 = ((vD)^{-0.2}\ v^2/D)_2$$

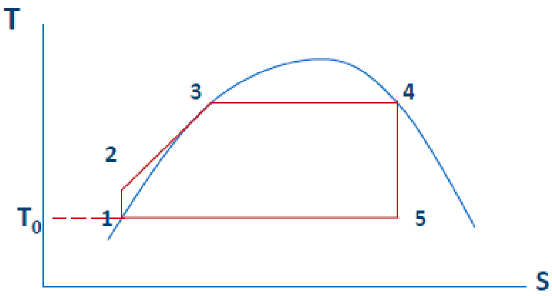
Noting that $v_2 = v_1$ and switching from diameters to radii:

$$(2R_1)^{0.8} = (1.081\ R_2)^{0.8}$$
$$R_1/R_2 = 1.081 / 2 = 0.5405$$

3

(a)

Simple cycle diagram



Relevant state data

At 40 bar: $h_4 = 2800.8$ kJ/kg $s_4 = 6.07$ kJ/kg/K

At 0.1 bar and ideal turbine: $x_5 = (s_5 - s_{f5})/(s_{g5} - s_{f5}) = (s_4 - s_{f5})/(s_{g5} - s_{f5})$

$s_{f5} = 0.649$ kJ/kg/K $s_{g5} = 8.149$ kJ/kg/K

$h_{5f} = 191.8$ kJ/kg $h_{5g} = 2583.9$ kJ/kg

$x_5 = (6.07 - 0.649)/(8.149 - 0.649) = 0.723$

$h_{s5} = h_{5f} + x\, h_{5fg} = 191.8 + 0.723\,(2583.9\text{-}191.8) =$

$= 1920.8$ kJ/kg

For a real turbine: $w_T = (h_4 - h_{5s})\,\eta = (2800.8 - 1920.8)\ 0.8 = 704$ kJ/kg

To deliver 30 MW: $P = \dot{m} w_T$ $\dot{m} = 30{,}000 / 704 = 42.6$ kg/s

(b)

Additional relevant data: $h_2 \approx h_1 + w_p = 191.8 + 45 = 236.8$ kJ/kg

Reactor power (Q_{in}): $Q_{in} = \dot{m}(h_4 - h_2) = 42.6\ (2800.8\ - 236.8) = 109.23$ MW

(c)

The amount of energy needed for 25 years of full power operation at CF = 80%:

$E = 109.2 \times 25 \times 365 \times 0.8 = 797{,}353$ MWd

From the graph, the reactivity limited burnup is 200 MWd/kg, therefore fuel inventory required: $m = E/B = 797{,}353/200 = 3987$ kg

(d)

Trip duration = 60,000 / 40 = 1500 h = 5.4×10^6 s.

Energy (mechanical propeller shaft rotation) needed for the trip

$E = 30 \times 10^6 \times 5.4 \times 10^6 = 1.62 \times 10^{14}$ J

Given 25.7% thermal efficiency, fission energy to be produced by the core:

$E_c = 1.62\times 10^{14} / 0.258 = 6.3 \times 10^{14}$ J

Assume energy released per fission is 200 MeV = $200 \times 10^{6} \times 1.6\times10^{-19} = 3.2\times10^{-11}$ J
Therefore, the required number of fissions = $6.3 \times10^{14} / 3.2\times10^{-11} = 1.97\times10^{25}$ fissions.
U235 is consumed by both fission and capture, thus the number of atoms consumed:
$N = 1.97\times10^{25} \times(40+7)/40 = 2.313\times10^{25}$ atoms
Molecular weight of U235: A = 0.235 kg/mole
Mass of U235 = $2.313\times10^{25} \times A/N_a = 2.313\times10^{25} \times 0.235 /(6.022\times10^{23}) = 9.03$ kg

(e)

Method	Disadvantages
Multi-batch fuel management	- Prohibitively short fuel cycle
Burnable poisons	- Residual reactivity penalty - Displacement of HM and/or coolant - Effects on fuel properties (e.g. thermal conductivity) - Uneven spatial and temporal depletion
Soluble boron	- System complexity, capital and maintenance costs - Dilution accident - Potential for positive MTC - Corrosion of components, complicated water chemistry
Control rods	- Power shape distortion - Inadvertent withdrawal/ejection accident
Geometry (movable fuel/reflectors)	- System complexity - Development costs due to lack of experience
High conversion ratio	- Complex core/fuel design - Alternative (liquid metal, gas) coolants with limited experience

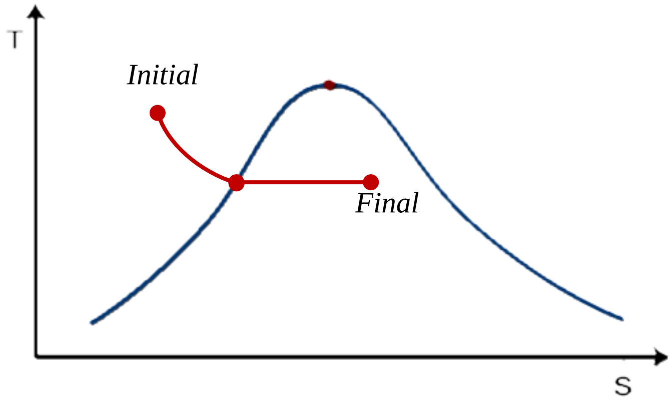
4

(a) Sources of energy inside a PWR containment include the following:
  - Decay heat of fission products and minor actinides
  - Energy stored in the primary coolant
  - Energy stored in the secondary coolant (in case of a steam line break inside the containment)
  - Exothermal chemical reactions – Zr-H2O, Zr-CO2, H2-O2, CO-O2.

(b) Systems and phenomena to remove energy from the containment
  - Containment cooling using forced convection of coolant through dedicated heat exchangers inside and outside the containment.
  - Containment spray – circulating water from the containment sump through external heat exchanger.
  - Residual heat removal system – circulating primary water through an external heat exchanger and re-injecting it back into the core.
  - AP-1000 uses the containment walls as heat exchange surface for condensing steam within the containment, which is cooled externally by water sprinkled from the containment top mounted tank or by natural circulation of air. (Bonus)
  - Means for preventing chemical reactions: hydrogen recombiners/burners, inert (N_2) containment atmosphere (popular in BWRs)

(c) Containment failure modes:
  - Leak due to over-pressure. Energy generated is not removed and results in high steam pressure.
  - External (weather, aircraft crash) or internal (equipment, debris due to H_2 explosion) missiles.
  - Core melt through containment after RPV failure.
  - Direct containment heating by molten core particles if ejected from pressurized failing RPV.
  - Ex-vessel steam explosion.
  - Containment by-pass, e.g multiple steam generator tube rupture.

(d)



(e)

State of primary water before the break:

Enthalpy at 150 bar, 300 C $h_1 = 1338.3$ kJ/kg

Final state: 4 bar, saturated:

$T = 143.61$ C, $h_f = 604.7$ kJ/kg, $h_g = 2133.4$ kJ/kg,
$v_f = 0.001084$ m^3/kg, $v_g = 0.4624$ m^3/kg

Balance of masses:

$V_c = V_f + V_g$ - neglecting the volume of all other structures

$V_c = v_f m_f + v_g m_g$ (1)

$m_f + m_g = m_p + m_{ice}$ - only partial vapour pressure is considered, thus no need to account for non-condensable gases, but neglecting the initial containment air humidity.

(2)

Balance of energies: $E_{Initial\_State} + E_{Decay\_heat} = E_{Final\_State}$

$$m_p h_1 + \int_0^{4h\times 3600s} 0.066 t^{-0.2} \times P_0 \, dt - h_{sf\,ice}\, m_{ice} - C_{ice}\, m_{ice}\, \Delta T_{ice} = h_f m_f + h_g m_g \qquad (3)$$

Solving (1) (2) and (3) for m_f, m_g and m_{ice}

$m_f = 1.1427 \times 10^6$ kg, $m_g = 105\,453$ kg, $m_{ice} = 748\,155$ kg