Question 1.

(a)

Total power generated in the pin cell:

$$q = \pi R_{fo}^2 \int_{-L/2}^{L/2} E_f\, \Sigma_f\, \phi_0 \cos\left(\frac{\pi z}{L}\right) dz = 2R_{fo}^2 L\, E_f\, \Sigma_f\, \phi_0$$

Axial distribution of volumetric fission energy release rate within fuel:

$$q'''(z) = (1-\alpha-\beta) E_f\, \Sigma_f\, \phi_0 \cos\left(\frac{\pi z}{L}\right)$$

Axial linear power in the fuel:

$$q'_{fuel}(z) = \pi R_{fo}^2\, q'''(z) = \pi R_{fo}^2 (1-\alpha-\beta)\, E_f\, \Sigma_f\, \phi_0 \cos\left(\frac{\pi z}{L}\right)$$

Axial linear power in cladding:

$$q'_{clad}(z) = \frac{q\alpha}{L} = 2R_{fo}^2\, E_f\, \Sigma_f\, \phi_0 \alpha$$

Axial linear power in coolant:

$$q'_{coolant}(z) = \frac{q\beta}{L} = 2R_{fo}^2\, E_f\, \Sigma_f\, \phi_0 \beta$$

Energy balance in dz in coolant: $$\left(q'_{fuel} + q'_{clad} + q'_{coolant}\right) dz = \dot{m} c_p dT$$

Substituting and integrating:

$$T_{coolant}(z) = T_{in} + \frac{(1-\alpha-\beta)}{\dot{m} c_p} \pi R_{fo}^2 E_f\, \Sigma_f\, \phi_0 \int_{-L/2}^{z} \cos\left(\frac{\pi z}{L}\right) dz + \frac{1}{\dot{m} c_p} \frac{q(\alpha+\beta)}{L} \int_{-L/2}^{z} dz$$

$$T_{coolant}(z) = T_{in} + \frac{(1-\alpha-\beta)}{\dot{m} c_p} R_{fo}^2 E_f\, \Sigma_f\, \phi_0 L \left(1 + \sin\left(\frac{\pi z}{L}\right)\right) + \frac{1}{\dot{m} c_p} \frac{q(\alpha+\beta)}{L} \left(\frac{L}{2} + z\right)$$

$$T_{coolant}(z) = T_{in} + \frac{(1-\alpha-\beta)}{\dot{m} c_p} R_{fo}^2 E_f\, \Sigma_f\, \phi_0 L \left(1 + \sin\left(\frac{\pi z}{L}\right)\right) + \frac{2R_{fo}^2 E_f\, \Sigma_f\, \phi_0 (\alpha+\beta)}{\dot{m} c_p} \left(\frac{L}{2} + z\right)$$

$$T_{coolant}(z) = T_{in} + \frac{R_{fo}^2 E_f\, \Sigma_f\, \phi_0}{\dot{m} c_p} \left[(1-\alpha-\beta) L \left(1 + \sin\left(\frac{\pi z}{L}\right)\right) + 2(\alpha+\beta) \left(\frac{L}{2} + z\right)\right]$$

(b)

The heat conduction equation in the cladding should include the heat source:

$$\frac{1}{r}\frac{d}{dr}\left(k_c r \frac{dT}{dr}\right) + \ q''' = 0$$

Heat transfer to the coolant is required only for a fraction of the heat. Therefore, heat balance for the film ΔT is needed as a BC at the cladding outer surface:

$$q'' = \frac{q'_{clad} + q'_{fuel}}{2\pi R_{co}} = h_c \Delta T \ = h_c\left(T_{clad}(R_{co}, z) - T_{coolant}(z)\right)$$

$$T_{clad}(R_{co}, z) = T_{coolant}(z) + \frac{q'_{clad} + q'_{fuel}}{2\pi R_{co} h_c}$$

where $T_{coolant}(z)$ is obtained in part (a) of the question.

The second boundary condition is at the pellet-cladding interface, stating that the outgoing heat flux from the fuel pellet is equal to the incoming heat flux into the cladding at $r = R_{fo}$:

$$q'' = -k_c \frac{dT}{dr}$$

$$\frac{q'_{fuel}}{2\pi R_{fo}} = -k_c \frac{dT}{dr}_{r=R_{fo}}$$

(c)

- Axially graded fuel enrichment or fuel density;
- Axially graded burnable poisons;
- Reduced axial coolant ΔT to reduce moderator density variation, for example, through increased flow rate or using coolant with higher heat capacity;
- Neutronically-efficient axial reflectors;
- Reduced reliance on control rods.

Question 2

(a)

Coolant thermal conductivity will affect the heat transfer coefficient and therefore temperature drop across the liquid boundary layer. The coolant flow is typically turbulent, and the heat transfer coefficient can be approximated by Dittus-Boelter correlation:

$$Nu = 0.023\ Re^{0.8} Pr^{0.4}$$

Thermal conductivity appears in both Nu and Pr numbers

$$h = \frac{k}{D} 0.023\ Re^{0.8} \left(\frac{\mu c_p}{k}\right)^{0.4} = A\ k^{0.6}$$

Therefore, the heat transfer coefficient will increase by a factor of $1.2^{0.6}$ = 1.1156 and ΔT across the liquid film will be reduced by the same factor if Coolant 2 is used instead of Coolant 1. New film ΔT would be 20K / 1.1156 = 17.93K, bringing down the temperatures of all core components, including the limiting maximum fuel temperature: T_{CL} = 1000 – (20 – 17.9) = 997.9K. Core power can now be increased to bring the maximum fuel temperature back to 1000K. Since the core power distribution is uniform, the peak fuel temperature is expected at the core exit (where the coolant temperature is the highest, 550K). ΔT across the fuel pin is proportional to fuel linear power rating. If the fuel thermal conductivity is assumed constant, the ratio of new to the original q' will be:

$\frac{q_2'}{q_1'} = \frac{1000-(550+17.9)}{1000-(550+20)} \approx 1.005$ or about 0.5% increase.

(b)

For Coolant 3, the same considerations can be applied, but c_p now appears in both heat transfer coefficient expressions and would affect the coolant axial temperature rise across the channel. For the heat transfer coefficient effect, as in (a):

$$h = \frac{k}{D} 0.023\ Re^{0.8} \left(\frac{\mu c_p}{k}\right)^{0.4} = B\ {c_p}^{0.4}$$

The factor by which film ΔT is reduced: $1.2^{0.6}$ = 1.076, which translates to a reduction of 20 – 18.6 = 1.4K.

From the calculation of the coolant axial temperature rise across the core, for the same inlet temperature (fixed at 500K), power and coolant mass flow rate, the core outlet temperature would be lower:

$(T_{out} - T_{in})\dot{m}c_p = Q$ $\qquad T_{out} = T_{in} + \frac{Q}{\dot{m}c_p}$, or new $T_{out} = 500 + \frac{550-500}{1.2} = 541.7\text{K}$

In other words, this effect leads to a more substantial reduction in T_{out} of 8.3K.

Cumulatively, (1.4 + 8.3) = 9.7K of temperature difference can be recaptured for the purpose of power uprate: $\frac{q_2'}{q_1'} = \frac{1000-(500+41.7+18.6)}{1000-(500+50+20)} \approx 1.0226$ or about 2.3% increase in power.

(c)

Power conversion efficiency would only be affected by the average temperature of heat addition. Instead of using the temperature differences available as a result of changes in coolant properties for the core power uprate, these temperature differences could have been used to increase the core average coolant temperature, which would also be the temperature of heat addition to the power conversion cycle.

For Coolant 2, about 2K would be available for increasing the coolant temperature uniformly across the core and therefore also on average.

For Coolant 3, the same can be said about the heat transfer coefficient effect (1.4K of available ΔT), i.e. it is gained uniformly across the whole core and thus also on average. However, although 8.3K coolant ΔT can be gained at the core outlet, only about half of that ΔT can be gained on average. Still, Coolant 3 would be the more preferable option.

Question 3.

(a)

Higher flow rate means a smaller temperature rise across the core. Noting that the coolant outlet temperature is to remain the same, this means the coolant inlet temperature will be higher. This has several positive effects.

- Higher average temperature of heat addition to the power cycle which should increase the thermodynamic efficiency of power conversion.
- Alternatively, the heat exchanger could be smaller – i.e. the extra mean temperature difference (MTD) gained across the heat exchanger can be "spent" on reducing the heat transfer area if that was an option. In a PWR steam generator, for example, a greater number of leaking tubes could be afforded to be plugged before replacement of the entire component would be required.
- The MTD across the heat exchanger would increase also because the heat transfer coefficient, being a strong function of flow rate ($h \sim Re^{0.8}$), will increase, reducing the film ΔT and overall thermal resistance across the heat exchanger wall.
- A more axially symmetric power distribution and larger operational margin on axial flux difference.
- For the fixed inlet subcooling, the CHF would generally increase with higher flow rate. However, T-inlet in this case will also increase with the opposite effect on CHF. Without specific properties of the coolant and the proposed extent of flow rate increase, it is hard to say conclusively whether the overall effect will be positive (higher CHF margin and simpler safety case), neutral or negative (requiring other safety provisions to compensate for the lost safety margin).

(b)

- Higher flow rate would lead to an increase in frictional pressure losses across the circuit and therefore require higher pumping power, i.e. a bigger (more expensive) pump plus higher operating costs of the pump.
- The temperature rise across the core will be reduced, effectively increasing the core average coolant temperature and reducing its density. Since the coolant temperature coefficient of reactivity should be negative to assure the core stability, such an increase will result in a negative reactivity insertion which will have to be compensated by either more frequent refuelling or higher enrichment.
- Since the average coolant temperature will rise, the core average fuel temperature should increase as well (even if only slightly). Therefore, additional negative reactivity from the fuel Doppler effect will have to be compensated as above.
- The above effect will be partially offset by the improved heat transfer coefficient which will reduce the temperature difference between the fuel and the coolant.
- Higher flow velocity will result in higher vibrations due to increased turbulence and therefore may require mechanical redesign of fuel assemblies – e.g. increasing the number of spacer grids, which will introduce additional pressure losses and make the fuel more expensive, increase neutron absorption and reduce local moderation, which will have to be compensated by higher enrichment. Similarly, vibrations issue will need to be assessed and, if problematic, mitigated for all the piping across the circuit, including steam generator/heat exchanger tubes.

- Lower moderator density would mean a harder neutron spectrum and reduced reactivity worth of the core reactivity control materials, such as control rods, soluble boron and burnable poisons, requiring a larger number of control rods or higher poison loading.
- Increasing the flow rate would mean a change in operating conditions of the reactor and require modifications to the reactor licence/safety case, which may be costly and time consuming.

(c)

- Reduce the pressure drop of the core. This might be done by increasing the flow area, e.g. increasing the fuel pins lattice pitch.
- This will also increase the H/HM ratio and compensate for reactivity loss due to an increase in coolant temperature and a decrease in coolant density.
- The increase in the lattice pitch, however, will increase the core volume and reduce the power density.
- Increasing the flow area can be also accomplished by reducing the fuel pin diameter, which will also increase the H/HM with its associated benefits. This option, however, will reduce the total amount of HM in the core (if the core volume is fixed) leading to shorter fuel cycle length.
- A smaller pin diameter will also lead to higher heat flux (for a fixed power density) and therefore potentially reduce the MDNBR.
- Reducing the pin diameter will also increase the surface to volume ratio of the fuel (because volume reduces faster than surface area with shrinking diameter), leading to slightly increased resonance absorption and reducing the magnitude of the improved moderation effect.

  Other ways of reducing pressure drop:

- Change the core aspect ratio, i.e. reduce the height with larger number of shorter fuel assemblies. This might challenge the ability to manufacture large diameter pressure vessels.
- Reduce pressure drop due to shock losses at the entry to fuel assemblies (optimise debris filters) or use fewer spacer grids. These will make the fuel more prone to debris induced failures and challenge the mechanical strength of the assembly.
- Reduce pressure losses of other components, e.g. pipes and steam generator tubes. This may lead to higher capital cost of these components.

Question 4

(a)

Need to establish reactivity of boron-free core at B=0. From points 2 and 3 in the table, one can obtain the slope and intersection with the axes of the boron-free linear reactivity curve:

$\rho = 15000 - 300B$, where B is in MWd/kg and ρ is in pcm.

The initial reactivity of boron-free core is thus 15000 pcm and boron reactivity worth:

$BW = \frac{\partial \rho}{\partial BC} \approx \frac{15000-0}{0-1800} = -8.3\, pcm/ppm$

(b)

Knowing the shape parameters of the boron-free, linear reactivity curve for a single batch, it can be combined to construct the reactivity curve of a 4-batch core.

Single-batch core burnup is B_1 = 15000/300 = 50 MWd/kg, which follows directly from the shape of the reactivity curve. The discharge burnup is related to B_1 as:

$$B_n = B_1 \frac{2n}{n+1} = 50 \frac{2 \times 4}{4+1} = 80 \text{ MWd/kg}$$

Naturally, the cycle burnup is B discharge divided by the number of batches (or cycles of residence):

$$B_c = \frac{B_n}{n} = \frac{80}{4} = 20 \text{ MWd/kg}$$

(c)

Core reactivity is estimated as the arithmetic average of the constituent batch reactivities at BOC:

$$\rho_{core} = \frac{1}{n} \sum_{i=1}^{n} \rho_i$$

At the beginning of cycle, each of the 4 batches will have the following reactivity:

Fresh fuel: $\rho_1 = 15000 - 300 \times 0$

Once-burnt fuel: $\rho_2 = 15000 - 300 \times B_c$

Twice-burnt fuel: $\rho_3 = 15000 - 300 \times 2B_c$

Thrice-burnt fuel: $\rho_4 = 15000 - 300 \times 3B_c$

$$\rho_{core} = \frac{1}{n} \sum_{i=1}^{n} \rho_i = \frac{1}{4}(15000 + 9000 + 3000 - 3000) = 6000 \text{ pcm}$$

Assuming BW is independent of fuel burnup (which is technically not true), the initial CBC:

$$CBC = \frac{\rho_{core}}{BW} = \frac{6000 \text{ pcm}}{8.3 \text{ pcm/ppm}} = 720 \text{ ppm}$$

(d)

The assumption of constant BW is unrealistic.

BW depends both on boron concentration and on fuel burnup.

Since boron is predominantly a thermal absorber, it neutronically competes for neutron absorption with other thermal absorbers (e.g. fissile U235) and with itself. Therefore, at the beginning of fuel life, when the fuel reactivity is the highest and so are the boron concentration and concentration of U235, BW should be expected to be the lowest (in magnitude).

With the depletion of fissile material and dilution of boron itself, BW should become progressively more negative.

Due to the above considerations, the BW estimated from the measurements presented in the table is reasonable because CBC of a fresh, single-batch core should account for the boron self-shielding effect, while burnup effects have not manifested themselves yet.

In part (b), only the boron-free core reactivities were manipulated. Therefore, limitations in calculating BW are irrelevant.

In part (c), the BW of fresh fuel measured at high boron concentration was used for estimating CBC of a partially burned core with lower boron content. This suggests that the actual BW would be more negative and less boron would be required in practice to achieve the critical condition.