Version ES/5

EGT3
ENGINEERING TRIPOS PART IIB

Wednesday 27 April 2022   2 to 3.40

Module 4I10

NUCLEAR REACTOR ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Nuclear Energy Data Book (21 pages)
Engineering Data Books

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 The axial distribution of neutron flux in a cylindrical PWR fuel pin with pellet radius R_{fo} is given by $\phi(z) = \phi_0 \cos(\pi z/L)$ where L is the active core height. The fuel macroscopic fission cross section is Σ_{f} and the energy released per fission is E_{f}. Most of the released energy is deposited in the fuel at the fission site. However, a fraction, α, of the total fission energy is deposited directly and axially uniformly in the cladding, while a fraction, β, is deposited directly and axially uniformly in the water coolant. The fuel cladding outer radius is R_{co} and all other symbols in what follows have their usual meaning.

(a) Show that the axial distribution of bulk coolant temperature is given by

$$T_{\text{coolant}}(z) = T_{\text{in}} + \frac{R_{\text{fo}}^2 E_{\text{f}} \Sigma_{\text{f}} \phi_0}{\dot{m} c_p} \left[(1 - \alpha - \beta) L \left(1 + \sin \frac{\pi z}{L} \right) + 2(\alpha + \beta) \left(\frac{L}{2} + z \right) \right]$$

[50%]

(b) State the equations along with the boundary conditions necessary to derive the radial temperature distribution within the cladding, $T_{\text{clad}}(r, z)$, at axial elevation z. Explain the physical meaning of each equation. Assume that the cladding thermal conductivity is constant and that thermal contact between the pellet and the cladding is perfect (i.e. there is no temperature drop across the gap). [30%]

(c) Suggest fuel design options for reducing axial power peaking. [20%]

2 Reactor designers are considering three alternative coolants with identical properties except that Coolant 2 has 20% higher thermal conductivity than Coolant 1, while Coolant 3 has 20% higher specific heat capacity than Coolant 1 (as shown in Table 1 below). The core power distribution can be assumed to be uniform. The core power is also known to be limited by the maximum fuel temperature of 1000K. The core operating conditions are such that boiling of the coolant is not expected. The coolant inlet temperature is fixed at 500K. The coolant outlet temperature is 550K and the temperature drop across the coolant laminar boundary layer is 20K when Coolant 1 is used.

In answering the following questions, clearly state any assumptions you are making.

(a) Estimate by what factor the core power can be increased if Coolant 2 is used instead of Coolant 1. [35%]

(b) Estimate by what factor the core power can be increased if Coolant 3 is used instead of Coolant 1. [35%]

(c) Discuss whether Coolant 2 or Coolant 3 would offer greater benefit with regards to improving the thermodynamic efficiency of the power conversion cycle. [30%]

Table 1

	Coolant 1	Coolant 2	Coolant 3
Thermal conductivity	k	$1.2k$	k
Specific heat capacity	c_p	c_p	$1.2c_p$
Dynamic viscosity	μ	μ	μ
Density	ρ	ρ	ρ

3 Heat from a reactor core is removed by a liquid coolant which also serves as a moderator. The heat is transported by forced circulation to a heat exchanger with a power conversion cycle, as shown in Fig. 1 below. It is proposed to increase the mass flow rate of the coolant in the primary circuit. The thermal power generated by the core, the geometry of all the circuit components and the core outlet coolant temperature are to remain the same.

(a) List and briefly explain all the positive effects that this change would have on the economics of the power plant. [35%]

(b) List and briefly explain all the negative effects that this change would have on the economics of the power plant. [35%]

(c) Propose changes in the core design which can help mitigate the negative effects that you have listed and identify the main drawbacks of these changes. [30%]

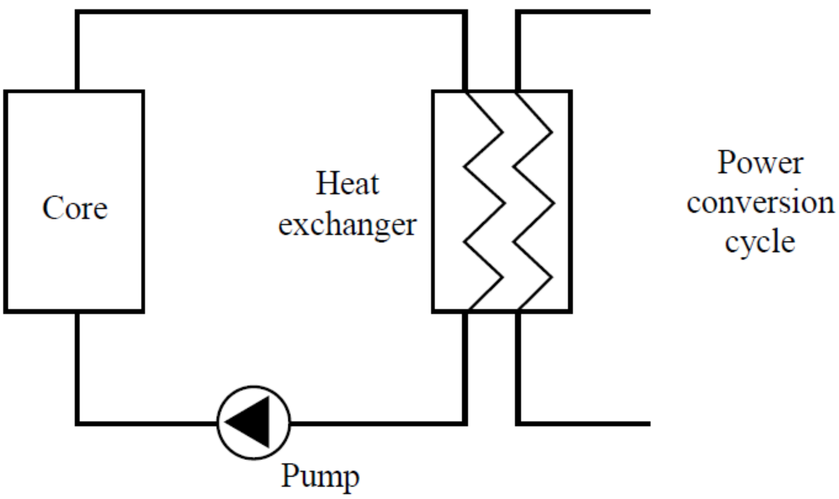

Fig. 1

4 Table 2 below lists three reactivity measurements of a single-batch core at different operating conditions. In answering the following questions, assume that soluble boron reactivity worth is independent of boron concentration and burnup, all Linear Reactivity Model assumptions are valid, and the effects of neutron leakage can be neglected.

(a) Estimate the soluble boron reactivity worth. [25%]

(b) If the same fuel is used in a 4-batch core, estimate the equilibrium cycle burnup and discharge fuel burnup. [25%]

(c) If the same fuel is used in a 4-batch core, estimate the core critical boron concentration at the beginning of an equilibrium cycle. [25%]

(d) Discuss the validity of the assumptions of boron reactivity worth being independent of boron concentration and burnup. How would your answers to (a), (b) and (c) change if more realistic boron reactivity worth dependence on boron concentration and burnup were assumed? [25%]

Table 2

Reactivity (pcm)	Burnup (MWd/kg)	Boron Concentration (ppm)
0	0	1800
12000	10	0
6000	30	0

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Nuclear Energy Data Book
Reactor Physics and Reactor Engineering

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Contents

1. NE1: Reactor Physics

General Data

Speed of light in vacuum	c	$299.792458 \times 10^{6}\ \mathrm{m\,s^{-1}}$
Avogadro's number	L	$6.022141 \times 10^{23}\ \mathrm{mol^{-1}}$
Magnetic permeability in vacuum	μ_0	$4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Planck constant	h	$6.626176 \times 10^{-32}\ \mathrm{J\,s}$
Boltzmann constant	k	$1.380662 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Elementary charge	e	$1.6021892 \times 10^{-19}\ \mathrm{C}$

Definitions

Unified atomic mass constant	u	$1.6605655 \times 10^{-27}\ \mathrm{kg}$ $(931.5016\ \mathrm{MeV})$
Electron volt	eV	$1.6021892 \times 10^{-19}\ \mathrm{J}$

Curie Ci $3.7 \times 10^{10}\ \mathrm{Bq}$

Barn b $10^{-28}\ \mathrm{m^2}$

Nomenclature

Symbols

- ϕ Flux distribution (in space)
- φ Flux distribution (in energy)
- σ Microscopic cross-section
- Σ Macroscopic cross-section
- e Enrichment
- D Diffusion coefficient
- N Number of atoms per unit volume
- V Volume

Superscripts

- F Fuel
- M Moderator

Subscripts

- a Absorption
- c Capture
- C Coolant
- f Fission
- fe Fertile
- s Scattering
- F Fuel
- F Fast (energy range)
- I Intermediate/resonance (energy range)
- M Moderator
- T Thermal (energy range)

Densities and Mean Atomic Weights

	"Nuclear" graphite	Aluminium Al	Cadmium Cd	Gold Au	Uranium U
Density / $\mathrm{kg\,m^{-3}}$	1600	2700	8600	19000	18900
Atomic weight	12	27	112.4	196	238

Thermal Neutron Cross-sections (in barns)

	"Nuclear" graphite	${}^{16}_{8}O$	${}^{113}_{48}Cd$	${}^{235}_{92}U$	${}^{238}_{92}U$	${}^{1}_{1}H$ unbound
Fission	0	0	0	580	0	0
Capture	4×10^{-3}	10^{-4}	27×10^{3}	107	2.75	0.332
Elastic scatter	4.7	4.2		10	8.3	38

To a first approximation the energy spectrum within the thermal range can be assumed to be proportional to the Maxwell-Boltzmann distribution:

$$\varphi(E) \propto \varphi_M(E) = \frac{1}{(kT)^2} E \exp(-E/kT)$$

where T = the temperature (in K)

Fission Energy

Kinetic energy of fission fragments	167 ± 5 MeV
Prompt γ-rays	6 ± 1 MeV
Kinetic energy of neutrons	5 MeV
Decay of fission products β	8 ± 1.5 MeV
γ	6 ± 1 MeV
Neutrinos (not recoverable)	12 ± 2.5 MeV
Total energy per fission	204 ± 7 MeV

Subtract neutrino energy and add neutron capture energy ⇒ ~200 MeV / fission

Fission Product Build-up and Decay

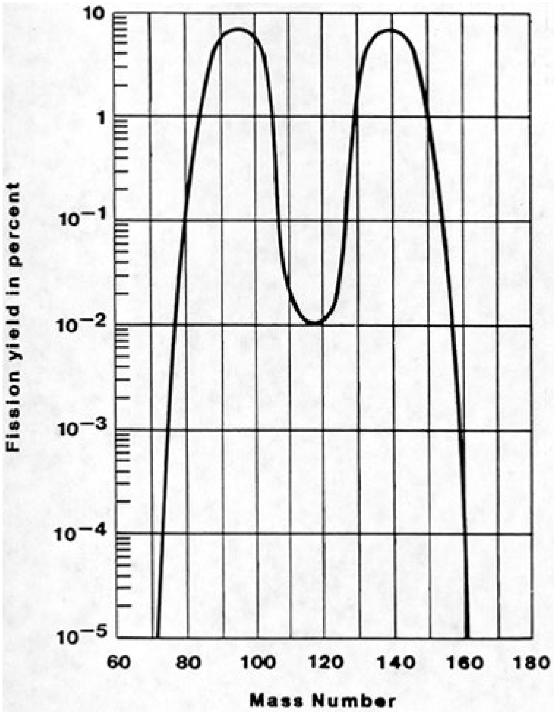
Using a lumped model, the concentration of a fission product will vary over time according to:

$$\frac{dN}{dt} = \gamma_{\mathrm{FP}} \Sigma_{\mathrm{f}} \phi - \lambda N - \sigma_{\mathrm{a}} N \phi$$

where N = the concentration of the product in question
γ_{FP} = the *fission yield* (the proportion of reactions producing the product in question)
λ = the isotope's decay constant
σ_{a} = the isotope's microscopic absorption cross-section

Fission Product Yield

Nuclei with mass numbers from 72 to 158 have been identified, but the most probable split is unsymmetrical, into a nucleus with a mass number of about 138 and a second nucleus that has a mass number between about 95 and 99, depending on the target.



The primary fission products decay by β^- emission. Some important decay chains (with relevant half-lives) from thermal-neutron fission of U-235 are:

Kr-90 $\rightarrow$ Rb-90 $\rightarrow$ Sr-90 $\rightarrow$ Y-90 $\rightarrow$ Zr-90

33 s 2.7 min 28 yr 64 hr

Sr-90 is a serious health hazard, because it is bone-seeking.

Sb-131 $\rightarrow$ Te-131 $\rightarrow$ I-131 $\rightarrow$ Xe-131

23 min 24 min 8 days

I-131 is a short-lived health hazard. It is thyroid-seeking.

I-135 $\rightarrow$ Xe-135 $\rightarrow$ Cs-135 (nearly stable) Fission yield $\gamma = 0.064$

6.7 hr 9.2 hr

Xe-135 is a strong absorber of thermal neutrons, with $\sigma_a = 2.65$ Mb.

Nd-149 $\rightarrow$ Pm-149 $\rightarrow$ Sm-149 Fission yield $\gamma = 0.011$

1.7 hr 53 hr

Sm-149 is a strong absorber of thermal neutrons, with $\sigma_a = 41$ kb.

Se-87 $\rightarrow$ Br-87 $\rightarrow$ either {Kr-86 + n} or {Kr-87 $\rightarrow$ Rb-87}

17 s 55 s 78 min

This chain leads to a “delayed neutron”.

Neutrons

Most neutrons are emitted within 10^{-13} s of fission, but some are only emitted when certain fission products, e.g. Br-87, decay.

The total yield of neutrons depends on the target and on the energy of the incident neutron. Some key values are:

Target nucleus	Fission induced by			
	Thermal neutron		Fast neutron	
	$\bar{\nu}$	η	$\bar{\nu}$	η
U-233	2.50	2.29	2.70	2.45
U-235	2.43	2.07	2.65	2.30
U-238	—	—	2.55	2.25
Pu-239	2.89	2.08	3.00	2.70

ν = average number of neutrons emitted per fission

η = average number of neutrons emitted per neutron absorbed

The Four Factor Formula

For a thermal reactor the *infinite multiplication factor* k_∞ is given by

$$k_\infty = \varepsilon\, p\, f\, \eta_T$$

where ε = the fast fission factor

p = the resonance escape probability

f = the thermal utilisation

η_T = the number of fission neutrons produced per thermal neutron absorbed in the fuel

$$p = 1 - \frac{V_\mathrm{F} N_\mathrm{fe}}{V_\mathrm{M} \xi^\mathrm{M} \Sigma_\mathrm{s}^\mathrm{M}} I$$

where I = the *resonance integral* defined by

$$I = \int_I \frac{\sigma_\mathrm{a}^\mathrm{fe}(E)\, \varphi_\mathrm{F}(E)}{E\, \varphi_\mathrm{M}(E)} dE$$

$$f = \frac{1}{1 + \varsigma\left(V_\mathrm{M} \bar{\Sigma}_{\mathrm{a}T}^\mathrm{M} \big/ V_\mathrm{F} \bar{\Sigma}_{\mathrm{a}T}^\mathrm{F}\right)}$$

where ς = the *thermal disadvantage factor* defined by

$$\varsigma = \frac{\bar{\varphi}_{\mathrm{M}T}}{\bar{\varphi}_{\mathrm{F}T}}$$

Reactivity Coefficients

Fuel Temperature Coefficient $\alpha_{\mathrm{F}} = \frac{1}{k_\infty}\frac{\partial k_\infty}{\partial \overline{T}_{\mathrm{F}}}$

α_{F} can be approximated as $\alpha_{\mathrm{F}} = \frac{1}{p}\frac{\partial p}{\partial \overline{T}_{\mathrm{F}}} = -\frac{1}{p}\frac{V_{\mathrm{F}} N_{\mathrm{fe}}}{V_{\mathrm{M}}\xi^{\mathrm{M}}\Sigma_{\mathrm{s}}^{\mathrm{M}}}\frac{\partial I}{\partial \overline{T}_{\mathrm{F}}}$

α_{F} is also known as the *prompt coefficient*.

Moderator Temperature Coefficient $\alpha_{\mathrm{M}} = \frac{1}{k_\infty}\frac{\partial k_\infty}{\partial \overline{T}_{\mathrm{M}}}$

To a good approximation $\alpha_{\mathrm{M}} = -\beta_{\mathrm{M}}\left[\left(\frac{1}{p}-1\right)-(1-f)\right]$

where β_{M} is the volumetric coefficient of thermal expansion at constant pressure given by

$$\beta_{\mathrm{M}} = -\frac{1}{N_{\mathrm{M}}}\frac{\partial N_{\mathrm{M}}}{\partial \overline{T}_{\mathrm{M}}}$$

Thermal Transients

An approximate lumped parameter model for thermal transients is

$$M_{\mathrm{F}}c_{\mathrm{F}}\frac{d\overline{T}_{\mathrm{F}}}{dt} = P - \frac{1}{R_{\mathrm{F}}}\left[\overline{T}_{\mathrm{F}} - \overline{T}_{\mathrm{C}}\right]$$

$$M_{\mathrm{C}}c_{\mathrm{C}}\frac{d\overline{T}_{\mathrm{C}}}{dt} = \frac{1}{R_{\mathrm{F}}}\left[\overline{T}_{\mathrm{F}} - \overline{T}_{\mathrm{C}}\right] - 2W_{\mathrm{C}}c_{\mathrm{C}}\left[\overline{T}_{\mathrm{C}} - T_i\right]$$

where M_{F} = the total fuel mass
M_{C} = the mass of coolant within the core
c_{F} = the fuel specific heat capacity
c_{C} = the coolant specific heat capacity
$\overline{T}_{\mathrm{F}}$ = the volume-average fuel temperature
$\overline{T}_{\mathrm{C}}$ = the volume-average coolant temperature
T_i = the coolant inlet temperature
P = the total power generated
R_{F} = the overall thermal resistance
W_{C} = the total coolant mass flow rate through the core

Composite Reactivity Coefficients

Isothermal Temperature Coefficient $\alpha_T \equiv \frac{d\rho_{\mathrm{fb}}}{d\overline{T}}$

$$\alpha_T = \alpha_{\mathrm{F}} + \alpha_{\mathrm{C}}$$

where α_{C} is the *coolant temperature coefficient*

Power Coefficient $\alpha_P \equiv \frac{d\rho_{\mathrm{fb}}}{dP}$

$$\alpha_P = R_{\mathrm{F}}\alpha_{\mathrm{F}} + \frac{1}{2W_{\mathrm{C}}c_{\mathrm{C}}}(\alpha_{\mathrm{F}} + \alpha_{\mathrm{C}})$$

Nuclear Reactor Kinetics

Name	*Symbol*	*Concept*
Effective multiplication factor	k_{eff}	$\frac{\text{production}}{\text{removal}} = \frac{P}{R}$
Excess multiplication factor	k_{ex}	$\frac{P-R}{R} = k_{eff} - 1$
Reactivity	ρ	$\frac{P-R}{P} = \frac{k_{ex}}{k_{eff}}$
Lifetime	l	$\frac{1}{R}$
Reproduction time	Λ	$\frac{1}{P}$

Lumped Reactor Kinetics Equations

$$\frac{dn}{dt} = S + \frac{\rho-\beta}{\Lambda} n + \sum_{i=1}^{6} \lambda_i c_i$$

$$\frac{dc_i}{dt} = \frac{\beta_i}{\Lambda} n - \lambda_i c_i$$

where n = neutron population

c_i = precursor population in group i

β = delayed neutron precursor fraction $= \sum_{i=1}^{6} \beta_i$

λ_i = decay constant for precursors in group i

S = independent source rate

Delayed Neutrons

A reasonable approximation for thermal-neutron fission of U-235 is:

Precursor half-life, s	55	22	5.6	2.1	0.45	0.15	Total
Mean life time of precursor ($1/\lambda_i$), s	80	32	8.0	3.1	0.65	0.22	
Number of neutrons produced per 100 fission neutrons ($100\,\beta_i$)	0.03	0.18	0.22	0.23	0.07	0.02	0.75

Neutron Diffusion Equation

$$\frac{dn}{dt} = -\nabla \underline{j} + (\eta - 1)\Sigma_a \phi + S$$

where $\underline{j} = -D\nabla\phi$ (Fick's Law)

$$D = \frac{1}{3\Sigma_s(1-\bar{\mu})}$$

with $\bar{\mu}$ = the mean cosine of the angle of scattering

Laplacian ∇^2

Slab geometry: $$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

Cylindrical geometry: $$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}+\frac{\partial^2}{\partial z^2}$$

Spherical geometry: $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial \psi^2}$$

Bessel's Equation of 0th Order

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+R=0$$

Solution is: $$R(r)=A_1J_0(r)+A_2Y_0(r)$$

$J_0(0)=1;\ Y_0(0)=-\infty;$

The first zero of $J_0(r)$ is at $r=2.405$

$J_1(2.405)=0.5183$, where $J_1(r)=\frac{1}{r}\int_0^r xJ_0(x)dx$.

Diffusion and Slowing Down Properties of Moderators

Moderator	Density, $\mathrm{g\,cm^{-3}}$	Σ_a, $\mathrm{cm^{-1}}$	D, cm	$L^2=D/\Sigma_a$, $\mathrm{cm^2}$
Water	1.00	22×10^{-3}	0.17	$(2.76)^2$
Heavy Water	1.10	85×10^{-6}	0.85	$(100)^2$
Graphite	1.70	320×10^{-6}	0.94	$(54)^2$

The *slowing down decrement* ξ is the most widely used measure of an isotope's ability to slow neutrons down by elastic scattering:

$$\xi=1+\frac{\alpha}{1-\alpha}\ln\alpha$$

where $$\alpha=\left[\frac{A-1}{A+1}\right]^2$$

with $A=M/m$ where M is the mass of the nucleus and m the mass of the neutron.
The *slowing down power* of a moderator is then $\xi\Sigma_s$ and the *slowing down ratio* is $\xi\Sigma_s/\Sigma_c$.

Reactivity Worth

$$\rho=-\frac{\int\phi\delta\Sigma_a\phi' dV}{\int\phi\nu\Sigma_f\phi' dV}$$

where ϕ = flux distribution prior to control poison insertion

ϕ' = flux distribution after control poison insertion

$\delta\Sigma_a$ = change in Σ_a due to control poison insertion

Health Physics: γ-ray Dose Rate

$$D = \frac{1.6 \times 10^{-13} A \Sigma E_\gamma t}{4\pi \rho R^2}$$

where D = the dose (in Gy)
A = the activity (in Bq)
Σ = the absorption cross-section for γ-rays in human tissue
E_γ = the (total) γ energy (in MeV)
t = the exposure time (in s)
ρ = the density of tissue
R = the distance from the source (in m)

Atomic Masses and Naturally Occurring Isotopic Abundances (%)

	electron	0.00055 u	90.80%	$^{20}_{10}Ne$	19.99244 u
	neutron	1.00867 u	0.26%	$^{21}_{10}Ne$	20.99385 u
99.985%	$^{1}_{1}H$	1.00783 u	8.94%	$^{22}_{10}Ne$	21.99138 u
0.015%	$^{2}_{1}H$	2.01410 u	10.1%	$^{25}_{12}Mg$	24.98584 u
0%	$^{3}_{1}H$	3.01605 u	11.1%	$^{26}_{12}Mg$	25.98259 u
0.0001%	$^{3}_{2}He$	3.01603 u	0%	$^{32}_{15}P$	31.97391 u
99.9999%	$^{4}_{2}He$	4.00260 u	96.0%	$^{32}_{16}S$	31.97207 u
7.5%	$^{6}_{3}Li$	6.01513 u	0%	$^{60}_{27}Co$	59.93382 u
92.5%	$^{7}_{3}Li$	7.01601 u	26.2%	$^{60}_{28}Ni$	59.93079 u
0%	$^{8}_{4}Be$	8.00531 u	0%	$^{87}_{35}Br$	86.92196 u
100%	$^{9}_{4}Be$	9.01219 u	0%	$^{86}_{36}Kr$	85.91062 u
18.7%	$^{10}_{5}B$	10.01294 u	17.5%	$^{87}_{36}Kr$	86.91337 u
0%	$^{11}_{6}C$	11.01143 u	12.3%	$^{113}_{48}Cd$	112.90461 u
98.89%	$^{12}_{6}C$	12.00000 u		$^{226}_{88}Ra$	226.02536 u
1.11%	$^{13}_{6}C$	13.00335 u		$^{230}_{90}Th$	230.03308 u
0%	$^{13}_{7}N$	13.00574 u	0.72%	$^{235}_{92}U$	235.04393 u
99.63%	$^{14}_{7}N$	14.00307 u	0%	$^{236}_{92}U$	236.04573 u
0%	$^{14}_{8}O$	14.00860 u	99.28%	$^{238}_{92}U$	238.05076 u
99.76%	$^{16}_{8}O$	15.99491 u	0%	$^{239}_{92}U$	239.05432 u
0.04%	$^{17}_{8}O$	16.99913 u		$^{239}_{93}Np$	239.05294 u
0.20%	$^{18}_{8}O$	17.99916 u		$^{239}_{94}Pu$	239.05216 u
				$^{240}_{94}Pu$	240.05397 u

2. Reactor Engineering

2.1 Economics

Life time Levelised Electricity Generating Cost (EGC) = $\dfrac{\sum\left[(I_t + M_t + F_t + W_t)\times(1+r)^{-t}\right]}{\sum\left[E_t \times (1+r)^{-t}\right]}$

Where:

I_t = Investment in year t
M_t = Operations & Maintenance in year t
F_t = Fuel costs in year t
W_t = Waste & Decommissioning cost in year t
E_t = Energy generated in year t
r = Discount rate (Real i.e. net of inflation)

2.2 Power Cycles

Ideal steam power cycle

Carnot cycle efficiency $\eta = \Delta T/T$

Where T is the heat source temperature and ΔT the temperature difference over which the cycle operates.

Ideal gas power cycle

Efficiency $\eta = 1 - [1/R_p]^{(\gamma - 1/\gamma)}$

Where R_p is the pressure ratio and γ is the ratio of the specific heats of the gas.

2.3 Heat Transfer

Coolants

The key fluid parameters for some useful reactor coolants:

Coolant	Pressure	Temp.	Density	Dyn. Visc.	Sp. Ht.	Cond.
	MPa	Deg C	kg/m^3	kg/s/m ×10^{-3}	kJ/kg/K	kW/m/K
Light Water	8.8	300	714	0.0897	5.76	0.000541
Carbon Dioxide	4	450	29.5	0.030	1.2	0.00007
Helium	4	450	3.1	0.036	5.2	0.000028
Sodium	0.1	550	817	0.23	1.26	0.061
Lead	0.1	500	10400	1.7	0.145	0.018
Bismuth	0.1	500	9700	1.2	0.135	0.015
LBE	0.1	500	10100	1.2	0.142	0.015
Sat Steam	22	360	144.1	0. 0275	25.1	0.000168

Coolant heat transport Figure of Merit $FoM = (C_p^{2.8} \rho^2)/ \mu^{0.2}$

Coolant	Pressure	Temp	FoM	Rel Water	Pr	Rel Pr
	MPa	Deg C				
Light Water	8.8	300	442963961	100	0.96	100
Carbon Dioxide	4	450	11639	0.0026	0.51	53.9
Helium	4	450	11920	0.0027	0.67	70.0
Sodium	0.1	550	6809827	1.54	0.005	0.5
Lead	0.1	500	1737030	0.39	0.01	1.4
Bismuth	0.1	500	1326300	0.30	0.01	1.1
LBE	0.1	500	1656586	0.37	0.01	1.2
Sat Steam	22	360	1407670277	317.8	4.11	430.2

Fuel channel & element temperatures

Fourier's Law for heat conduction:

$$\overline{q''} = -k\overline{\nabla}T$$

Heat conduction (Fourier-Biot) equation:

$$\nabla(k\overline{\nabla}T) + q''' = \rho C_p \frac{\partial T}{\partial t}$$

Plain geometry:

Temperature difference across fuel (k_F) with half thickness a, & q''' volumetric rating

$$T_{Max} - T_{FO} = q''' a^2 / 2k_F$$

Cladding of thickness: t, & conductivity k_C

$$T_{Ci} - T_{CO} = q''' a\, t / k_C$$

Cylindrical geometry

For solid fuel pellets radius R_{FO} the peak fuel temperature is function of linear rating

$$\int_T^{T_{Max}} k_f(T)dT = \frac{q'}{4\pi}$$ q' linear rating and q''' volumetric rating

or for constant fuel conductivity k_f

Temperature difference across fuel $T_{Max} - T_{FO} = q''' R_{FO}^2 / 4k_f$

Also $T_{Max} - T_{FO} = q' / (4\pi\, k_f)$

Cladding - Temperature difference across clad

$$T_{Ci} - T_{CO} = q' / (2\pi\, k_C)[\ln(R_{CO} / R_{Ci})]$$

Spherical geometry

For solid fuel:

Temperature difference across fuel $T_{Max} - T_{FO} = q'''R_{FO}^{2} / (6k_f)$

Cladding of fuelled region inner radius R_1, outer radius R_2 and generating total heat Q.

$$T_{Ci} - T_{CO} = Q[R_2 - R_1]/[4\pi k_C R_1 R_2]$$

For annular, cylindrical fuel elements and chopped cosine flux profile, the maximum fuel temperature at position z along the channel:

$$T_F(z) = T_{in} + \frac{q'_0}{\rho A V C_p}\frac{L_e}{\pi}\left[\sin\frac{\pi z}{L_e} + \sin\frac{\pi L}{2L_e}\right] + \left[\frac{q'_0}{2\pi R_{Co} h_T} + \frac{q'_0}{2\pi k_C}\ln\left(\frac{R_{Co}}{R_{Ci}}\right) + \frac{q'_0}{2\pi R_G h_G} + \frac{q'_0}{4\pi k_F}\right]\cos\left[\frac{\pi z}{L_e}\right]$$

at z_c when: $$\tan\left(\frac{\pi z_c}{L_e}\right) = \left[\frac{L_e}{\rho A V C_p}\right] \Big/ \left[\frac{1}{2R_{Co} h_T} + \frac{1}{2R_C h_G} + \frac{1}{2k_C}\ln\left(\frac{R_{Co}}{R_{Ci}}\right) + \frac{1}{4k_f}\right]$$

where

L_e	effective core height	q'_0	peak linear heat rating
h_T	coolant heat transfer coefficient	k_C	clad thermal conductivity
h_G	gas gap heat transfer coefficient	k_f	fuel mean thermal conductivity
ρ	coolant density	A	flow area
V	coolant flow velocity	Cp	coolant specific heat

Single phase heat transfer and hydraulics

There is range of correlations for convective heat transfer to fluids. The most commonly used correlations for forced convection in nuclear energy applications are:

Gas and liquid coolants (excluding liquid metals) - turbulent flow:

$Nu = 0.023Re^{0.8}Pr^{0.33}$ $\quad Re > 10{,}000 \quad 0.7 > Pr > 120$

Liquid metal coolants – turbulent flow:

$Nu = 6 + 0.006.Pe$ $\quad$ square pin array

$Nu = 0.047(1 - e^{-3.8(P/D-1)})(Pe^{0.77} + 250)$ $\quad$ triangular pin array

Where both Nusselt and Reynolds numbers are based on hydraulic diameter:

$Nu = h_T D_h / k$ $\qquad Pe = RePr = (\rho V D_h C_p)/k$

Hydraulic diameter $D_h = 4 \times$Flow Area/Wetted Perimeter

P/D = Pitch/Diameter of pin array

Moody friction factor f

Laminar flow: $f = \frac{64}{Re}$

Turbulent flow: $f = \frac{0.184}{Re^{0.2}}$ (smooth pipes)

2.4 Two Phase Flow Quantities

Void Fraction $a = \frac{A}{A_v + A_l}$

Flow based Quality $X = \frac{m_v}{\dot{m}_v + \dot{m}_l}$ Slip ratio $S = \frac{u_v}{u_l}$

Relationship between Void Fraction and Quality

$$a = \frac{X}{x + \left(\frac{\rho_v}{\rho_t}\right)(1-x)s}$$ where (*v*) denotes vapour &(*l*) liquid

2.5 In-core Fuel Management Equilibrium Cycle Burn-up Length Ratio

For M-batch refuelling: $\frac{B_M}{B_1} = \frac{2M}{M+1}$

Where: B_M is the irradiation burn-up for M batch refuelling.

2.6 Decay Heat Estimation

Decay Heat (Beta & Gamma): $P(t) = 0.066 P_0 \left(t^{-0.2} - (t + t_0)^{-0.2}\right)$

where $P_d(t)$ = the power generation due to β decay and γ-radiation

P_0 = the power before shutdown,

t_0 = the time of operation before shutdown (in s)

t = the elapsed time since shutdown (in s)

2.7 Reactor Materials Properties

Material	Temp, K	Density, g/cm³	Linear expansion, K×10⁻⁶	Specific Heat, kJ/kg/K	Thermal Conduct, W/m/K	UTS, MPa	Yield, MPa	Young's Modulus, GPa
Graphite Nuclear grade	300	1.7	-	0.71	156	13.8	58(C)	-
	500		3.6	1.25	118	15.9		9.0
	800		5.0	1.67	73	17.2		
	1400		6.3	1.88	36	19.3		
	2500		8.5	-	31	27.6		
Steel, carbon A 533B	300	7.86	-	0.50	52	500	340	207
	600		10.2	0.59	43	530	280	182
	750		10.4	0.63	38	450	240	172
	800		10.4	0.67	35	390	200	169
Steel, stainless type 347	300	7.95	-	0.50	14	520	210	-
	500	7.86	16.9	0.52	17	420	-	173
	700	7.71	17.4	0.55	20	400	150	166
	800	-	18.5	0.57	22	390	-	157
Uranium Carbide	300	13.63	-	0.15	-	-	372(C)	214
	800		-		23			
	1250		10.3		23			
Uranium Dioxide	300	10.98	-	0.23	8.0		960(C)	183
	500		9.0	0.28	6.1			-
	800		10.1	0.30	4.1			165
	110		-	0.31	2.6			
	1400		12.8	0.32	2.2			
	2300			0.42	2.3			
Zircalloy-2	300	6.56	-	0.28	12.7	490	300	95
	500		-	0.31	15.2	280	170	90
	600		6.5	0.33	16.5	210	117	78
	800			0.35	18.9	-		
	1000			0.37	21.6	-		

Source: Glasstone & Sesonske

3. Fuel Cycle, Waste & Decommissioning

3.1 Enrichment of Isotopes

Separative Capacity D:

$$D = W(2x_w - 1)\ln\frac{x_w}{1-x_w} + P(2x_p - 1)\ln\frac{x_p}{1-x_p} - F(2x_f - 1)\ln\frac{x_f}{1-x_f}$$

Where x_w is the mol fraction of the tails
x_p is the mol fraction of the product
x_f is the mol fraction of the feed
W is flow rate of tails (mols/s)
P is flow rate of product
F is flow rate of feed

Separative Work:

$$S = E_w(2x_w - 1)\ln\frac{x_w}{1-x_w} + E_p(2x_p - 1)\ln\frac{x_p}{1-x_p} - E_f(2x_f - 1)\ln\frac{x_f}{1-x_f}$$

where E_w is mass of tails (kg or Te)
E_p is mass of product
E_f is mass of feed
All fractions are mass fractions

Value function: $v(x) = (2x-1)\ln\left(\frac{x}{1-x}\right) \approx -\ln(x)$ for small x

where x is the mol or mass fraction of the component in question.

3.2 Hold up and decay (Extended Bateman's Equation)

$$N_i = \lambda_1\lambda_2\ldots\lambda_{i-1}P\sum_{j=1}^{i}\frac{[1-\exp(-\lambda_j T)]\exp(-\lambda_j\tau)}{\lambda_j\prod_{\substack{k=1\\k\neq j}}^{i}(\lambda_k - \lambda_j)}$$

where N_i = number of atoms of nuclide I at time $T+\tau$, T = filling time,
λ_j = decay constant of nuclide j, τ = decay hold-up time after filling,
P = parent nuclide arising rate (atoms/unit time)

Note: decay constant λ_j and filling time P must have the same units of time.

4. Fusion

4.1 Major fusion reactions:

D-D: $^2_1D + ^2_1D \rightarrow ^3_2He\,(0.8\,\text{MeV}) + n\,(2.45\,\text{MeV})$

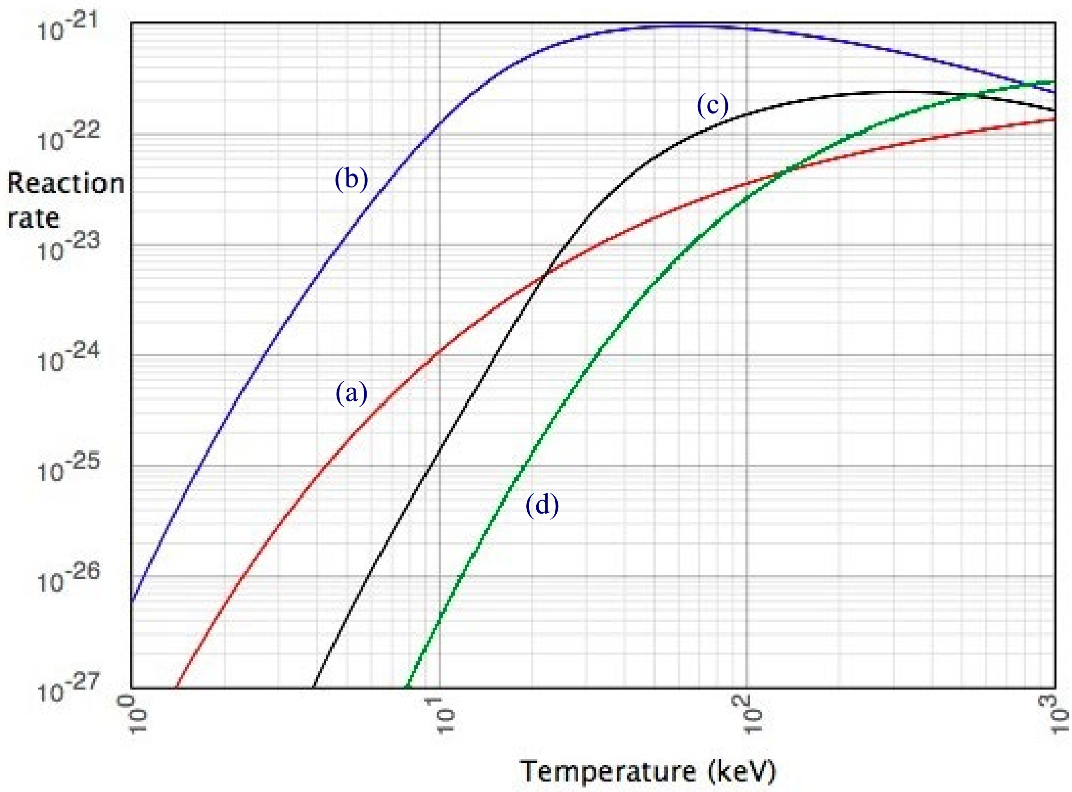
$^2_1D + ^2_1D \rightarrow ^3_1H\,(1.0\,\text{MeV}) + ^1_1H\,(3.0\,\text{MeV})$

D-T: $^2_1D + ^3_1T \rightarrow ^4_2He\,(3.5\,\text{MeV}) + n\,(14.1\,\text{MeV})$

D-^{3}He: $^2_1D + ^3_2He \rightarrow ^4_2He\,(3.66\,\text{MeV}) + ^1_1H\,(14.64\,\text{MeV})$

p-B: $^1_1H + ^{11}_5B \rightarrow 3\,^4_2He + 8.7\,\text{MeV}$

The reaction rate $\langle \sigma v \rangle (\text{m}^3\text{s}^{-1})$ for these reactions is shown as a function of plasma temperature in keV in the graph below. (a) D-D (including both reactions); (b) D-T; (c) D-^{3}He; (d) p-B.



4.2 Tritium breeding reaction:

Li-n: $^{6}_{3}Li + n \rightarrow ^{4}_{2}He\,(2.05\,\mathrm{MeV}) + ^{3}_{1}\mathrm{T}\,(2.75\,\mathrm{MeV})$

4.3 Plasma parameters:

Plasma β: $\beta = \dfrac{\langle P\rangle}{B^2/2\mu_0}$

Normalised β: $\beta_N = \beta\dfrac{aB_T}{I_P}$ (%, m, T, MA)

Greenwald density limit n_G: $n_G = \dfrac{I_P}{\pi a^2}$ (10^{20} m^{-3}, MA, m)

Plasma parameter Λ_D: $\Lambda_D = \dfrac{4\pi}{3}\dfrac{\varepsilon_0}{e^3}\dfrac{T_e^{3/2}}{n_e^{1/2}} \gg 1$

4.4 Tokamak ELMy H-mode confinement time scaling law IPB98(y,2):

$$\tau_e = H \times 0.145 I^{0.93} R_0^{1.39} a^{0.58} \kappa^{0.78} n_{20}^{0.41} B_0^{0.15} A^{0.19} P^{-0.69} \;(\mathrm{MA, m, m, 10^{20}\,m^{-3}, T, MW})$$

4.5 Lawson criteria:

$$n_e \tau_E \geq \frac{12}{E}\frac{k_B T}{\langle \sigma v\rangle} \quad (\approx 1.5\times 10^{20}\,\mathrm{s\ m^{-3}\ for\ D\text{-}T})$$

$$n_e T_e \tau_E \geq \frac{12}{E}\frac{k_B T^2}{\langle \sigma v\rangle} \quad (\approx 10^{21}\,\mathrm{keV\ s\ m^{-3}\ for\ D\text{-}T})$$

4.6 Bremsstrahlung radiation:

$$P_{\mathrm{brem}} \approx 0.85\,\mathrm{Pr}^2 \frac{Z_{\mathrm{eff}}^2}{\mathrm{T}_{10\mathrm{keV}}^{3/2}} (\mathrm{MW\,m^{-3}, atm, 10\ keV})$$

$$Z_{\mathrm{eff}} = \frac{\sum Z_i^2 n_i}{\sum Z_i n_i} \quad \text{(Effective plasma ion charge)}$$

4.7 Larmour (gyro) radius:

$$\rho_L = \frac{v_{th}}{\omega_{cy}} = \frac{v_{th}}{ZeB/m} = \frac{(2mT)^{1/2}}{ZeB}$$

4.8 Molar masses of common fusion isotopes (and neutrons):

Isotope	Symbol	**Molar mass (g)**
Neutron	${}^{1}_{0}n$	1.0087
Hydrogen	${}^{1}_{1}H$	1.0079
Deuterium	${}^{2}_{1}D$	2.0141
Tritium	${}^{3}_{1}T$	3.01605
Helium-3	${}^{3}_{2}He$	3.01603
Helium	${}^{4}_{2}He$	4.0026
Boron-11	${}^{11}_{5}B$	**11.0093**

4.9 Nomenclature

β	ratio of thermal/magnetic pressure (%)	β_N	ratio to Troyon stability limit
B	Magnetic field (Tesla)	I_P	plasma current (MA)
E	Energy released per fusion (MeV)	$\langle\sigma_v\rangle$	average fusion cross section
n	number density		
Pr	Pressure (either Pa, or bars)		
T_e	Temperature in energy units (eV, or keV)		
Z	Ion charge number		
ρ_L	Lamour radius		
v_{th}	thermal velocity	w_{cy}	cyclotron frequency

5. Nuclear Safety

5.1 UK NII/ONR system of Safety Categorisation and Classification

A **safety categorisation** scheme could be determined on the following basis:

a. Category A – any function that plays a principal role in ensuring nuclear safety.
b. Category B – any function that makes a significant contribution to nuclear safety.
c. Category C – any other safety function.

The **method** for categorising safety functions should take into account:

a. the consequence of failing to deliver the safety function;
b. the extent to which the function is required, either directly or indirectly, to prevent, protect against or mitigate the consequences of initiating faults;
c. the potential for a functional failure to initiate a fault or exacerbate the consequences of an existing fault;
d. the likelihood that the function will be called upon.

A safety classification scheme for **safety measures** (structures, systems and components) could be determined on the following basis:

a. Class 1 – any structure, system or component that forms a principal means of fulfilling a Category A safety function.
b. Class 2 – any structure, system or component that makes a significant contribution to fulfilling a Category A safety function, or forms a principal means of ensuring a Category B safety function.
c. Class 3 – any other structure, system or component.

5.2 Long term Health Risks related to Radiation

The International Commission for Radiological Protection has recommended that the following expression should be used to assess the risk of death from radiation.

- Risk of death from developing a fatal cancer = Radiation dose(in Sv) x Risk Factor
- They recommend a risk factor for adults of 0.04 per Sievert.

5.3 Safety Criteria

I. UK Radiation dose limits for normal (non accident) operations

Risk Group	Industrial workers	Members of the general public
Basic Safety Level	20mSv	1mSv
Basic Safety Objective	1mSv	0.02

II. UK Risk Criteria for nuclear accidents

Risk Group	Industrial Workers	Member of the general public
Basic Safety Level	10^{-4}	10^{-4}
Basic Safety Objective	10^{-6}	10^{-6}

III. ALARP cost benefit criteria

1. Where the benefit is the prevention of death, the current convention used by UK Health & Safety Executive, when conducting a Cost Benefit Analysis is to adopt a benchmark value of about £1,000,000 (2001 prices) for the **value of preventing a fatality** (**VPF**).

2. However due to the aversion caused by deaths from cancer and radiation a premium should be applied to this value. **A factor of 2** is recommended to be applied for any deaths caused by radiation and therefore £2m for VPF should be used for nuclear facilities and or radioactive installations.

3. The disproportion factor applies a weighting factor so that more money should be spent to improve the safety of higher risk facilities in comparison to lower risks facilities.

    - The values to be used are at least a factor of 10 if the risks are predicted to be near the **Basic Safety Level** (**BSL**), and
    - A factor of 3 if the risks are predicted to be near the **Basic Safety Objective** (**BSO**).

IV. Dose Staircase Criteria

Targets for the predicted frequency of any single accident, which could give doses to a person on the site, are for: **Effective dose, mSv**	**Predicted frequency per annum BSL/ BSO**
2 – 20	$1 \times 10^{-1}/1 \times 10^{-3}$
20 – 200	$1 \times 10^{-2}/1 \times 10^{-4}$
200 – 2000	$1 \times 10^{-3}/1 \times 10^{-5}$
> 2000	$1 \times 10^{-4}/1 \times 10^{-6}$