1.

Thermal power produced by a microreactor core is removed by a coolant with specific heat capacity c_p. The coolant temperature at the core outlet is limited to T_{out}. The coolant passes through a heat exchanger where heat is transferred with effective heat transfer coefficient U to a secondary circuit. The coolant is then recirculated into the core by means of a pump with operational characteristic given by $W = A\,\dot{m}H$, where W is the power consumed by the pump, $\dot{m}$ is the mass flow rate, H is the pressure head (in Pa) provided by the pump and A is a constant.

The primary coolant loop flow pressure loss can be characterised by the expression $\Delta p = B\,\dot{m}^2$, where B is a constant.

(a) Using the data provided in Table 1 below, estimate the heat transfer area of the heat exchanger, if the effective mean temperature of the secondary coolant was to be kept at 200 °C and the pump consumes 5 kW of power.

(35%)

(b) The heat exchanger upfront cost is £8500 per m^2 of heat exchange area, while the electricity cost for operating the pump is £0.1/kWh, charged annually at the year end. If the upfront cost of the pump can be neglected and the reactor lifetime is 3 years, would it be economically beneficial to increase the size of the pump to 10 kW? The cost of borrowing money for this project is 10% per annum.

(35%)

(c) For a realistic system, list any other positive and negative effects on the reactor economics that would be expected from increasing the size of the pump.

(30%)

Reactor power, MW_{th}	5
Coolant specific heat capacity, c_p, J kg^{-1} K^{-1}	4,000
Coolant outlet temperature, T_{out}, °C	350
Effective heat transfer coefficient, U, W m^{-2} K^{-1}	5,000
A, m^3 kg^{-1}	1
B, kg^{-1} m^{-1}	5

Table 1

(a)

Heat to be transferred to the secondary side through surface area S:

$$Q = SU(\bar{T}_P - \bar{T}_S) \approx SU\left(\frac{T_{out} + T_{in}}{2} - \bar{T}_S\right)$$

Or $$S \approx \frac{Q}{U\left(\frac{T_{out} + T_{in}}{2} - \bar{T}_S\right)}$$

The mass flow of the primary coolant needed can be obtained from the energy balance for heat and pumping:

$$B\,\dot{m}^2 = \frac{W}{A\,\dot{m}} \qquad \text{or} \qquad \dot{m} = \sqrt[3]{\frac{W}{A\,B}}$$

$$Q = \dot{m}c_p(T_{out} - T_{in}) = \sqrt[3]{\frac{W}{A\,B}}c_p(T_{out} - T_{in});$$

$$T_{in} = T_{out} - \sqrt[3]{\frac{A\,B}{W}}\frac{Q}{c_p} = 350 - \sqrt[3]{\frac{1 \times 5}{5000}} \times \frac{5 \times 10^6}{4000} \approx 225\text{ °C}$$

Substituting T_{in} into the expression for heat exchange area:

$$S \approx \frac{Q}{U\left(\frac{T_{out} + T_{in}}{2} - \bar{T}_S\right)} = \frac{5 \times 10^6}{5000\left(\frac{350 + 225}{2} - 200\right)} = 11.43\,m^2$$

(b) Repeating calculation in (a) for $W = 10kW$

$T_{in} = 251$ °C

$S = 9.96\,m^2$

Upfront cost saving on heat exchanger = $\Delta S \times 8500 = (11.43 - 9.96) \times 8500 = £12{,}495$

The costs are incurred upfront at year 0, therefore this does not need discounting if this year is chosen as a reference. The savings would be at the expense of the additional cost of pumping power:

5 kW of extra power to be paid over 3 years at £0.1/kWh

Each year costs will have to be discounted with present value factor $PV = (1 + i)^{-t}$

Incremental pumping costs per year: $365 \times 24 \times 0.1 \times 5 = £4380$

Over 3 years with discounting: $£4380 \times (1 + 1.1^{-1} + 1.1^{-2}) = £11{,}981$

The answer suggests that the bigger pump is worthwhile. However, if the money can be borrowed cheaper, then this conclusion could be reversed. At zero borrowing costs, there is no benefit from paying for electricity later. Then, the total cost of running the bigger pump would be

$£4380 \times 3 = £13{,}140$ which is higher than the savings from the smaller heat exchanger.

(c)

A bigger pump will increase the flow rate in the primary circuit. This will have multiple implications for the reactor economics.

- As T_{out} is fixed, higher flow rate will increase T_{in} and, therefore, the core average temperature will be higher. As all reactors are generally designed to have negative temperature coefficients (MTC, DC), the core reactivity will be lower. This will have to be compensated by either higher enrichment or a shorter fuel cycle.

- A higher flow rate may have an effect on structural materials corrosion/erosion. Higher corrosion may require more expensive materials and/or more frequent inspection increasing the costs.

- A higher core average temperature can either reduce the costs of HX (as shown above) or increase the average temperature of heat addition to the power conversion cycle, increasing its efficiency and leading to cost savings.

- A higher flow velocity may impact the mechanical design of components due to induced vibrations which will require redesign, leading to additional costs.

- A higher core average coolant temperature would increase the core average fuel temperature reducing the margin to melting but, on the other hand, it may improve (increase) critical heat flux. Depending on the coolant and reactor thermal limits, this may relax the requirements on safety equipment and/or reduce expenditure on preparing the reactor safety case.

2.

A rectangular prism shaped research reactor core is composed of 100 cylindrical fuel pins arranged in a square lattice with a moderator to fuel volume ratio of 2. The core base is a square with 80 cm sides as shown in Fig. 1 below. The reactor is to produce 1 MW of thermal power. It is cooled by light water at 50 °C. The coolant temperature rise across the core is negligible. Radial heat transfer between the fuel pellet surface and the coolant can be characterised by a heat transfer coefficient $h = 5000$ W m^{-2} K^{-1}. The thickness of the cladding and the gap are much smaller than the fuel pin radius.
Assume that the fuel is uniform and no burnable or control poisons are used. Assume the Diffusion Theory can be applied, and extrapolated boundaries can be neglected, leading to cosine-shape power distribution of the form: $q'''(z) \sim \cos\frac{\pi z}{L}$ in each direction.

(a) Explaining your reasoning, show that the maximum fuel centreline temperature can be approximated by

$$T_{CL} = T_{coolant} + q'_{max}\left(\frac{1}{2\pi Rh} + \frac{1}{4\pi k}\right)$$

where q'_{max} is the maximum linear heat rate, R is the fuel pin radius and k is the fuel average thermal conductivity. (20%)

(b) Given the expression in (a), estimate the core height (L) if the maximum fuel temperature should not exceed 1500 ℃ in normal operation and fuel average thermal conductivity $k = 3$ W m^{-1} K^{-1}. (30%)

(c) If the critical heat flux for this core is constant $q''_C = 10^5$ W m^{-2}, estimate the Minimum Departure from Nucleate Boiling Ratio (MDNBR) for this core. (20%)

(d) How can MDNBR be improved? (30%)

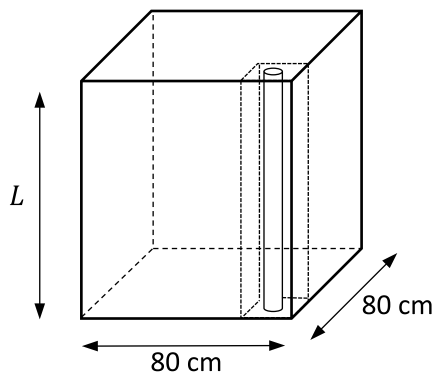

Fig. 1

(a) The coolant temperature is uniform throughout the core. The power distribution is a superposition of three cosines in each direction with maximum heat generation, and thus linear power, occurring in the centre of the prism.

Starting from a general expression in Nuclear Databook:

$$T_{CL}(z) = T_{in} + \frac{q_0' \, L_e}{\pi \, \dot{m} \, c_p} \left[sin\left(\frac{\pi z}{L_e}\right) + sin\left(\frac{\pi L}{2L_e}\right) \right] +$$

$$+ \frac{q_0' \, cos\, \pi \left(\frac{z}{Le}\right)}{2\pi} \left(\frac{1}{R_{co} h_F} + \frac{1}{k_c} ln \frac{R_{co}}{R_{ci}} + \frac{1}{R_g h_g} + \frac{1}{2k_f} \right)$$

We note that T_{in} does not change significantly as we integrate the heat generation along the channel. Therefore, the second term on the RHS disappears and $T_{coolant} \approx T_{in}$ at any z.

$$T_{CL}(z) = T_{coolant} + \frac{q_0' \, cos\, \pi \, \left(\frac{Z}{L_e}\right)}{2\pi} \left(\frac{1}{R_{co} h_F} + \frac{1}{k_c} ln \frac{R_{co}}{R_{ci}} + \frac{1}{R_g h_g} + \frac{1}{2k_f} \right)$$

We further note that extrapolation distance can be neglected. Thus, $L_e = L$.

Cosine has the maximum value of 1 at the core centre: $q'_{max} = \max\left\{ q_0' cos\, \pi \, \left(\frac{z}{L}\right) \right\} = q_0'$

The heat transfer across the coolant laminar boundary layer, cladding and gap is characterised by a single heat transfer coefficient. Since the cladding and gap thicknesses are much smaller than the pellet radius, R can be taken as the pin radius. Therefore:

$$\frac{1}{R_{co} h_F} + \frac{1}{k_c} ln \frac{R_{co}}{R_{ci}} + \frac{1}{R_g h_g} \approx \frac{1}{Rh}$$

Finally, pulling it all together, we can write for the fuel maximum centreline temperature

$$T_{CL} = T_{coolant} + q'_{max} \left(\frac{1}{2\pi Rh} + \frac{1}{4\pi k} \right)$$

where q'_{max} is the maximum linear power in the central pin at half the core height and R is the pin radius.

(b) Determine the fuel radius. The lattice unit cell size is 80/10 = 8cm. Unit cell area A = 8^2 = $64 cm^2$. Fuel pin cross sectional area $S = \pi R^2$ and Vm/Vf = 2. Therefore:

$$R = \sqrt{\frac{64}{3\pi}} = 2.6 \, cm$$

$$q'_{max} = \frac{(T_{CL} - T_{coolant})}{\frac{1}{2\pi Rh} + \frac{1}{4\pi k}} = \frac{(1500 - 50)}{\frac{1}{2\pi \times \, 2.6 \times 0.5} + \frac{1}{4\pi \, \times 0.03}} = 522.6 \frac{W}{cm}$$

Determine the value of q'_{max} by normalising power density to total power.

The reactor is a prism with a square base and extrapolation distances can be neglected.

$$1MW = q'''_{max} \int_{-40}^{+40} \cos\frac{\pi x}{80} \int_{-40}^{+40} \cos\frac{\pi y}{80} \int_{-L/2}^{L/2} \cos\frac{\pi z}{L}\, dz\, dy\, dx = q'''_{max} \frac{160}{\pi}\frac{160}{\pi}\frac{2L}{\pi} =$$

$$= q'''_{max} \frac{51200\, L}{\pi^3} = \frac{q'_{max}}{(8\, cm)^2}\frac{51200\, L}{\pi^3} = \frac{522.6}{64} \times \frac{51200\, L}{\pi^3} = 1MW$$

$$L = 74.2\ cm$$

(c)

Since the coolant temperature is approximately constant throughout the core, the maximum heat flux and maximum power density should be in the same location at the core centre.

Given the maximum linear heat rate $q'_{max} = 522.6 \frac{W}{cm}$, find the heat flux at the same location:

$$q''_{max} = \frac{q'_{max}}{2\pi R} = \frac{522.6}{2\pi \times 8} = 10.4\ \frac{W}{cm^2} = 104{,}000\ \frac{W}{m^2}$$

$$MDNBR = \frac{q''_C}{q''_{max}} = \frac{10^5}{104{,}000} = 0.96$$

This result suggests that DNB will occur in the most limiting location and the integrity of the cladding can be breached.

(d)

The result in (b) is not acceptable and needs to be improved for MDNBR to increase above 1 with sufficient margin allowing for transient conditions and uncertainties related to modelling and manufacturing.

MDNBR can be improved by either increasing the critical heat flux or reducing the maximum operating heat flux or both.

If the core power is fixed, the operating heat flux can be reduced by increasing the heat transfer area, for example through increasing the number of pins or simply increasing each pin diameter. More exotic variations could be annular internally and externally cooled fuel pins or twisted plate or cruciform shaped fuel rods.

The maximum heat flux can also be reduced by flattening the power distribution using variable fuel enrichment across the core, efficient reflectors or burnable poisons.

The critical heat flux can be increased by increasing water subcooling, as more energy will then be required for the coolant in the cladding vicinity to reach sufficient superheat.

Increasing the flow rate will increase turbulence and promote bubble detachment.

Generally, larger flow area (or hydraulic diameter) should also increase the critical heat flux.

3.

A PWR core produces 3000 MW of thermal power. The coolant flow rate is 15,000 kg s^{-1}. The core average linear heat generation rate is $q' = 18\ kW\ m^{-1}$. The thermal conductivity of the fuel is $k = 3\ W\ m^{-1}\ K^{-1}$ and can be assumed constant. At the Hot Zero Power (HZP) condition, the coolant temperature is 280 °C. The coolant specific heat capacity is $c_p = 5{,}400\ J\ kg^{-1}\ K^{-1}$

(a) Estimate the core average coolant temperature and core average fuel temperature at Hot Full Power (HFP) conditions. (40%)

(b) The fuel Doppler reactivity coefficient is given by $DC = -\frac{3\times10^{-4}}{\sqrt{T}}\ K^{-1}$, where T is in K, while the Moderator Temperature Coefficient of reactivity is $MTC = -5 \times 10^{-4}\ K^{-1}$. Estimate the reactivity decrement associated with the core heat-up from HZP to HFP.

(20%)

(c) List possible ways in which reactor operators can affect MTC and DC. (20%)

(d) List possible ways in which reactor designers can affect MTC and DC. (20%)

Clearly state any assumptions you are making.

3.

(a) At HZP, the core inlet and outlet coolant temperatures are approximately the same and equal to 280 °C.

At HFP, the core inlet temperature stays the same at 280 °C. The core outlet temperature can be obtained from the heat balance.

$$Q = \dot{m}c_p(T_{out} - T_{in})$$

$$T_{out} = T_{in} + \frac{Q}{\dot{m}c_p} = 280 + \frac{3000 \times 10^6}{15 \times 10^3 \times 5400} = 317 \text{ °C}$$

Assume that the axial power distribution is symmetric and $c_p \neq f(T)$.

Then, the core average coolant temperature is $\bar{T} = \frac{(T_{out}+T_{in})}{2} = \frac{(280 + 317)}{2} = 298.5$ °C

The radial temperature distribution within the fuel rods is non-uniform with very large gradient due to the poor thermal conductivity of UO2. Therefore, the fuel temperature requires averaging. The simplest way to average is volume weighting.

$$\bar{T}_f = \frac{\int T(r)dV}{\int dV}$$

Obtain $T(r)$ by integrating the heat conduction equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{q'''}{k} = 0$$

Integrating once

$$kr\frac{dT}{dr} + q'''\frac{r^2}{2} + C_1 = 0$$

Assuming conventional solid fuel pellets are used, a zero heat flux boundary condition at the fuel centre gives $C_1 = 0$

Assuming further that the temperature difference between the fuel and coolant is dominated by the temperature rise across the pellet, the fuel surface temperature is approximately equal to the coolant temperature and can be taken as the second boundary condition, i.e. $T_f(R) = T_{coolant}$.

A second integration and use of this boundary condition then gives

$$T_f(r) = T_{coolant} + \frac{q'''}{4k}R^2\left(1 - \frac{r^2}{R^2}\right) = T_{coolant} + \frac{q'}{4\pi k}\left(1 - \frac{r^2}{R^2}\right)$$

The fuel average temperature is therefore:

$$\bar{T}_f = \frac{\int T(r)dV}{\int dV} = \frac{\int T(r)\, 2\pi r dr}{\int 2\pi r dr} = T_{coolant} + \frac{q'''R^2}{8k} = T_{coolant} + \frac{q'}{8\pi k} = 298.5 + \frac{18000}{8\pi \times 3}$$

$$\bar{T}_f = 537.2 \text{ °C}$$

where integration is for the whole fuel pellet between $r = 0$ and $r = R$.

Volume weighted averaging is not necessarily the most appropriate for calculating the Doppler Effect. Since the Doppler Effect is dominated by resonance absorption in U238, it is more appropriate to consider temperature weighting using the radial neutron capture rate in U238, which is strongly peaked towards the fuel surface due to spatial resonance self-shielding.

(b) The reactivity decrement due to coolant heat-up:

$$\Delta\rho = \Delta T \times MTC = (298.5 - 280) \times (-5 \times 10^{-4}) = -0.00925 = -925\,pcm$$

The reactivity decrement due to the Doppler Effect is not uniform with temperature as $DC = f(T)$. Therefore:

$$\Delta\rho = \int_{T_{HZP}}^{T_{HFP}} DC(T)dT = -\int_{280+273}^{537.2+273} \frac{3 \times 10^{-4}}{\sqrt{T}}\,dT = -3 \times 10^{-4} \times 2\sqrt{T}\Big|_{553}^{810.2} = -0.00297$$

$$= -297\,pcm$$

Total reactivity decrement = 297 + 925 = 1222 pcm or 1.222%

(c) Reactor operators can change operating conditions (such as power, coolant temperature, pressure and flow rate). These will have an effect on core materials temperatures and densities. Changes in both MTC and DC are controlled by relative changes in the fission and absorption rates in various core materials.

- Changing the coolant temperature would change the coolant density and thus H/HM, possibly changing the slope of the reactivity vs H/HM curve and consequently MTC. The hydrogen cross section for H atoms bound in water is also temperature-dependent and therefore would result in a shift in the neutron spectrum.
- Changing power will change the fuel temperature, therefore DC as DC=f(T). Also, the equilibrium Xe concentration will change resulting in a slight change in the spectrum.
- The coolant flow rate controls the core average coolant temperature for a fixed T_{inlet}. Therefore, changing it will have a similar effect on $T_{coolant}$ as changing T_{inlet}.
- Changing the system pressure will slightly affect the coolant density leading to the same effect as described above.
- Inserting control rods would reduce reactivity leading to a drop in the coolant temperature with similar effect as above. It may also change the power distribution such that neutron leakage can increase, leading to a slight change in spectrum because fast neutrons will leak more.
- Adding boron to the coolant would have a similar effect as a control rod insertion, but with a more uniform spectrum hardening effect.

(d) Reactor designers can choose fuel enrichment, burnable poison type and location, fuel lattice H/HM as well as core dimensions and aspect ratio.

- Higher enrichment makes the spectrum slightly harder (=> more negative DC) but also reduces the amount of U238 – the primary resonance absorber (=> less negative DC).
- A smaller pin diameter would increase the fuel surface area (for the same mass of fuel), thus increasing resonance absorption and DC.
- Lattice H/HM would shift the spectrum affecting MTC as discussed above.
- Burnable poisons can preferentially absorb neutrons in certain energy ranges, affecting both MTC and DC.
- Core dimensions and aspect ratio would control the core leakage which would affect the spectrum as discussed above. More leaky core would have more negative MTC.

4.

The EPR and AP1000 are examples of modern reactor designs.

(a) List the differences between the two reactor designs in their approach to cooling a molten reactor core in a severe accident along with advantages and disadvantages of each approach. (25%)

(b) List the differences between the two reactor designs in their approach to responding to a Large-break Loss of Coolant Accident along with advantages and disadvantages of each approach. (25%)

(c) List the main sources of stored energy within a PWR containment and comment on the significance of each source. (25%)

(d) List all the barriers that prevent the release of fission products into the environment in the case of a severe accident in an LWR along with strategies to maintain their integrity. (25%)

4.

(a) The EPR uses a core catcher. The molten core debris are allowed to melt predictably through the pressure vessel with high confidence on the final location and geometry of the debris. This allows the designing of reliable cooling, criticality control and containment systems. However, decontamination and decommissioning post-accident may be complicated because larger areas would be in contact with radioactive substances.

The AP1000 relies on in-vessel retention of the molten core. This is achieved through flooding of the reactor vessel as well as the cavity where it is located, which provides reliable cooling of the core debris from all directions. In this case, radioactive material is contained within the vessel without spreading to other areas. However, reliable prediction of molten core behaviour is challenging, complicating the safety case which would require extensive analyses and validation.

(b) In both cases, pressuriser and pressurised accumulators provide initial core cooling.

The EPR takes the evolutionary approach typical of previous generations of PWRs. Accumulators allow sufficient time for the Emergency Diesel Generators (EDGs) to start which will then power motor-driven pumps for injecting make up water into the pressure vessel. The water-steam mixture will continue escaping from the primary system through the break, heating and pressurising the containment. EDG power will also be used for circulating water from the containment sump through an external heat exchanger and then recirculating this water through containment sprays. Reliability of the system is achieved through x4 redundancy of safety trains which provide all of the above functions. The multiplicity and high reliability requirements of the safety systems make the reactor expensive.

The AP1000 response to a LB-LOCA is almost entirely passive. IRWST water will flow under gravity into the core. The steam escaping through the break into the containment will condense passively on the containment walls and flow under gravity back into the IRWST. The containment walls are cooled by spraying water on them from the outside water tank located on the roof of the shield building. Once the supply of water is exhausted, natural circulation of air is sufficient to cool the containment walls because the decay heat will be sufficiently reduced by then. This approach allows substantial savings on safety systems and other components, improving the plant economics. It however still requires stored energy (e.g. batteries) for instrumentation and valve alignment, therefore it is not fully passive.

(c) Primary coolant – a large source of stored energy with rapid release into the containment due to high pressure. Requires prompt action to keep the core cooled.

Secondary coolant – much smaller source of energy due to lower pressure which can add to the problem of containment pressurisation but offers no direct threat to the integrity of the core.

Decay heat – by far the largest source of energy which requires provisions for continuous long-term cooling.

Exothermal reaction of Zr with steam – promotes self-sustainable cladding oxidation and loss of coolable core geometry.

Combustion of non-condensable gases (H and CO) – products of chemical reactions of Zr with steam (H) or molten core debris with concrete (CO). Although the amount of energy release is relatively small, spontaneous combustion may happen very rapidly, generating pressure wave and mechanical loads that can compromise the integrity of safety equipment.

The distribution of these sources of energy is:

Sources	Energy (GJ)	Fraction (%)
1. Primary coolant	324.8	11.1
2. Secondary coolant	82.7	2.8
3. Decay heat (1 day)	2241.0	76.3
4. Chemical reaction	118.0	4.0
– Zr and H_2O	36.3	1.2
– Zr and CO_2	95.4	3.3
5. Combustion	37.3	1.3
– H_2 combustion		
– CO combustion		
Total	2935.5	100.0

(d)