## Q1

(a) [30%]

$$q_{ch,max} = \frac{Q_{tot}}{N_{ch}} \cdot P_r = \frac{1400}{500} \cdot 1.667 = 4.667 [MW]$$

$$q_{rod,max} = \frac{q_{ch,max}}{N_{ch}} = \frac{4.667}{19} = 0.2456[MW]$$

$$q_{rod,max}^{\prime\prime} = \frac{q_{rod,max}^{\prime}}{\pi \cdot D_f} = \frac{q_{rod,max}}{\pi \cdot D_f \cdot L_f} \cdot P_{ax} = \frac{0.2456}{\pi \cdot 0.01 \cdot 5} \cdot 1 = 1.5635 [MW]$$

Water conditions at the channel inlet for  $T_{in} = 200 \text{ °C}$ 

$$\rho_{l} = 859.4 [kg/m^{3}] \quad \mu_{l} = 1.36 \times 10^{-4} [Pa \cdot s] \quad C_{p} = 4467 [J/kg \cdot ^{\circ}C]$$

$$k = 0.665 [W/m \cdot ^{\circ}C] \quad Pr = 1$$

$$40000 [m^{3}] \quad 1 \quad [hr]$$

$$\dot{m}_{ch} = v \cdot \rho = \frac{\dot{V}_{tot}}{N_{ch}} \cdot \rho = \frac{40000 \left[\frac{m^2}{hr}\right] \cdot \frac{1}{3600} \left[\frac{m^2}{sec}\right]}{500} \cdot 859.4 \left[\frac{kg}{m^3}\right] = 19.098 \left[\frac{kg}{s}\right]$$

$$T_{out,max} = T_{in} + \frac{q_{ch,max}}{\dot{m}_{ch} \cdot C_p} = 200 + \frac{4.667 \times 10^6}{19.098 \cdot 4467} = \mathbf{254.71}^\circ \mathrm{C}$$

(b) [30%]

$$A_f = \frac{(\pi \cdot D_{ch}^2 - N_{rod} \cdot \pi \cdot D_{rod}^2)}{4} = \frac{(\pi \cdot 0.1^2 - 19 \cdot \pi \cdot 0.01^2)}{4} = 6.362 \times 10^{-3} \ [m^2]$$
$$G = \frac{\dot{m}_{ch}}{A_f} = \frac{19.098}{6.362 \times 10^{-3}} \approx 3002 \left[\frac{kg}{s \cdot m^2}\right]$$

The mass flux remains constant between the channels as well as between the fuel pins.

The hydraulic diameter -  $D_H = 0.028 [m]$ 

The maximal wall temperature would be at the point where the temperature of the coolant is maximal. As the axial peaking factor is 1 which implies a uniform heat distribution, the maximal cladding temperature would be at the channel outlet.

$$Re = \frac{\rho \cdot v \cdot D_H}{\mu} = \frac{G \cdot D_H}{\mu} = \frac{3002 \cdot 0.028}{1.36 \times 10^{-4}} = 618059$$
$$h = \frac{k}{D_H} \cdot 0.023 \cdot Re^{0.8} \cdot Pr^{0.33} = \frac{0.665}{0.028} \cdot 0.023 \cdot 618059^{0.8} \cdot 1^{0.33} = 23453.9 \left[\frac{W}{m^2 \cdot C}\right]$$

$$T_w = T_{out} + \frac{q_{rod,max}''}{h} = 254.71 + \frac{1.5635 \times 10^6}{23453.9} = 321.4^{\circ}\text{C}$$

(c) [40%]

$$q_{cr}^{\prime\prime} = 3.95 \cdot C \cdot \left(\frac{G}{1356}\right)^m \cdot (T_{sat} - T_b)^{0.22}$$

By linear interpolation using the values given in the table for P = 5 [*MPa*]

 $\rightarrow C = 0.731 \times 10^6$  m = 0.2112

 $T_b = 254.71 \text{ °C}$   $T_{sat}(P = 5[MPa]) = 263.94 \text{ °C}$ 

 $q_{cr}^{\prime\prime} = 3.95 \cdot 0.731 \times 10^{-6} \cdot \left(\frac{3002}{1356}\right)^{0.2112} \cdot (263.94 - 254.71)^{0.22} = 5.5687 \, [MW]$ 

$$q_{rod}^{\prime\prime} = 1.5635 \,[MW]$$

$$MDNBR = \frac{5.5687}{1.5635} = 3.562$$

The MDNBR design limit is 2.19, thus there is scope for raising the reactor power here.

#### **Assessor's Comments:**

#### 15 attempts, Average mark 55.7%, Maximum 95%, Minimum 5%.

A popular question attempted by almost all candidates. This first part of the question required the candidate to estimate the maximum coolant temperature. Most of the candidates answered correctly; however, some candidates calculated the average outlet temperature rather than the maximal outlet temperature. The second part of the question required the peak cladding temperature to be determined. A common mistake made by candidates concerned the power shape. The question stated an axial power peaking factor of 1, which implies a uniform power distribution; however, some candidates assumed a cosine power shape. The last part required the minimal departure from nucleate boiling to be estimated. This could be done by a straightforward calculation and use of the formula given in the question. This part was answered with variable degrees of success.

## Q2

# (a) [10%]

Starting from general heat conduction equation in cylindrical coordinates:

$$\frac{1}{r}\left(\frac{\partial}{\partial r}\left(rk(T)\frac{\partial T}{\partial r}\right)\right) + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(k(T)\frac{\partial T}{\partial \varphi}\right) + \frac{\partial}{\partial z}\left(k(T)\frac{\partial T}{\partial z}\right) + q^{\prime\prime\prime} = \rho C_p \frac{\partial T}{\partial t}$$

The governing assumptions:

- 1. Steady state
- 2. Uniform properties (k(T) = const)
- 3. Outer wall temperature of the fuel is constant  $(T(\varphi) = const @ r = R_{out})$
- 4. Sufficiently far from the rod ends

$$\frac{1}{r}\left(\frac{d}{dr}\left(rk\frac{dT}{dr}\right)\right) = -q^{\prime\prime}$$

First integration gives

$$rk\frac{dT}{dr} = -\frac{q^{'''}}{2} \cdot r^2 + C_a$$
$$k\frac{dT}{dr} = -\frac{q^{'''}}{2} \cdot r + \frac{C_a}{r}$$

Integrating again

$$k \int dT = \int \left( -\frac{q'''}{2} \cdot r + \frac{C_a}{r} \right) dr$$

$$k \cdot T(r) = -\frac{q^{\prime\prime\prime}}{4} \cdot r^2 + C_a \cdot \ln(r) + C_b$$

(b) [30%]

The equation derived in (a) applies.

As the fuel rod in the question is made from two regions, each one abides by the above equation:

$$(1.1) k_{UO_2} \frac{dT}{dr} = -\frac{q_{UO_2}^{\prime\prime\prime}}{2} \cdot r + \frac{C_1}{r}$$

$$(1.2) k_{UO_2} \cdot T_{UO_2}(r) = -\frac{q_{UO_2}^{\prime\prime\prime}}{4} \cdot r^2 + C_1 \cdot \ln(r) + C_2$$

$$(2.1) k_{MOX} \frac{dT}{dr} = -\frac{q_{MOX}^{\prime\prime\prime}}{2} \cdot r + \frac{C_3}{r}$$

$$(2.2) k_{MOX} \cdot T_{MOX}(r) = -\frac{q_{MOX}^{\prime\prime\prime}}{4} \cdot r^2 + C_3 \cdot \ln(r) + C_4$$

First, assume the arrangement of material.  $UO_2$  has a higher melting point. Thus, it will be assumed to be the inner fuel, as the first guess.

For the given system (Eqs. (1) and (2)) the boundary conditions (BCs) are:

- 1.  $\frac{\partial T}{\partial r} = 0$  @ r = 0
- 2.  $T = T_{max} @ r = 0$
- 3.  $-k_{UO_2} \frac{\partial T_{UO_2}}{\partial r} = -k_{MOX} \frac{\partial T_{MOX}}{\partial r} @ r = R_i$

- 4.  $T = 400 \,^{\circ}\text{C} @ r = R_o$
- 5.  $T_{UO_2}(R_i) = T_{MOX}(R_i)$

From the first two BCs (1 and 2), it is possible to obtain  $C_1$  and  $C_2$  in Eq. 1:

$$C_1 = 0$$
 and  $C_2 = k_{UO_2} T_{max}$ 

 $C_3$  can be found by applying the third BC to Eqs. 1 and 2:

$$-k_{UO_2} \frac{\partial T_{UO_2}}{\partial r} = -k_{MOX} \frac{\partial T_{MOX}}{\partial r}$$
$$-\frac{q_{UO_2}^{\prime\prime\prime}}{2} \cdot R_i = -\frac{q_{MOX}^{\prime\prime\prime}}{2} \cdot R_i + C_3 \cdot \frac{1}{R_i} \rightarrow C_3 = \frac{q_{MOX}^{\prime\prime\prime}}{2} \cdot R_i^2 - \frac{q_{UO_2}^{\prime\prime\prime}}{2} \cdot R_i^2$$
$$C_3 = \frac{R_i^2}{2} \cdot \left(q_{MOX}^{\prime\prime\prime} - q_{UO_2}^{\prime\prime\prime}\right)$$

 $C_4$  can be easily extracted from the fourth BC applied to Eq. 2:

$$T_{MOX}(R_o) = -\frac{q_{MOX}^{\prime\prime\prime}}{4k_{MOX}} \cdot R_o^2 + \frac{R_i^2}{2k_{MOX}} \cdot \left(q_{MOX}^{\prime\prime\prime} - q_{UO_2}^{\prime\prime\prime}\right) \cdot \ln(R_o) + \frac{C_4}{k_{MOX}} = 400 \,^\circ\text{C}$$
$$\frac{C_4}{k_{MOX}} = 400 + \frac{q_{MOX}^{\prime\prime\prime}}{4k_{MOX}} \cdot R_o^2 - \frac{R_i^2}{2k_{MOX}} \cdot \left(q_{MOX}^{\prime\prime\prime} - q_{UO_2}^{\prime\prime\prime}\right) \cdot \ln(R_o)$$

Thus, Eqs. 1 and 2 can be rewritten, to obtain the following forms:

(1) 
$$T_{UO_2}(r) = -\frac{q_{UO_2}^{\prime\prime\prime}}{4k_{UO_2}} \cdot r^2 + T_{max}$$
  
(2) 
$$T_{MOX}(r) = -\frac{q_{MOX}^{\prime\prime\prime}}{4k_{MOX}} (r^2 - R_o^2) + \frac{R_i^2}{2k_{MOX}} \cdot \left(q_{MOX}^{\prime\prime\prime} - q_{UO_2}^{\prime\prime\prime}\right) \cdot \ln\left(\frac{r}{R_o}\right) + 400$$

(c) [40%]

The two equations must meet the final BC:

$$-\frac{q_{UO_2}^{\prime\prime\prime}}{4k_{UO_2}} \cdot R_i^2 + T_{max} = -\frac{q_{MOX}^{\prime\prime\prime}}{4k_{MOX}} (R_i^2 - R_o^2) + \frac{R_i^2}{2k_{MOX}} \cdot \left(q_{MOX}^{\prime\prime\prime} - q_{UO_2}^{\prime\prime\prime}\right) \cdot \ln\left(\frac{R_i}{R_o}\right) + 400$$

Substituting all the parameters with  $k_{UO_2} = 3[W/m \cdot {}^{\circ}C], k_{MOX} = 2.5[W/m \cdot {}^{\circ}C], q''_{MOX} = 1.5 \cdot q''_{UO_2}$  (as specified in the question),  $R_i = 0.004 m$ ,  $R_o = 0.005 m$  and based on the primary assumption that the central material is  $UO_2$  and because we are looking for the maximal linear power, we will use  $T_{max} = 2800 \, {}^{\circ}C$ . Thus

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$$\begin{aligned} -\frac{q_{UO_2}^{\prime\prime\prime}}{4\cdot 3} \cdot 0.004^2 + 2800 \\ &= -\frac{1.5 \cdot q_{UO_2}^{\prime\prime\prime}}{4\cdot 2.5} (0.004^2 - 0.005^2) + \frac{(0.004)^2}{2\cdot 2.5} \cdot \left(1.5 \cdot q_{UO_2}^{\prime\prime\prime} - q_{UO_2}^{\prime\prime\prime}\right) \cdot \ln\left(\frac{4}{5}\right) + 400 \\ &\therefore \quad -q_{UO_2}^{\prime\prime\prime} \cdot 1.333 \cdot 10^{-6} + 2800 = q_{UO_2}^{\prime\prime\prime} \cdot 1.35 \cdot 10^{-6} - q_{UO_2}^{\prime\prime\prime} \cdot 3.57 \cdot 10^{-7} + 400 \\ &\therefore \quad q_{UO_2}^{\prime\prime\prime} \approx 1031.81 \frac{MW}{m^3} \end{aligned}$$

The maximal linear power is given by

$$q' = \pi R_i^2 \cdot q_{UO_2}^{\prime\prime\prime} + \pi (R_o^2 - R_i^2) \cdot q_{MOX}^{\prime\prime\prime} = \pi R_i^2 \cdot q_{UO_2}^{\prime\prime\prime} + \pi (R_o^2 - R_i^2) \cdot 1.5 \cdot q_{UO_2}^{\prime\prime\prime}$$
  
$$\therefore \quad q' = \pi [0.004^2 + 1.5 \cdot (0.005^2 - 0.004^2)] \cdot 1031.81 \cdot 10^6$$
  
$$q' = 95.63 \frac{kW}{m}$$

Finally, we need to test whether the initial assumption did not lead to melting of the outer fuel. First, find the radius at which the outer fuel temperature peaks:

$$\frac{dT_{MOX}}{dr} = -\frac{q_{MOX}^{\prime\prime\prime}}{2k_{MOX}} \cdot r + \frac{R_i^2}{2k_{MOX}} \cdot \left(q_{MOX}^{\prime\prime\prime} - q_{UO_2}^{\prime\prime\prime}\right) = 0$$
  
$$\therefore \quad r_{max}^2 = R_i^2 \frac{1}{q_{MOX}^{\prime\prime\prime}} \left(q_{MOX}^{\prime\prime\prime} - q_{UO_2}^{\prime\prime\prime}\right) = R_i^2 \left(1 - \frac{q_{UO_2}^{\prime\prime\prime}}{q_{MOX}^{\prime\prime\prime}}\right) = 0.004^2 \left(1 - \frac{1}{1.5}\right)$$
  
$$\therefore \quad r_{max} = 2.31 \times 10^{-3} m$$

This is lower than the inner fuel radius. Thus, the maximal plutonium temperature would be at  $r = R_i$ :

$$T_{MOX-max}(R_i) = -\frac{1.5 \cdot q_{UO_2}^{\prime\prime\prime}}{4k_{MOX}} (R_i^2 - R_o^2) + \frac{R_i^2}{2 \cdot k_{MOX}} \cdot \left(1.5 \cdot q_{UO_2}^{\prime\prime\prime} - q_{UO_2}^{\prime\prime\prime}\right) \cdot \ln\left(\frac{R_i}{R_o}\right) + 400$$
  
$$\therefore \quad T_{MOX-max}(R_i) = q_{UO_2}^{\prime\prime\prime} \left[\frac{1.5}{4k_{MOX}} (R_o^2 - R_i^2) + \frac{R_i^2}{2 \cdot k_{MOX}} \cdot (1.5 - 1) \cdot \ln\left(\frac{R_i}{R_o}\right)\right] + 400$$

 $\therefore$   $T_{MOX-max}(R_i)$ 

$$= 1031.81 \cdot 10^{6} \left[ \frac{1.5}{4 \cdot 2.5} (0.005^{2} - 0.004^{2}) + \frac{0.004^{2}}{2 \cdot 2.5} \cdot (1.5 - 1) \cdot \ln\left(\frac{4}{5}\right) \right] + 400$$
$$T_{MOX-max}(R_{i}) \approx 1424.57^{\circ}\text{C}$$

The MOX layer does not exceed the melting temperature. Therefore, the answer stands:

$$q'_{max} = 95.63 \ \frac{kW}{m}$$

(d) [20%]

Starting from the general

$$T(r) = -\frac{q^{\prime\prime\prime}}{4k} \cdot r^2 + C_a \cdot \ln(r) + C_b$$

With two BC:

1. 
$$\frac{\partial T}{\partial r} = 0$$
 @  $r = 0$ 

2.  $T = T_{max} @ r = 0$ 

Thus  $C_a = 0$  and  $C_b = T_{max}$  giving the final temperature profile form of

$$T(r) = -\frac{q^{\prime\prime\prime}}{4k} \cdot r^2 + T_{max}$$

Noting that  $T = 400 \text{ °C} @ r = R_o$  again, the linear power for the temperature independent conductivity coefficient is found from

$$T(R_o) = -\frac{q'''}{4k} \cdot R_0^2 + T_{max} \Rightarrow 400 = -\frac{q'''}{4 \cdot 3} \cdot 0.005^2 + 2800$$
$$q''' = 1152 \frac{MW}{m^3}$$

$$q' = \pi R_o^2 \cdot q_{UO_2}^{\prime\prime\prime} = 1152 \times 10^3 \cdot \pi \cdot 0.005^2 = 90.48 \frac{\kappa w}{m^3}$$

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	Duplex	Solid UO <sub>2</sub>
Linear power, kW/m	95.63	90.48

Yes, there is a clear uprate in power between the two cases, even though the conductivity of the MOX layer is lower than that of the  $UO_2$  layer. The conductivity difference is not large, however, and the MOX layer generates much more power per unit of volume (50% higher), thus enabling an increase in linear power to be achieved.

#### **Assessor's Comments:**

### 15 attempts, Average mark 48%, Maximum 90%, Minimum 15%.

Another popular question attempted by almost all candidates. The question was the most technical but conceptually simple. The question asked the candidate to determine which fuel configuration will achieve maximal linear power. All the candidates managed to answer the first part without any problem; however, only a few completed the question correctly. The main source of mistakes was wrong use of boundary conditions required to obtain the temperature distribution. Very few candidates reached the end of the question, as wrong boundary conditions led to mistakes or unsuccessful integration of the temperature profile obtained in first part.

## Q3

(a) [25%]

(i) From measurements 2 and 7 we can obtain the  $^{135}$ Xe worth:

$$\alpha_{Xe} = \frac{d\rho}{d(Power)} = \frac{1.001 - 0.998}{(1.001 \cdot 0.998) \cdot 30\%} \times 10^5 = 10 \ \frac{pcm}{\%} power$$

Note that calculation of the <sup>135</sup>Xe worth using states 5 and 1 gives the **stationary xenon** 

**poisoning** value and not the <sup>135</sup>Xe worth.

(ii) The control rod worth is obtained from states 1 and 2:

$$\rho_{CR} = \frac{1.004 - 0.998}{0.998 \cdot 1.004} \times 10^5 \approx 600 \, pcm$$

(iii) The moderator temperature coefficient can be obtained from states 6 and 5:

$$\alpha_M = \frac{d\rho}{dT_M} = \frac{1.025 - 1.049}{(1.025 \cdot 1.049) \cdot 200} \times 10^5 = -11.2 \ \frac{pcm}{_{\circ C}}$$

(iv) The boron worth is calculated from states 1 and 3:

$$\alpha_B = \frac{d\rho}{dN_B} = \frac{0.989 - 1.004}{(0.989 \cdot 1.004) \cdot 150} \times 10^5 = -10 \ \frac{pcm}{ppm}$$

(b) [25%]

 $B_1 > B_2 > B_3$ 

At CZP, the reactor operates with the highest excess of reactivity, as the moderator temperature coefficient (MTC), Doppler coefficient (DC) and Xe feedbacks are yet to have an impact on the core. Therefore, in order to maintain the reactor at  $\rho = 0$  a larger quantity soluble boron would be need in the core.

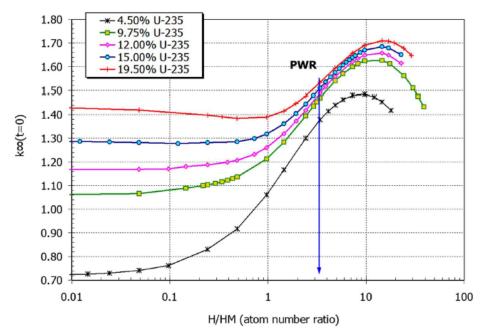
At HFP right after start-up (condition 2), the negative safety coefficients (MTC and DC) reduce the reactivity ( $\rho < 0$ ). There is still no (or little) Xe in the system; thus, it does not play a factor yet. Therefore, in order to compensate for the changes in reactivity and maintain  $\rho = 0$  the boron concentration must be reduced. ( $B_2 < B_1$ )

After two days (condition 3), Xe reaches its equilibrium state, which leads to an additional reduction in reactivity ( $\rho < 0$ ). Thus, to compensate for the strong negative reactivity introduced by the Xe, the boron concentration must be reduced further to maintain  $\rho = 0$ . ( $B_3 < B_2$ )

## (c)(i) [10%]

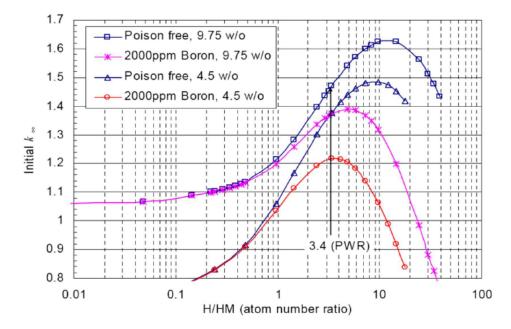
Water-to-fuel ratio is essentially H/HM ratio.

Light water absorbs a non-negligible amount of neutrons. Thus, its density change due to thermal expansion will lead to fewer absorptions, which will lead to a positive effect on reactivity acting against the loss of moderation effect. The relationship between H/HM and  $k_{\infty}$  is shown below:



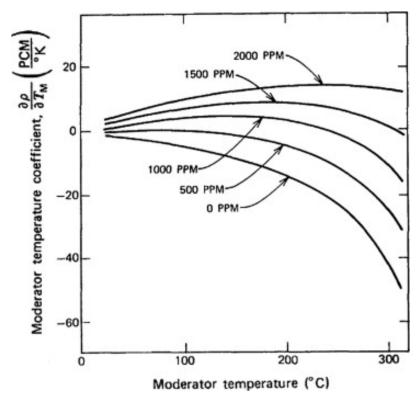
There is a distinct maximum. The reactor would operate to the left-hand side of this maximum. In this way we will ensure that with any thermal expansion of water (i.e., reduction in H/HM) the reactivity will decrease, allowing control of the reactor.

Similarly, we should take care to remain to the left of the maximum under all circumstances, as the curve shifts with the amount of soluble boron in the water, as shown in the figure below.



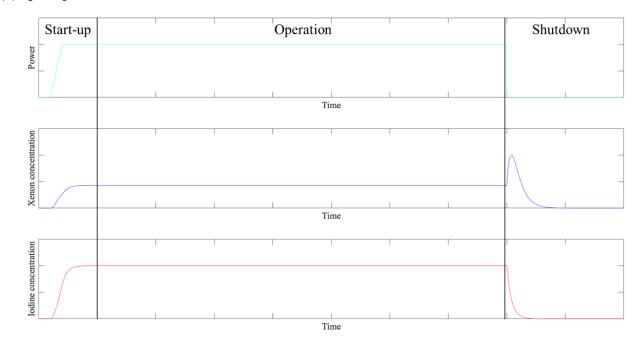
[Detailed figures such as these are not required for full credit in the examination]

(c)(ii) [15%]



[Detailed figure such as this is not required for full credit in the examination]

(d) [25%]



# At reactor start-up:

At start-up the <sup>135</sup>I and <sup>135</sup>Xe concentrations are negligible. Both grow towards secular equilibria at which the rates of production (from fission in the case of <sup>135</sup>I and from both fission and <sup>135</sup>I decay

in the case of  $^{135}$ Xe) match the rates of removal (by decay in the case of  $^{135}$ I and by both decay and transmutation by neutron capture in the case of  $^{135}$ Xe).

## At reactor shut-down:

At shut-down the production of <sup>135</sup>I through fission stops and thus the concentration decays away exponentially. The production of <sup>135</sup>Xe through fission also stops but the production through <sup>135</sup>I decay continues. The only removal mechanism for <sup>135</sup>Xe is now decay, but due to its longer half-life the <sup>135</sup>Xe concentration rises initially post shut-down. As the <sup>135</sup>Xe production rate is falling continuously (with the <sup>135</sup>I concentration) the <sup>135</sup>Xe concentration reaches a maximum a few hours after the shut-down and then declines towards zero.

### **Assessor's Comments:**

### 15 attempts, Average mark 78.4%, Maximum 90%, Minimum 47%.

This was another popular question attempted by most of the candidates. It required familiarity and understanding of safety coefficients and reactivity control mechanisms in a light water reactor. In the first part all candidates correctly obtained all but one of the coefficients. None of the candidates calculated xenon worth correctly. Xenon worth is estimated per unit of power, and thus should have been calculated from states 2 and 7 and not states 5 and 1, which give the stationary xenon poisoning value. The following section were answered well by almost all candidates, with some small mistakes in explanations and graph plots.

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# Q4

(a) [20%]

Equilibrium flow quality:

$$x_e = \frac{h - h_f}{h_{fg}} \tag{1}$$

The SFEE at the outlet of the tube bundle:

 $h\,\dot{m}_i = h_g \dot{m_s} + h_f \,\dot{m_f}$ 

 $r=\frac{1-x_e}{x_e}$ 

Noting that:  $r = \frac{\dot{m}_f}{\dot{m}_s}$  and  $\dot{m}_i = \dot{m}_f + \dot{m}_s \implies \dot{m}_f = \frac{r}{1+r} \dot{m}_i$  and  $\dot{m}_s = \frac{\dot{m}_i}{1+r}$ 

Hence: 
$$h \, \dot{m}_i = h_g \frac{\dot{m}_i}{1+r} + h_f \frac{r}{1+r} \, \dot{m}_i$$

and 
$$x_e = \frac{h_g \frac{1}{1+r} + h_f \frac{r}{1+r} - h_f}{h_{fg}} = \frac{h_g + h_f r - (1+r) h_f}{(1+r) h_{fg}} = \frac{h_g - h_f}{(1+r) h_{fg}} = \frac{1}{1+r}$$

or

note that  $h_{fg} = h_g - h_f$ 

(b) [30%]

SFEE for the tube bundle:

$$\frac{Q}{N} = \dot{m}_i (h_o - h_i) \tag{2}$$

In the moisture separator the SFEE:

$$\dot{m}_i h_o = \dot{m}_s h_g + \dot{m}_f h_f \tag{3}$$

The mass balance:

$$\dot{m}_i = \dot{m}_f + \dot{m}_s \tag{4}$$

Combining Eqs. 3 and 4:

$$\dot{m}_i h_o = (\dot{m}_i - \dot{m}_s) h_f + \dot{m}_s h_g = \dot{m}_i h_f + \dot{m}_s h_{fg}$$
(5)

Substituting Eq. 5 into Eq. 2:

$$\frac{Q}{N} = \dot{m}_{i}h_{f} + \dot{m}_{s}h_{fg} - \dot{m}_{i}h_{i} \implies \dot{m}_{s} = \frac{1}{h_{fg}}\left(\frac{Q}{N} - \dot{m}_{i}(h_{f} - h_{i})\right)$$
(6)

(c) [40%]

First, estimate the total power transferred in the heat exchanger from the primary side:

$$Q_{SG} = \dot{m}C_p(T_{out} - T_{in}) = \frac{17500}{4} \cdot 5.762 \times 10^3 \cdot (315 - 285) = 756.262 \, MW$$

Primary water properties at  $T_{avg} = 300$  °C and p = 155 bar

Relevant data from the Thermofluids Data Book noting that the quantities below are much more strongly dependent on temperature than on pressure, and therefore taking those at core average temperature but saturated pressure is a reasonable assumption.

Specific heat capacity at constant pressure	$5.762 \ \frac{kJ}{kg \ K}$
Density	712.25 $\frac{kg}{m^3}$
Dynamic viscosity	$8.97 \times 10^{-5} Pa \cdot s$
Thermal conductivity	$0.541 \frac{W}{m K}$

The primary heat transfer coefficient can be approximated from Dittus-Boelter correlation as there is no phase change:

$$Nu = 0.023 \ Re^{0.8} \ Pr^{0.33}$$

$$Re = \frac{4\dot{m}}{\pi D\mu} = \frac{4\left[\frac{17500}{4 \cdot 10000}\right]}{\pi \cdot 0.014 \cdot 8.97 \times 10^{-5}} = 443,576$$
$$Pr = \frac{C_p \cdot \mu}{k} = \frac{5762 \cdot 8.97 \times 10^{-5}}{0.541} = 0.955$$
$$Nu = 0.023 \cdot 443,576^{0.8} \ 0.955^{0.33} = 746$$
$$=> h_P = \frac{k \cdot Nu}{D} = \frac{0.541 \cdot 746}{0.014} = 29\frac{kW}{m^2 K}$$

The secondary water heat transfer coefficient is provided:

$$h_S = 24 \ \frac{kW}{m^2 \ K}$$

The overall heat transfer coefficient is:

$$U = \left(\frac{1}{h_P} + \frac{1}{h_S} + \frac{t}{k}\right)^{-1} = \left(\frac{1}{29} + \frac{1}{24} + \frac{0.001}{10/1000}\right)^{-1} = 5.677 \ kW/m^2K$$

The heat transfer across the steam generator can be expressed using the Log-Mean-Temperature-Difference (LMTD):

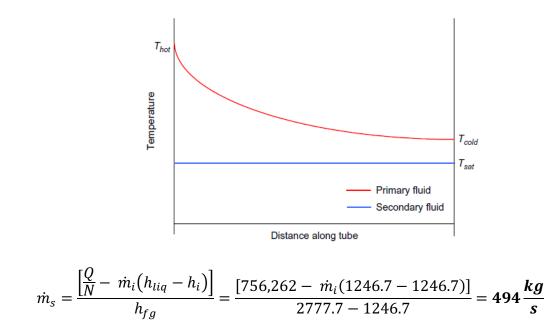
$$A = N_{tubes} L\pi D = 1000 \cdot 20 \cdot \pi \cdot 0.014 = 8796 m^2$$
$$Q = U \times A \times LMTD \rightarrow LMTD = \frac{756,262}{5.677 \times 8796} = 15.1 \text{ °C}$$

The inlet temperature to the steam generator can be obtained from the LMTD expression:

$$LMTD = \frac{(T_H - T_{sat}) - (T_C - T_{in})}{\ln\left[(T_H - T_{sat})/(T_C - T_{in})\right]} = \frac{(315 - 281.87) - (285 - T_{in})}{\ln\left[(315 - 281.87)/(285 - T_{in})\right]} = 15.1 \text{ °C}$$

To find  $T_{in}$  this equation needs to be solved iteratively. However, as we are told its value in the question, we can just check that the equation is satisfied for that value. Substituting  $T_{in} = 280$  °C in the expression for the LMTD yields 14.9 °C. Close enough!

The inlet temperature to the steam generator tube bundle is relatively close to the saturation temperature at 66 bar because the recirculation rate is typically high and subcooling of feedwater is typically low. This implies that the subcooled length is short compared to the tube length, i.e. the secondary fluid reaches saturation temperature in a relatively short distance. Thus, assuming that the water enters the SG tube bundle as a saturated liquid, there is no need to know the feedwater mass flow rate entering the steam generator as the corresponding term obtained in part (b) of this question cancels out.



# (d) [10%]

Primary calorimetry:

The water comes out of the core at a range of temperatures depending of the power of the assembly channel it flowed though. The mixing process does not achieve a uniform state before the water enters the hot leg. Once in the hot leg, the water tends to stratify; therefore, it is very important where the temperature is measured. The temperature difference between the inlet and outlet in a PWR is about 30 °C. If temperature is measured with an error of 1 °C, this will lead to a 3% error in power estimation.

Secondary calorimetry:

Steam does not stratify and the water entering the steam generator is at a uniform cold temperature. Therefore, this measure will give a better indication of core power.

## **Assessor's Comments:**

### 12 attempts, Average mark 60.9%, Maximum 85%, Minimum 20%.

This was the easiest and least labour-intensive question. It is surprising that it was not more popular given the candidates' previous knowledge of the subject. The question tested the candidates' knowledge of the analysis of steam generators. Most candidates tackled the first two sections well and obtained the required expressions for the recirculation ratio and steam mass flow rate. In part (c), most candidates did not check their solution against the temperature assumption given in the question. The last part of the question was not answered correctly by any candidate. Secondary calorimetry is more accurate as there is less temperature stratification in the vapor coming out of the steam generator. This was surprising as the concept was described in the very last lecture of the term.