4I11 Exam 2013-14 Crib

Q1 Fusion technology I: Divertor design

Model answer:

(a) *D-T fusion produces ionised alpha particles which are exhausted together with the stripped electrons – two ions per fusion;*

Divertor design challenges: Under the very high load from ions exhausted from plasma:

i. *Designed to allow cryo-pump to remove exhaust gases from vessel*
ii. *Have high heat transfer capability*
iii. *Must not sputter or melt*
iv. *Means of energy recovery*

(b) *150 MW at 10 MW m^{-2} means that the power must be spread across around 15 m^2. Erosions rate is given as 0.1e-9 m/s*

i. *Tungsten removal is erosion rate times the ion production rate associated with 150MW of heat being deposited*

*0.1e-9 * 15 m^2 * 3600 * 24 * 365 = 0.047 m^3 yr.*

This is 905 kg yr^{-1}, or nearly a tonne per year.

ii. *This metal must ultimately end up as dust in the machine and must be cleaned out regularly or there is a risk of contaminating the plasma to the point that it suffers radiative collapse.*

0.1 nm s^{-1} of erosion is also ~3 mm yr^{-1}, so the divertor armour will need to be replaced frequently. In addition, similar (although slower, but across a much larger area) erosion will be occurring in the rest of the first wall of the reactor, adding to the total tungsten dust.

(c) *Calculating the ion handling requirement*

i. *Reaction rate = Power / energy per reaction*

*= 150 MW/(2 ions*10eV* ev/J)*

*150e6/(2*10*1.602e-19) = 4.68e25 s^{-1} ions (the additional factor of 2 comes from taking the energy carried by the electrons as well as the ions into account).*

ii. *Comparing this with the fuelling rate calculated from the total power/ energy per fusion*

Fusion reaction rate is 2GW/Energy per fusion

$$= \quad 2.5e9/(17.6e6*1.602e\text{-}19) = 8.87e20\ s^{-1}.$$

Each reaction involves two ions so the fuelling rate is;

$$2*8.87e20 \qquad = 1.77e21\ s^{-1}.$$

Dividing divertor heat load ion handling rate divided by the fuelling ion rate, we get:

$$= 4.68e25/1.77e21 \sim 26{,}000.$$

Therefore, the particle input into the reactor is wholly dominated by the demands of divertor protection rather than fuelling.

Q2 Fusion technology II: Physics of plasma heating

Model Answer:

(a) *D-T reaction 14.1 MeV neutron + 3.5 MeV alpha particle = 17.6 MeV total energy per fission. Neutrons travel through the plasma without interacting and energy can be captured in the wall of the Tokomak. The alpha particle become ionised and interacts with the plasma depositing 3.5 of 17.6 MeV i.e. 20% of energy in the plasma.*

(b) *Three main ways to heat the plasma in current Tokomaks:*
   a. *Resistive (or ohmic) heating from current circulating through the plasma, induced by external magnetic coils – solenoid through the centre of the machine.*
   b. *RF heating - powerful beams of radio waves resonate with the cyclotron motion of the ions or electrons winding them up.*
   c. *Neutral beam injection – energetic particles injected from outside the plasma which are neutralised to allow them to penetrate the magnetic fields surrounding the plasma – once in the plasma they quickly become ionised, are trapped in the plasma and deposit their energy.*

(c) *Resistive heating is given by:*

$$P = I^2 R \propto I^2 T^{-3/2}$$

By definition, the energy confinement time (τ) is the plasma thermal stored energy (W) divided by the heating power density (P):

$$\tau = \frac{W}{P} = \frac{3n.k.T}{P}$$

Hence stored energy is given by:

$$nT \propto W = P\tau$$

Where heating (P) is ohmic, substituting from H-mode in data book:

$$W \, \alpha \, T \, \alpha \, P.I^{0.93}.P^{-0.69} = P^{0.31}.I^{0.93}$$

(dropping the fixed terms from τ *and W).*

(d)

i. *Substituting the ohmic heating this then gives:*

$$T \propto (I^2 T^{-1.5})^{0.31} I^{0.93}$$

$$T \propto I^{1.55} T^{-0.465}$$

$$T^{1.465} \propto I^{1.55}$$

$$T \propto I^{1.06}$$

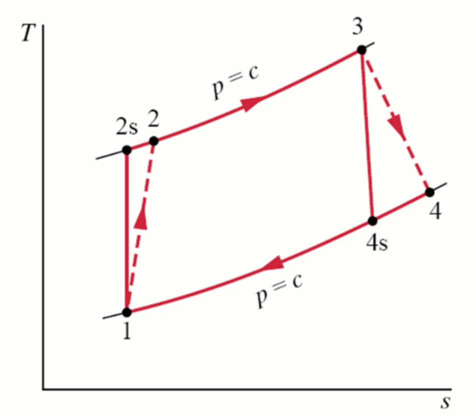
There is a stability current limit ($I<2\pi Ba^2/R$) above which the plasma is unstable. Due to the above relation, for a power plant plasma, T is therefore also limited (to ~3.3 keV, lecture (3)).

ii. *At high temperatures, both self-heating from fusion-produced alpha particles and radiation from the plasma (principally Bremsstrahlung radiation) become significant. Both of these terms rapidly become much larger than the ohmic heating term.*
The temperature required for alpha heating to exceed Bremsstrahlung radiation losses (4.4 keV)_, is greater than the temperature at which the plasma becomes unstable due to the current limit.
Additional heating e.g. neutral beam injection, is required to increase the temperature to the point that alpha heating begins to dominate and the plasma is ignited. In addition, just using ohmic heating, the heating power into the plasma can only be varied by changing the plasma current. Having additional heating allows the separation of heating control from current control.

Q3 Advanced reactors power conversion cycles

Model Answer

a.



M. J. Moran, H. N. Shapiro, B.R. Munson, D.P. DeWitt, Introduction to Thermal Systems Engineering: Thermodynamics, Fluid Mechanics, and Heat Transfer 2002

b. Turbine and compressor efficiencies:

$$\eta_t = \frac{W_{real}}{W_{ideal}} = \frac{\dot{m}c_p(T_3 - T_4)}{\dot{m}c_p(T_3 - T_{4s})}; \quad \eta_c = \frac{W_{ideal}}{W_{real}} = \frac{\dot{m}c_p(T_{2s} - T_1)}{\dot{m}c_p(T_2 - T_1)}$$

Assume Helium can be treated as an ideal gas and for adiabatic processes: $Tp^{\frac{1-\gamma}{\gamma}} = const$;

Thus: $T_3 = 750 + 273 = 1023\ K$; $T_1 = 120 + 273 = 393\ K$;

$$T_{4s} = T_3 r^{(1-\gamma)/\gamma} = 676.8\ K$$

$$T_{2s} = T_1 r^{(\gamma-1)/\gamma} = 594.0\ K$$

Where r is the pressure ratio of the cycle: $r = \frac{p_2}{p_1} = \frac{p_3}{p_4} = \frac{7}{2.5} = 2.8$

The turbine and compressor outlet temperatures are:

$$T_4 = T_3 - \eta_t(T_3 - T_{4s}) = 1023 - 0.9\,(1023 - 676.8) = 711.4\ K$$

$$T_2 = T_1 + \frac{(T_{2s} - T_1)}{\eta_t} = 393 + \frac{(594 - 393)}{0.9} = 616.3\ K$$

The turbine and compressor work

$$\dot{W}_T = \dot{m}(h_3 - h_4) = \dot{m}c_p(T_3 - T_4)$$

$$\dot{W}_{CP} = \dot{m}(h_2 - h_1) = \dot{m}c_p(T_2 - T_1)$$

Net cycle power = $\dot{W}_T - \dot{W}_{CP} = 400\ MW = \dot{m}c_p[(T_3 - T_4) - (T_2 - T_1)]$

From Data book (p. 12), $c_p = 5.19\ kJ/kg$

$$\dot{m} = \frac{400 \times 10^6}{c_p[(T_3 - T_4) - (T_2 - T_1)]} = 873.1\ kg/s$$

c. Cycle efficiency: $$\eta = \frac{\dot{W_T} - \dot{W_{CP}}}{\dot{Q_R}}$$

Heat from the reactor: $$\dot{Q_R} = \dot{m} c_p (T_3 - T_2) = 1842.9\ MW$$

$$\eta = \frac{[(T_3 - T_4) - (T_2 - T_1)]}{(T_3 - T_2)} = 21.7\ \%$$

Heat to the boiler: $$\dot{Q_B} = \dot{m} c_p (T_4 - T_1) = 1443\ MW$$

It is split between heating up the liquid water up to saturation, then the phase transition.

From steam tables (Data Book, p. 18), saturated steam at 200 °C should be at 1.55 MPa of pressure. From Data Book, p. 27 and interpolating between the specific heat values for 20 and 30 °C, we obtain the average c_p = (4181+4497)/2 = 4339 *J/kg/K*.

Heat of evaporation at 1.55 MPa (Data Book, p. 18): h_{fg} = 2792 – 852 = 1940 *kJ/kg*

$$\dot{Q_B} = \dot{m}_s[c_p(200 - 25) + h_{fg}]$$

$$\dot{m} = \frac{\dot{Q_B}}{[c_p(200 - 25) + h_{fg}]} = \frac{1443 \times 10^6}{[4339\ (200 - 25) + 1940 \times 10^3]} = 534.5\ kg/s$$

d. The important properties to consider when choosing the gas coolant are:
   - Larger $\rho\, c_p$ would reduce the mass flow rate (flow velocity) and/or the size of the components for a given cycle power output. CO_2, for example, has roughly five times lower specific heat than Helium, however, it is about 10 time denser, making it preferable coolant in this respect.
   - The ratio of specific heats ($\gamma = c_p / c_v$) would affect the cycle efficiency. Higher γ of monoatomic noble gases (γ = 1.67) would be beneficial in this respect in comparison with CO_2 for example, which has specific heats ratio of about 1.4. From lecture notes, for a given pressure ratio, the cycle power conversion efficiency is given by: $\eta = 1 - r^{(1-\gamma)/\gamma}$.
   - Chemical compatibility with the fuel and structural materials, to avoid corrosion for example.
   - Low neutron absorption cross section is beneficial to avoid radioactive contamination of the power cycle components and also to avoid penalty on fuel enrichment.
   - Gas molecules with multiple atoms should be stable (should not dissociate) at the cycle operating temperatures and in radiation field.

Q4 Fast spectrum reactors

Model Answer

a. Fission rate = Total power/Energy per fission = $2.5 \times 10^9/(2 \times 10^8 \times 1.6 \times 10^{-19})$
$= 7.8 \times 10^{19}$ fissions per second.

b. Assumption: all captures in ^{238}U result in production of ^{239}Pu
Assumption: proportion of different neutron reactions do not change with irradiation and composition of the core.
Net loss of ^{239}Pu = (Fission + Capture) in ^{239}Pu – Capture in ^{238}U
$= 0.2945 + 0.0474 - 0.1040 = 0.2379$.
Net loss of ^{239}Pu per fission $= 0.2379/(0.0062 + 0.0388 + 0.2945) = 0.7007$
Net rate of loss of ^{239}Pu $= 0.7007 \times 7.8 \times 10^{19} = 5.466 \times 10^{19}$ atoms per second
$= 5.466 \times 10^{19} \times 3600 \times 8760 \times 0.8 = 1.38 \times 10^{27}$ atoms per year
$= 1.38 \times 10^{27} \times (239/6.022 \times 10^{26}) = 548$ kg per year

c. Number of ^{240}Pu atoms formed = number of captures in ^{239}Pu.
Number of captures in ^{239}Pu/Number of fissions =
$=0.0474/(0.0062 + 0.0388 + 0.2945) = 0.1396$
Rate of formation of ^{240}Pu $= 7.8 \times 10^{19} \times 0.1396 = 1.09 \times 10^{19}$ atoms per second
$= 1.09 \times 10^{19} \times 3600 \times 8760 \times 0.8 \times (240/6.022 \times 10^{26})$
$= 110$ kg per year.

d. Source strength = number of atoms × (0.6931/half-life) × 2.21
$= 1.09 \times 10^{19} \times 3600 \times 8760 \times 0.8 \times 2.21 \times 0.6931/(6569 \times 8760 \times 3600)$
$= 2.033 \times 10^{15}$ neutrons per second.

e. Source strength of ^{239}Pu
$= 9000 \times 0.218 \times (6.022 \times 10^{26}/239) \times (.6931/24100 \times 8760 \times 3600) \times 2.16$
$= 9.7 \times 10^{15}$ neutrons per second.

f. When the reactor is subcritical the power is proportional to the neutron source strength. If no other neutron sources are present, the ^{240}Pu will make the shutdown power at the end of the year about 20% higher than at the beginning, assuming the shutdown reactivity is the same in both cases.