Version GTP/3

EGT3 / EGT2
ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Friday 29 April 2022   2 to 3.40

Module 4M16

NUCLEAR POWER ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachment: 4M16 Nuclear Power Engineering data sheet (8 pages).
Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) You have been asked to use one-group neutron diffusion theory to investigate the feasibility of a uranium dioxide (UO_2) fuelled, heavy water (D_2O) cooled and moderated reactor. The fuel will be made from natural uranium, in which the isotopic abundance of uranium-235 is 0.00715. A colleague has estimated that the moderator-to-fuel volume ratio should be 16. The density of UO_2 is $10^4\ \mathrm{kg\,m^{-3}}$ and that of D_2O is $1.1\times10^3\ \mathrm{kg\,m^{-3}}$.

Assuming that the macroscopic absorption cross-section of structural materials (e.g. cladding) is $0.026\ \mathrm{m^{-1}}$ and that their volume is negligible compared to that of the fuel and the moderator/coolant, show that the macroscopic fission cross-section of the contents of the core is $0.5437\ \mathrm{m^{-1}}$ and find the macroscopic absorption cross-section of the core contents. Take the atomic weights of natural uranium, oxygen and deuterium to be 238.07 u, 16.00 u and 2.014 u.

Nuclear data: deuterium (D): $\sigma_\mathrm{a} = 6\times10^{-4}$ barns; oxygen (O): $\sigma_\mathrm{a} = 2\times10^{-4}$ barns; U-235: $\sigma_\mathrm{c} = 107$ barns, $\sigma_\mathrm{f} = 580$ barns; U-238: $\sigma_\mathrm{c} = 2.75$ barns, $\sigma_\mathrm{f} = 0$ barns. [30%]

(b) Given that the average number of neutrons released in a U-235 fission reaction (ν) is 2.43, find the value of the parameter η, the average number of neutrons released per neutron absorbed, for this core design and comment on the significance of the result. [10%]

(c) The one-group neutron diffusion equation for a steady-state, source-free, homogeneous system can be written in the form

$$\nabla^2\phi + B^2\phi = 0$$

where ϕ is the neutron flux and $B^2 = (\eta - 1)\Sigma_\mathrm{a}/D$, with Σ_a being the macroscopic absorption cross-section and D the diffusion coefficient. The solution of this equation for a cylindrical geometry reactor of radius R and height H is of the form

$$\phi(r,z) = \phi_0\, J_0(\alpha r)\cos(\beta z)$$

where J_0 is an ordinary zero-order Bessel function and ϕ_0 the flux at the reactor centre.

If extrapolation distances can be neglected, how are the values of parameters α and β related to the geometric (R and H) and neutronic (η, Σ_a and D) properties of the core? [20%]

(d) What will the dimensions of such a reactor be if it is constructed with the core composition specified in (a), $H = 2R$ and the diffusion coefficient $D = 1\ \mathrm{cm}$? In practice, why will the core need to be significantly larger than this, and how could criticality be maintained? [20%]

(e) If the heavy water moderator/coolant is replaced by light water, the macroscopic absorption cross-section of the core contents will increase while the diffusion coefficient *D* will reduce significantly. Explain these changes qualitatively and discuss their implications for the feasibility of a natural uranium fuelled, light water moderated and cooled reactor. [20%]

2 To a good approximation, the equations governing the behaviour of iodine-135 and xenon-135 in a 'lumped' reactor model can be written as

$$\frac{dI}{dt} = \gamma_i \Sigma_f \phi - \lambda_i I$$

$$\frac{dX}{dt} = \lambda_i I - \lambda_x X - \sigma X \phi$$

where all symbols have their usual meanings.

(a) A thermal reactor is operating in steady state with a neutron flux (ϕ_0) of 5×10^{17} m^{-2}s^{-1}. Find the steady-state poisoning effect due to xenon-135. Take the average number of neutrons released in fission (ν) to be 2.43, $\lambda_i = 2.874 \times 10^{-5}$ s^{-1}, $\lambda_x = 2.027 \times 10^{-5}$ s^{-1}, $\gamma_i = 0.061$ and $\sigma = 2.75$ Mbarns. [20%]

(b) After operating for a prolonged period at this flux level, the flux in the reactor is increased rapidly to $2\phi_0$ and then held constant.

(i) Explain why, immediately following the change in flux, the iodine-135 population will start to increase, while the xenon-135 population will start to decrease. [10%]

(ii) Show that the iodine-135 population will vary as

$$I = \frac{\gamma_i \Sigma_f \phi_0}{\lambda_i}\left[2 - \exp(-\lambda_i t)\right]$$

Here, the change in flux is taken to occur at $t = 0$ and is modelled as a step change. [20%]

(iii) Find an expression for the corresponding variation of the xenon-135 population with time after the change in flux. [40%]

(iv) Discuss the implications of these variations for reactor control. [10%]

3 A Pressurized Water Reactor (PWR) has been designed for a three-batch fuel management strategy with partial refuelling every 548 days. A 32-day outage is required to refuel the reactor, so the PWR will operate for 516 days between outages. The PWR's core contains 180 fuel assemblies, and it runs at constant power between outages.

In equilibrium three-batch operation the enrichment of the fresh fuel loaded will be 3.6% uranium-235. The whole-core reactivity at the beginning-of-cycle is proportional to the quantity $(e - 2e_0)$, where e is the enrichment percentage of the fuel and e_0 is the enrichment percentage of natural uranium (0.715%).

It may be assumed that the reactivity of a PWR decreases linearly with burnup during operation.

(a) Show that if the same fuel was used in one-batch operation, the equilibrium cycle length (excluding the outage) would be 1032 days. [5%]

(b) If equilibrium three-batch operation is to be established immediately, what should the enrichments of the initial batches loaded at start-up be? [25%]

(c) The reactor operator is contemplating a change to an operational regime in which the PWR is refuelled every 365 days.

(i) If the fuel design is not to be changed and refuelling outages will last 21 days in the new operational regime, what fuel management batch strategy will be needed to achieve this once steady-state operation has been established? [10%]

(ii) What is the main advantage of this change in refuelling strategy? Are there any disadvantages? [10%]

(d) If the change in operational regime is instituted from steady-state three-batch operation, what will be the lengths (excluding outages) rounded to the nearest day of the first *two* cycles following the change? Comment on the operational and economic implications of this transition. [50%]

4 (a) Describe three methods of treating liquid wastes arising from the production of electricity by nuclear energy. What are the advantages and disadvantages of each method? [30%]

(b) A waste stream arising at a rate of $0.063\,\mathrm{m^3\,hr^{-1}}$ contains $29.8\,\mathrm{Bq\,g^{-1}}$ of strontium-91 and $0.973\,\mathrm{Bq\,g^{-1}}$ of yttrium-91. The relevant decay chain is:

$$^{91}\mathrm{Sr} \xrightarrow[t_{1/2}\,=\,9.5\text{ hours}]{} {}^{91}\mathrm{Y} \xrightarrow[t_{1/2}\,=\,58.5\text{ days}]{} {}^{91}\mathrm{Zr}\text{ (stable)}$$

The waste stream is first collected in a tank over a period of ten days, after which it is passed without further storage through an ion exchanger with a decontamination factor of ten. The density of the effluent is $1000\,\mathrm{kg\,m^{-3}}$.

Calculate the final total specific activity of the effluent and comment on the effectiveness of the process in treating the components of the waste stream. [70%]

END OF PAPER

Data Sheet S/15

Revised March 2012

MODULE 4M16

NUCLEAR POWER ENGINEERING

DATA SHEET

General Data

Speed of light in vacuum	c	$299.792458 \times 10^{6}\ \mathrm{m\,s^{-1}}$
Magnetic permeability in vacuum	μ_0	$4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Planck constant	h	$6.62606957 \times 10^{-34}\ \mathrm{J\,s}$
Boltzmann constant	k	$1.380662 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Elementary charge	e	$1.6021892 \times 10^{-19}\ \mathrm{C}$

Definitions

Unified atomic mass constant	u	$1.6605655 \times 10^{-27}\ \mathrm{kg}$ $(931.5016\ \mathrm{MeV})$
Electron volt	eV	$1.6021892 \times 10^{-19}\ \mathrm{J}$
Curie	Ci	$3.7 \times 10^{10}\ \mathrm{Bq}$
Barn	barn	$10^{-28}\ \mathrm{m^2}$

Atomic Masses and Naturally Occurring Isotopic Abundances (%)

	electron	0.00055 u	90.80%	$^{20}_{10}Ne$	19.99244 u
	neutron	1.00867 u	0.26%	$^{21}_{10}Ne$	20.99385 u
99.985%	$^{1}_{1}H$	1.00783 u	8.94%	$^{22}_{10}Ne$	21.99138 u
0.015%	$^{2}_{1}H$	2.01410 u	10.1%	$^{25}_{12}Mg$	24.98584 u
0%	$^{3}_{1}H$	3.01605 u	11.1%	$^{26}_{12}Mg$	25.98259 u
0.0001%	$^{3}_{2}He$	3.01603 u	0%	$^{32}_{15}P$	31.97391 u
99.9999%	$^{4}_{2}He$	4.00260 u	96.0%	$^{32}_{16}S$	31.97207 u
7.5%	$^{6}_{3}Li$	6.01513 u	0%	$^{60}_{27}Co$	59.93381 u
92.5%	$^{7}_{3}Li$	7.01601 u	26.2%	$^{60}_{28}Ni$	59.93078 u
0%	$^{8}_{4}Be$	8.00531 u	0%	$^{87}_{35}Br$	86.92196 u
100%	$^{9}_{4}Be$	9.01219 u	0%	$^{86}_{36}Kr$	85.91062 u
18.7%	$^{10}_{5}B$	10.01294 u	17.5%	$^{87}_{36}Kr$	86.91337 u
0%	$^{11}_{6}C$	11.01143 u	12.3%	$^{113}_{48}Cd$	112.90461 u
98.89%	$^{12}_{6}C$	12.00000 u		$^{226}_{88}Ra$	226.02536 u
1.11%	$^{13}_{6}C$	13.00335 u		$^{230}_{90}Th$	230.03308 u
0%	$^{13}_{7}N$	13.00574 u	0.72%	$^{235}_{92}U$	235.04393 u
99.63%	$^{14}_{7}N$	14.00307 u	0%	$^{236}_{92}U$	236.04573 u
0%	$^{14}_{8}O$	14.00860 u	99.28%	$^{238}_{92}U$	238.05076 u
99.76%	$^{16}_{8}O$	15.99491 u	0%	$^{239}_{92}U$	239.05432 u
0.04%	$^{17}_{8}O$	16.99913 u		$^{239}_{93}Np$	239.05294 u
0.20%	$^{18}_{8}O$	17.99916 u		$^{239}_{94}Pu$	239.05216 u
				$^{240}_{94}Pu$	240.05397 u

Simplified Disintegration Patterns

Isotope	$^{60}_{27}Co$	$^{90}_{38}Sr$	$^{90}_{39}Yt$	$^{137}_{55}Cs$	$^{204}_{81}Tl$
Type of decay	β^-	β^-	β^-	β^-	β^-
Half life	5.3 yr	28 yr	64 h	30 yr	3.9 yr
Total energy	2.8 MeV	0.54 MeV	2.27 MeV	1.18 MeV	0.77 MeV
Maximum β energy	0.3 MeV (100%)	0.54 MeV (100%)	2.27 MeV (100%)	0.52 MeV (96%) 1.18 MeV (4%)	0.77 MeV (100%)
γ energies	1.17 MeV (100%) 1.33 MeV (100%)	None	None	0.66 MeV (96%)	None

Thermal Neutron Cross-sections (in barns)

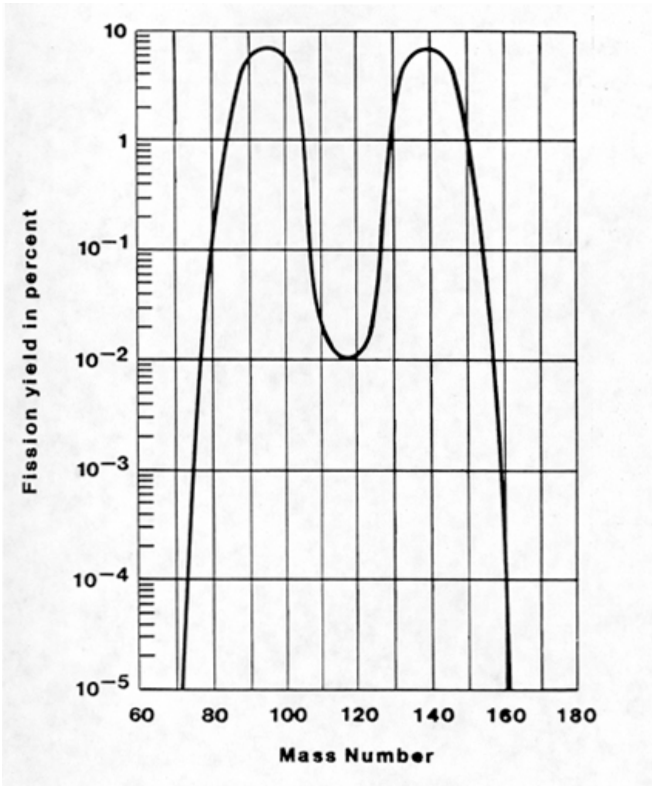
	"Nuclear" graphite	$^{16}_{8}O$	$^{113}_{48}Cd$	$^{235}_{92}U$	$^{238}_{92}U$	$^{1}_{1}H$ unbound
Fission	0	0	0	580	0	0
Capture	4×10^{-3}	10^{-4}	27×10^{3}	107	2.75	0.332
Elastic scatter	4.7	4.2		10	8.3	38

Densities and Mean Atomic Weights

	"Nuclear" graphite	Aluminium Al	Cadmium Cd	Gold Au	Uranium U
Density / $kg\,m^{-3}$	1600	2700	8600	19000	18900
Atomic weight	12	27	112.4	196	238

Fission Product Yield

Nuclei with mass numbers from 72 to 158 have been identified, but the most probable split is unsymmetrical, into a nucleus with a mass number of about 138 and a second nucleus that has a mass number between about 95 and 99, depending on the target.



The primary fission products decay by β^- emission. Some important decay chains (with relevant half lives) from thermal-neutron fission of U-235 are:

Kr-90 $\xrightarrow[33\ \text{s}]{}$ Rb-90 $\xrightarrow[2.7\ \text{min}]{}$ Sr-90 $\xrightarrow[28\ \text{yr}]{}$ Y-90 $\xrightarrow[64\ \text{hr}]{}$ Zr-90

Sr-90 is a serious health hazard, because it is bone-seeking.

Sb-131 $\xrightarrow[23\ \text{min}]{}$ Te-131 $\xrightarrow[24\ \text{min}]{}$ I-131 $\xrightarrow[8\ \text{days}]{}$ Xe-131

I-131 is a short-lived health hazard. It is thyroid-seeking.

I-135 $\xrightarrow[6.7\ \text{hr}]{}$ Xe-135 $\xrightarrow[9.2\ \text{hr}]{}$ Cs-135 (nearly stable) Fission yield $\gamma = 0.064$

Xe-135 is a strong absorber of thermal neutrons, with $\sigma_a = 3.5$ Mbarn .

Pm-149 $\xrightarrow[54\ \text{hr}]{}$ Sm-149 Fission yield $\gamma = 0.014$

Sm-149 is a strong absorber of thermal neutrons, with $\sigma_a = 53$ kbarn .

Se-87 $\xrightarrow[17\ \text{s}]{}$ Br-87 $\xrightarrow[55\ \text{s}]{}$ either {Kr-86 + n} or {Kr-87 $\xrightarrow[78\ \text{min}]{}$ Rb-87}

This chain leads to a “delayed neutron”.

Neutrons

Most neutrons are emitted within 10^{-13} s of fission, but some are only emitted when certain fission products, e.g. Br-87, decay.

The total yield of neutrons depends on the target and on the energy of the incident neutron. Some key values are:

Target nucleus	Fission induced by			
	Thermal neutron		Fast neutron	
	ν	η	ν	η
U-233	2.50	2.29	2.70	2.45
U-235	2.43	2.07	2.65	2.30
U-238	—	—	2.55	2.25
Pu-239	2.89	2.08	3.00	2.70

ν = number of neutrons emitted per fission

η = number of neutrons emitted per neutron absorbed

Delayed Neutrons

A reasonable approximation for thermal-neutron fission of U-235 is:

Precursor half life / s	55	22	5.6	2.1	0.45	0.15	
Mean life time of precursor ($1/\lambda_i$) / s	80	32	8.0	3.1	0.65	0.22	Total
Number of neutrons produced per 100 fission neutrons ($100\,\beta_i$)	0.03	0.18	0.22	0.23	0.07	0.02	0.75

Fission Energy

Kinetic energy of fission fragments	167 ± 5 MeV
Prompt γ-rays	6 ± 1 MeV
Kinetic energy of neutrons	5 MeV
Decay of fission products β	8 ± 1.5 MeV
γ	6 ± 1 MeV
Neutrinos (not recoverable)	12 ± 2.5 MeV
Total energy per fission	204 ± 7 MeV

Subtract neutrino energy and add neutron capture energy ⇒ ~200 MeV / fission

Nuclear Reactor Kinetics

Name	*Symbol*	*Concept*
Effective multiplication factor	k_{eff}	$\frac{\text{production}}{\text{removal}} = \frac{P}{R}$
Excess multiplication factor	k_{ex}	$\frac{P-R}{R} = k_{eff} - 1$
Reactivity	ρ	$\frac{P-R}{P} = \frac{k_{ex}}{k_{eff}}$
Lifetime	l	$\frac{1}{R}$
Reproduction time	Λ	$\frac{1}{P}$

Reactor Kinetics Equations

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c + s$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where $n =$ neutron concentration

$c =$ precursor concentration

$\beta =$ delayed neutron precursor fraction $= \sum \beta_i$

$\lambda =$ average precursor decay constant

Neutron Diffusion Equation

$$\frac{dn}{dt} = -\nabla . \underline{j} + (\eta - 1)\Sigma_{\mathrm{a}} \phi + S$$

where $\underline{j} = -D\nabla\phi$ (Fick's Law)

$D = \dfrac{1}{3\Sigma_{\mathrm{s}}(1 - \overline{\mu})}$

with $\overline{\mu} =$ the mean cosine of the angle of scattering

Laplacian ∇^2

Slab geometry: $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Cylindrical geometry: $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

Spherical geometry: $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \psi^2}$

Bessel's Equation of 0th Order

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+R=0$$

Solution is: $$R(r)=A_1J_0(r)+A_2Y_0(r)$$

$J_0(0)=1;\ Y_0(0)=-\infty;$

The first zero of $J_0(r)$ is at $r=2.405$.

$J_1(2.405)=0.5183$, where $J_1(r)=\frac{1}{r}\int_0^r xJ_0(x)dx$.

Diffusion and Slowing Down Properties of Moderators

Moderator	Density $\mathrm{g\,cm^{-3}}$	Σ_a $\mathrm{cm^{-1}}$	D cm	$L^2=D/\Sigma_a$ $\mathrm{cm^2}$
Water	1.00	22×10^{-3}	0.17	$(2.76)^2$
Heavy Water	1.10	85×10^{-6}	0.85	$(100)^2$
Graphite	1.70	320×10^{-6}	0.94	$(54)^2$

In-core Fuel Management Equilibrium Cycle Length Ratio

For M-batch refueling: $$\theta=\frac{T_M}{T_1}=\frac{2}{M+1}$$

Enrichment of Isotopes

Value function: $$v(x)=(2x-1)\ln\left(\frac{x}{1-x}\right)\approx-\ln(x)\text{ for small } x$$

For any counter-current cascade at low enrichment:

Enrichment section reflux ratio: $$R_n\equiv\frac{L''_n}{P}=\frac{x_p-x'_{n+1}}{x'_{n+1}-x''_n}$$

Stripping section reflux ratio: $$R_n=\left[\frac{x_p-x_f}{x_f-x_w}\right]\left[\frac{x'_{n+1}-x_w}{x'_{n+1}-x''_n}\right]$$

Bateman's Equation

$$N_i = \lambda_1 \lambda_2 \ldots \lambda_{i-1} P \sum_{j=1}^{i} \frac{[1-\exp(-\lambda_j T)]\exp(-\lambda_j \tau)}{\lambda_j \prod_{\substack{k=1 \\ k \neq j}}^{i} (\lambda_k - \lambda_j)}$$

where N_i = number of atoms of nuclide i T = filling time

λ_j = decay constant of nuclide j τ = decay hold-up time after filling

P = parent nuclide production rate

Temperature Distribution

For axial coolant flow in a reactor with a chopped cosine power distribution, Ginn's equation for the non-dimensional temperature is:

$$\theta = \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\frac{\pi L}{2L'}\right) = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

where L = fuel half-length

L' = flux half-length

$T_{c1/2}$ = coolant temperature at mid-channel

T_{co} = coolant temperature at channel exit

$Q = \frac{\pi \dot{m} c_p}{UA} \frac{L}{L'}$

with $\dot{m}$ = coolant mass flow rate

c_p = coolant specific heat capacity (assumed constant)

$A = 4\pi r_o L$ = surface area of fuel element

and for radial fuel geometry:

$$\frac{1}{U} = \underbrace{\frac{1}{h}}_{\text{bulk coolant}} + \underbrace{\frac{1}{h_s}}_{\text{scale}} + \underbrace{\frac{t_c}{\lambda_c}}_{\text{thin clad}} + \underbrace{\frac{r_o}{h_b r_i}}_{\text{bond}} + \underbrace{\frac{r_o}{2\lambda_f}\left(1 - \frac{r^2}{r_i^2}\right)}_{\text{fuel pellet}}$$

with h = heat transfer coefficient to bulk coolant

h_s = heat transfer coefficient of any scale on fuel cladding

t_c = fuel cladding thickness (assumed thin)

λ_c = fuel cladding thermal conductivity

r_o = fuel cladding outer radius

r_i = fuel cladding inner radius = fuel pellet radius

h_b = heat transfer coefficient of bond between fuel pellet and cladding

λ_f = fuel pellet thermal conductivity

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Answers

Q1 (a) $1.0323\ \text{m}^{-1}$

(b) 1.280

(c) $\alpha = \dfrac{2.405}{R}$; $\beta = \dfrac{\pi}{H}$; $\left(\dfrac{2.405}{R}\right)^2 + \left(\dfrac{\pi}{H}\right)^2 = \dfrac{(\eta - 1)\Sigma_{\text{a}}}{D}$

(d) $R = 0.534\ \text{m}$; $H = 1.069\ \text{m}$

Q2 (a) -0.0219

(b)(iii) $X = \dfrac{2\gamma_i \Sigma_f \phi_0}{\lambda_{\text{eff}}} + \dfrac{\gamma_i \Sigma_f \phi_0}{\lambda_i - \lambda_{\text{eff}}} \exp(-\lambda_i t) + \left[\dfrac{\gamma_i \Sigma_f \phi_0}{\lambda_x + \sigma\phi_0} - \dfrac{2\gamma_i \Sigma_f \phi_0}{\lambda_{\text{eff}}} - \dfrac{\gamma_i \Sigma_f \phi_0}{\lambda_i - \lambda_{\text{eff}}}\right] \exp(-\lambda_{\text{eff}} t)$

where $\lambda_{\text{eff}} = \lambda_x + 2\sigma\phi_0$

Q3 (b) 3.6%, 2.515%, 1.43%

(c)(i) 5-batch

(d) 310 days, 337 days

Q4 (b) $0.280\ \text{Bq}\,\text{g}^{-1}$