Q1

(a)   The M:F volume ratio is specified as 16, therefore in $1 \ \mathrm{m}^3$

$$V_\mathrm{F} = \frac{1}{17} = 0.05882 \ \mathrm{m}^3 \quad \text{and} \quad V_\mathrm{M} = \frac{16}{17} = 0.94118 \ \mathrm{m}^3$$

Atomic weight of UO_2 molecule: $A_{UO_2} = 238.07 + 2 \times 16.00 = 270.07 \ \mathrm{u}$

Atomic weight of D_2O molecule: $A_{D_2O} = 2 \times 2.014 + 16.00 = 20.028 \ \mathrm{u}$

Therefore the number of UO_2 molecules per unit volume [1.661×10^{-27} is the conversion factor between u and kg] is

$$N_{UO_2} = \frac{V_\mathrm{F} \rho_{UO_2}}{1.661 \times 10^{-27} A_{UO_2}} = \frac{0.05882 \times 10^4}{1.661 \times 10^{-27} \times 270.07} = 1.311 \times 10^{27}$$

Of these 0.715% are molecules containing U-235 atoms and the rest (99.285%) contain U-238 atoms.

The number of D_2O molecules per unit volume is

$$N_{D_2O} = \frac{V_\mathrm{M} \rho_{D_2O}}{1.661 \times 10^{-27} A_{D_2O}} = \frac{0.94118 \times 1.1 \times 10^3}{1.661 \times 10^{-27} \times 20.028} = 3.112 \times 10^{28}$$

$$\Sigma_\mathrm{a} = \sum_i \Sigma_{a_i} = \sum_i N_i (\sigma_{c_i} + \sigma_{f_i})$$

$$\therefore \quad \Sigma_{aU} = 1.311 \times 10^{27} \{0.00715 \times (107 + 580) + 0.99285(2.75 + 0)\} \times 10^{-28} = 1.00192 \ \mathrm{m}^{-1}$$

$$\therefore \quad \Sigma_{aO_F} = 1.311 \times 10^{27} \{2 \times 2 \times 10^{-4}\} \times 10^{-28} = 5.244 \times 10^{-5} \ \mathrm{m}^{-1}$$

$$\therefore \quad \Sigma_{aD_2O} = 3.112 \times 10^{28} \{2 \times 6 \times 10^{-4} + 2 \times 10^{-4}\} \times 10^{-28} = 4.3568 \times 10^{-3} \ \mathrm{m}^{-1}$$

$$\Sigma_\mathrm{a} = \Sigma_{aU} + \Sigma_{aO_F} + \Sigma_{aD_2O} + \Sigma_\mathrm{clad}$$

$$\therefore \quad \Sigma_\mathrm{a} = 1.00192 + 5.244 \times 10^{-5} + 4.3568 \times 10^{-3} + 0.026 = 1.0323 \ \mathrm{m}^{-1}$$

Only U-235 is fissile

$$\therefore \quad \Sigma_\mathrm{f} = 1.311 \times 10^{27} \times 0.00715 \times 580 \times 10^{-28} = 0.5437 \ \mathrm{m}^{-1} \qquad \text{[30\%]}$$

(b) $\eta = \dfrac{\nu \Sigma_\mathrm{f}}{\Sigma_\mathrm{a}} = \dfrac{2.43 \times 0.5437}{1.0323} = 1.280$

For the lattice design to be viable, η needs to be greater than 1. This value is above 1 but not very much above (η approaches 2 in PWRs and BWRs), so the design should be viable but the reactor will need frequent refuelling as Σ_f reduces during operation. [10%]

(c)   The values of α and β are determined by the boundary conditions for the flux solution. If extrapolation distances can be neglected, these are that $\phi = 0$ at $r = R$ and at $z = \pm \dfrac{H}{2}$.

The first of these implies that $J_0(\alpha R) = 0$

$$\therefore \quad \alpha R = 2.405 \quad \Rightarrow \quad \alpha = \frac{2.405}{R}$$

The second implies that
$$\cos\left(\frac{\beta H}{2}\right) = 0$$

$$\therefore \quad \frac{\beta H}{2} = \frac{\pi}{2} \quad \Rightarrow \quad \beta = \frac{\pi}{H}$$

Finally, for a steady-state solution to exist, the criticality condition

$$\alpha^2 + \beta^2 = B^2$$

must be met. Thus

$$\left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 = \frac{(\eta - 1)\Sigma_a}{D} \qquad [20\%]$$

(d) If $H = 2R$
$$\left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{2R}\right)^2 = \frac{8.251}{R^2} = \frac{(\eta - 1)\Sigma_a}{D}$$

Using values found in (a) and (b)

$$B^2 = \frac{(\eta - 1)\Sigma_a}{D} = \frac{(1.280 - 1) \times 1.0323}{0.01} = 28.90 \text{ m}^{-2}$$

$$\therefore \quad R^2 = \frac{8.251}{28.90} \quad \Rightarrow \quad R = 0.534 \text{ m}$$

$$\therefore \quad H = 2R = 1.069 \text{ m}$$

A reactor built and operated to this design would only very briefly be able to maintain criticality. Fissile material will deplete as the reactor operates and thus Σ_f will decrease, taking the reactor subcritical. In addition, reactor poisons, like xenon-135, will accumulate, increasing Σ_a and also reducing criticality.

If the reactor is substantially above minimum critical size, it will have initial excess reactivity that will enable it to operate for longer before refuelling is required. Criticality during operation can be maintained through the use of control poisons (e.g. control rods) to compensate the excess reactivity due to the fuel.

A batch refuelling strategy will also help criticality to be maintained (and reduce the burden on the control system), indeed the CANDU reactor (which is D_2O cooled and moderated and uses natural uranium fuel) is refuelled online and thus, in effect, uses a batch refuelling scheme with a number of batches equal to the number of fuel channels. [20%]

(e) The capture cross-section of H_2O is *much* larger than that of D_2O (the hydrogen nuclei in H_2O can readily accept another neutron). Thus, the change of coolant/moderator will lead to an increase in the macroscopic absorption cross-section of the core contents.

The scattering cross-section of H_2O is also larger than that of D_2O. Thus, the change of moderator increases Σ_s and therefore reduces D, which is inversely proportional to Σ_s.

Even if the M:F ratio can be reduced thanks to the more effective moderation provided by H_2O, the increase in Σ_a means that η will be less than 1, and it will not be possible to establish criticality in a reactor with this core composition however large the reactor. For a light water cooled and moderated reactor to be feasible η must be increased by enriching the fuel to increase the proportion that is fissile. [20%]

Assessor's Comments:

All candidates: 82 attempts, Average raw mark 10.3/20, Maximum 16, Minimum 0.

A popular question attempted by 84.5% of candidates.

Part (a) proved surprisingly difficult with many candidates failing: to account for the molecules (UO_2 and D_2O) correctly in finding number densities; to account for the moderator-fuel ratio; to recall that absorption includes fission (as well as capture); to account for the contribution to the absorption cross-section of the moderator; to recognise that the number densities used in calculating macroscopic cross-sections are 'per unit volume' quantities.

Many answers to part (b) showed a lack of appreciation of the significance of η being close to 1. Many candidates wasted (significant amounts of) time deriving the solution given in part (c). A fairly common error in part (c) was a failure to recognise the symmetry in z in the given solution, resulting in the imposition of an incorrect boundary condition and thus an incorrect expression for β.

In answering part (d) the vast majority of candidates incorrectly stated that the core would need to be larger to account for neutron leakage – the neutron diffusion equation **does** account for leakage. Few appreciated that a minimum volume critical reactor will become subcritical almost immediately because of the depletion of fissile material and the build-up of poisons like xenon-135. In answering part (e) many candidates failed to recognise that η would change if the moderator/ coolant was changed and only considered the impact of changes to D and Σ_a.

Q2

(a)   In steady state $\dfrac{dI}{dt} = 0$ and $\dfrac{dX}{dt} = 0$. Denote the equilibrium iodine-135 and xenon-135 populations I_0 and X_0.

$$\frac{dI}{dt} = \gamma_i \Sigma_f \phi_0 - \lambda_i I_0 = 0 \quad \Rightarrow \quad \lambda_i I_0 = \gamma_i \Sigma_f \phi_0$$

$$\therefore \quad I_0 = \frac{\gamma_i \Sigma_f \phi_0}{\lambda_i}$$

$$\frac{dX}{dt} = \lambda_i I_0 - \lambda_x X_0 - \sigma X_0 \phi_0 = 0 \quad \Rightarrow \quad \lambda_x X_0 + \sigma X_0 \phi_0 = \lambda_i I_0$$

$$\therefore \quad X_0 = \frac{\lambda_i I_0}{\lambda_x + \sigma \phi_0} = \frac{\gamma_i \Sigma_f \phi_0}{\lambda_x + \sigma \phi_0}$$

The poisoning effect of X-135 is given by $\rho_X = -\dfrac{\sigma X}{\nu \Sigma_f}$

$$\therefore \quad \rho_{X_0} = -\frac{\sigma X_0}{\nu \Sigma_f} = -\frac{\sigma}{\nu \Sigma_f} \frac{\gamma_i \Sigma_f \phi_0}{\lambda_x + \sigma \phi_0} = -\frac{\gamma_i \sigma \phi_0}{\nu(\lambda_x + \sigma \phi_0)}$$

$$\therefore \quad \rho_{X_0} = -\frac{0.061 \times 2.75 \times 10^{-22} \times 5 \times 10^{17}}{2.43 \times (2.027 \times 10^{-5} + 2.75 \times 10^{-22} \times 5 \times 10^{17})} = -0.02188 \qquad \text{[20%]}$$

(b)

(i)   Consider $\dfrac{dI}{dt} = \gamma_i \Sigma_f \phi - \lambda_i I$. Immediately following the step change in flux, the production rate of iodine-135 (the first term on the right-hand side) will increase because the flux (and therefore fission rate) has increased but the removal rate (the second term on the right-hand

side) will not change significantly (until the iodine population has started to grow). Thus the iodine-135 population increases.

Consider $\dfrac{dX}{dt} = \lambda_i I - \lambda_x X - \sigma X \phi$. Immediately following the step change in flux, the production rate through the decay of iodine-135 (the first term on the right-hand side) and the removal rate through the decay of xenon-135 (the second term on the right-hand side) will not change significantly (neither population has had time to change) but the removal rate through neutron capture and transmutation (the last term on the right-hand side) will in effect double following the step change in flux. Thus the xenon-135 population decreases initially.   [10%]

(ii)   Following the change in flux, the variation of the iodine-135 population is governed by

$$\frac{dI}{dt} = 2\gamma_i \Sigma_f \phi_0 - \lambda_i I$$

$$\therefore \quad \frac{dI}{dt} + \lambda_i I = 2\gamma_i \Sigma_f \phi_0$$

By inspection, the particular integral is

$$I_{\mathrm{PI}} = \frac{2\gamma_i \Sigma_f \phi_0}{\lambda_i}$$

Also by inspection, the complementary function is

$$I_{\mathrm{CF}} = A \exp(-\lambda_i t)$$

Therefore the general solution is

$$I = \frac{2\gamma_i \Sigma_f \phi_0}{\lambda_i} + A \exp(-\lambda_i t)$$

The boundary condition is that at $t = 0$, $I = I_0 = \dfrac{\gamma_i \Sigma_f \phi_0}{\lambda_i}$ as found in (a).

$$\therefore \quad \frac{\gamma_i \Sigma_f \phi_0}{\lambda_i} = \frac{2\gamma_i \Sigma_f \phi_0}{\lambda_i} + A \quad \Rightarrow \quad A = -\frac{\gamma_i \Sigma_f \phi_0}{\lambda_i}$$

$$\therefore \quad I = \frac{2\gamma_i \Sigma_f \phi_0}{\lambda_i} - \frac{\gamma_i \Sigma_f \phi_0}{\lambda_i} \exp(-\lambda_i t) = \frac{\gamma_i \Sigma_f \phi_0}{\lambda_i} \left[2 - \exp(-\lambda_i t) \right] \qquad\qquad [20\%]$$

(iii) Following the change in flux, the variation of the xenon-135 population is governed by

$$\frac{dX}{dt} = \lambda_i I - \lambda_x X - 2\sigma X \phi_0$$

$$\therefore \quad \frac{dX}{dt} + \left[\lambda_x + 2\sigma\phi_0 \right] X = \lambda_i I$$

$$\therefore \quad \frac{dX}{dt} + \lambda_{eff} X = \lambda_i I$$

where $\lambda_{eff} = \lambda_x + 2\sigma\phi_0$

Substituting the result for I from (ii)

$$\therefore \quad \frac{dX}{dt} + \lambda_{eff} X = \gamma_i \Sigma_f \phi_0 \left[2 - \exp(-\lambda_i t) \right]$$

By inspection, the particular integral will be of the form

$$X_{\mathrm{PI}} = A + B\exp(-\lambda_i t)$$

$$\therefore \quad -\lambda_i B\exp(-\lambda_i t) + \lambda_{eff}\left[A + B\exp(-\lambda_i t)\right] = \gamma_i \Sigma_f \phi_0 \left[2 - \exp(-\lambda_i t)\right]$$

$$\therefore \quad \lambda_{eff} A = 2\gamma_i \Sigma_f \phi_0 \quad \Rightarrow \quad A = \frac{2\gamma_i \Sigma_f \phi_0}{\lambda_{eff}} \left[= \frac{2\gamma_i \Sigma_f \phi_0}{\lambda_x + 2\sigma\phi_0}\right]$$

and $\quad \therefore \quad -\lambda_i B + \lambda_{eff} B = -\gamma_i \Sigma_f \phi_0 \quad \Rightarrow \quad B = \frac{\gamma_i \Sigma_f \phi_0}{\lambda_i - \lambda_{eff}} \left[= \frac{\gamma_i \Sigma_f \phi_0}{\lambda_i - \lambda_x - 2\sigma\phi_0}\right]$

Also by inspection, the complementary function is

$$X_{\mathrm{CF}} = C\exp(-\lambda_{eff}t)$$

Therefore the general solution is

$$X = \frac{2\gamma_i \Sigma_f \phi_0}{\lambda_{eff}} + \frac{\gamma_i \Sigma_f \phi_0}{\lambda_i - \lambda_{eff}}\exp(-\lambda_i t) + C\exp(-\lambda_{eff}t)$$

The boundary condition is that at $t = 0$, $X = X_0 = \dfrac{\gamma_i \Sigma_f \phi_0}{\lambda_x + \sigma\phi_0}$ as found in (a)

$$\therefore \quad \frac{\gamma_i \Sigma_f \phi_0}{\lambda_x + \sigma\phi_0} = \frac{2\gamma_i \Sigma_f \phi_0}{\lambda_{eff}} + \frac{\gamma_i \Sigma_f \phi_0}{\lambda_i - \lambda_{eff}} + C$$

$$\therefore \quad C = \frac{\gamma_i \Sigma_f \phi_0}{\lambda_x + \sigma\phi_0} - \frac{2\gamma_i \Sigma_f \phi_0}{\lambda_{eff}} - \frac{\gamma_i \Sigma_f \phi_0}{\lambda_i - \lambda_{eff}}$$

$$\therefore \quad X = \frac{2\gamma_i \Sigma_f \phi_0}{\lambda_{eff}} + \frac{\gamma_i \Sigma_f \phi_0}{\lambda_i - \lambda_{eff}}\exp(-\lambda_i t) + \left[\frac{\gamma_i \Sigma_f \phi_0}{\lambda_x + \sigma\phi_0} - \frac{2\gamma_i \Sigma_f \phi_0}{\lambda_{eff}} - \frac{\gamma_i \Sigma_f \phi_0}{\lambda_i - \lambda_{eff}}\right]\exp(-\lambda_{eff}t) \qquad [40\%]$$

(iv)   The time variation of the xenon-135 population means that the poisoning effect due to the xenon also changes with time. If criticality is to be maintained, as implied by the desire for constant flux, then this means that control action (e.g. the moving of control rods) is necessary to compensate for the change in poisoning. In this particular case the xenon-135 population initially decreases (as discussed in (i)) before rising to its new equilibrium level (which is higher than at the start of the transient, since the steady-state poisoning effect [as analysed in (a)] is a monotonic function of flux level – the higher the flux the greater the poisoning effect). [10%]

Assessor's Comments:

All candidates: 95 attempts, Average raw mark 12.6/20, Maximum 19, Minimum 0.

The most popular question attempted by all but two candidates.
Part (a) was generally done well as long as candidates could recall the definition of the xenon poisoning effect (or derive it). A few candidates contrived to get the wrong numerical answer but inserting the value of γ_i rather than λ_i into a correct expression.
Answers to part (b)(i) often failed to consider with sufficient clarity the implications of the lack of initial change in the iodine and xenon populations.
The success of answers to parts (b)(ii) and (iii) inevitably depended heavily on the candidate's ability to solve the differential equations in question.

Many answers to (b)(iv) discussed the impact of xenon poisoning when a reactor is shut down, rather than answering the question actually asked. Few candidates appreciated the significance of the flux (and hence power) being required to be constant during the specified transient and the reactor control implications of this.
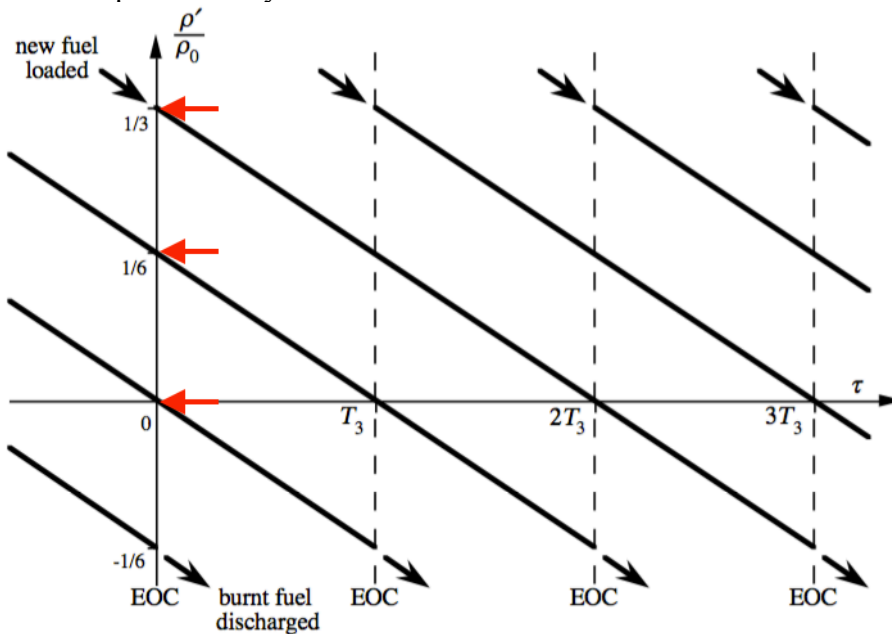
Q3

(a)   The question indicates that the linear reactivity model, $\rho_0[1 - \tau/T_1]$, where τ = burnup and T_1 is the cycle length (in units of burnup) for one-batch operation, can be used. As the reactor power is constant, burnup is proportional to time, so work in units of time.

From page 7 of the 4M16 data sheet:

$$\frac{T_M}{T_1} = \frac{2}{M+1}$$

$$\therefore \quad T_1 = \frac{(3+1)T_3}{2} = 2T_3 = 2 \times 516 = 1032 \text{ days} \qquad\qquad [5\%]$$

(b)   The three-batch equilibrium cycle is:

To establish equilibrium operation immediately, three different initial batches are needed in the start-up core with reactivities as indicated by the red arrows. These are the reactivities of a fresh, once-burnt and twice-burnt batch.

Thus, the *first batch* to be removed should have an initial reactivity = 0. As $\rho \propto (e - 2e_0)$,

$$\therefore \quad e_1 = 2e_0 = 2 \times 0.715 = 1.43\% \text{ U-235}$$

The *second batch* to be removed should have an initial reactivity which is half that of the equilibrium operation fuel,

$$\therefore \quad e_2 - 2e_0 = \tfrac{1}{2}(e_3 - 2e_0) = \tfrac{1}{2}(3.6 - 2 \times 0.715) = 1.085$$

$$\therefore \quad e_2 = 1.085 + 2 \times 0.715 = 2.515\% \text{ U-235}$$

And the *third batch* to be removed should have the same initial reactivity as the equilibrium operation fuel (and therefore the same enrichment), i.e. $e_3 = 3.6\%$ U-235.   [25%]

(c)

(i)   For an annual cycle with 21-day outages, $T_M = 365 - 21 = 344 \text{ days}$.

$$\frac{T_M}{T_1} = \frac{2}{M+1} \quad \Rightarrow \quad M+1 = \frac{2T_1}{T_M}$$

$$\therefore \quad M = \frac{2T_1}{T_M} - 1 = \frac{2 \times 1032}{344} - 1 = 5$$

i.e. a five-batch strategy.   [10%]

(ii)   In three-batch operation, the availability of the reactor (proportion of time at power) is

$$A_3 = \frac{516}{548} = 0.942$$

In five-batch operation, the availability is

$$A_3 = \frac{344}{365} = 0.942$$

So, availability is unaffected by the change in operational regime.

The discharge burnup of the fuel $B_M = MT_M$, so this change in operational regime will increase the discharge burnup by a factor of

$$\frac{5T_5}{3T_3} = \frac{5 \times 344}{3 \times 516} = 1.111$$

i.e. an 11% increase. This is the main advantage of the change.

An annual cycle also enables the reactor to be refuelled at a point in the year when electricity demand (and therefore price) is lowest, thus minimising the opportunity cost associated with the refuelling outage.

One might anticipate that the availability would decrease with a change from three- to five-batch operation, but the calculations above show that the availability is unchanged, so there is no significant disadvantage in this case.   [10%]

(d)   As the reactor contains 180 assemblies, in three-batch operation each batch consists of 60 assemblies, while in five-batch operation each batch consists of 36 assemblies.

The reactivity of each batch varies as $\rho_0[1 - t/T_1]$, where t is the total burnup of the batch. As the fuel design (enrichment) is unchanged, ρ_0 is unchanged.

Immediately after refuelling the first five-batch cycle the reactor contains:

     36 fresh assemblies
     60 assemblies with 516 days' burnup
     60 assemblies with $2 \times 516 = 1032$ days' burnup and
     $180 - 60 - 60 - 36 = 24$ assemblies with $3 \times 516 = 1548$ days' burnup.

Using the partial reactivity model (PRM), at the end-of-cycle (EOC):

$$\frac{36}{180}\left[1 - \frac{\tau_1}{T_1}\right] + \frac{60}{180}\left[1 - \frac{\tau_1 + 516}{T_1}\right] + \frac{60}{180}\left[1 - \frac{\tau_1 + 1032}{T_1}\right] + \frac{24}{180}\left[1 - \frac{\tau_1 + 1548}{T_1}\right] = 0$$

$$\therefore \quad \frac{\tau_1}{T_1} = 1 - \frac{1}{3} \times \frac{516}{T_1} - \frac{1}{3} \times \frac{1032}{T_1} - \frac{2}{15} \times \frac{1548}{T_1}$$

$$\therefore \quad \tau_1 = T_1 - 172 - 344 - 206.4 = 1032 - 722.4 = 309.6 \text{ days, i.e. } 310 \text{ days}$$

Thus, immediately after refuelling the second five-batch cycle the reactor contains:

       36 fresh assemblies
       36 assemblies with 310 days' burnup
       60 assemblies with $516 + 310 = 826$ days' burnup and
       $180 - 60 - 36 - 36 = 48$ assemblies with $1032 + 310 = 1342$ days' burnup.

Using the PRM again, at EOC:

$$\frac{36}{180}\left[1 - \frac{\tau_2}{T_1}\right] + \frac{36}{180}\left[1 - \frac{\tau_2 + 310}{T_1}\right] + \frac{60}{180}\left[1 - \frac{\tau_2 + 826}{T_1}\right] + \frac{48}{180}\left[1 - \frac{\tau_2 + 1342}{T_1}\right] = 0$$

$$\therefore \quad \frac{\tau_2}{T_1} = 1 - \frac{1}{5} \times \frac{310}{T_1} - \frac{1}{3} \times \frac{826}{T_1} - \frac{4}{15} \times \frac{1342}{T_1}$$

$$\therefore \quad \tau_2 = T_1 - 62 - 275.3 - 357.9 = 1032 - 695.2 = 336.8 \text{ days, i.e. } 337 \text{ days}$$

When five-batch equilibrium operation is established the cycle length (without outage) will be $365 - 21 = 344$ days. So, these first two transition cycles are not too far off this and getting closer. There may be some economic penalty associated with scheduling the outages at non-optimal times; ideally they should be scheduled at times when the electricity price is low so that lost revenue is minimized.

The discharge burnup in equilibrium five-batch operation is equivalent to $5 \times 344 = 1720$ days of full power operation. The batch-average maximum burnup accumulated by fuel discharged after the first transition cycle is $1548 + 310 = 1858$ days' worth. The batch-average maximum burnup accumulated by fuel discharged after the second transition cycle is $1342 + 337 = 1679$ days' worth. As there are 48 such assemblies and only 36 are discharged, 12 of these will continue into the third transition cycle. Their discharge burnup is therefore likely to exceed 2000 days' worth. It is possible that this will exceed the irradiation limit of the fuel. This is certainly an issue that will need to be monitored.   [50%]

Assessor's Comments:

All candidates: 81 attempts, Average raw mark 9.8/20,   Maximum 18, Minimum 0.

A popular question, attempted by 83.5% of candidates.
Many answers to part (b) incorrectly identified the target reactivities needed for the start-up core; others made a meal of finding the end-of-cycle reactivities in equilibrium and thus wasted valuable time.
Many answers to part (c) incorrectly assumed availability would be reduced by change from 3-batch to 5-batch operation. Candidates who made the effort to calculate the availability in each case (easily done) did not make this mistake.
There were many very strange implementations of the transition from 3-batch to 5-batch operation envisaged by candidates in trying to answer part (d). If subsequent calculations were correct then substantial partial credit was given, but many calculations failed to account correctly for the changes in batch size in determining partial reactivity contributions and/or did not keep accurate track of the burnup accumulated by individual batches.
In discussing the operational implications, very few candidates recognised the potential impact on burnup limits of the long dwells required of some assemblies.

Q4

(a)   Hold-up and decay is the simplest and cheapest method of liquid waste treatment suitable for short half-life nuclides. It can also be used for some medium-term half-life nuclides if the quantities are low. It simply involves storing the waste for sufficient time for the activity to decay to a safe level before discharge to the environment. At least two tanks are normally needed, one filling and one holding the material for the required period of time. Care may be needed with some daughter products as they can be more radioactive than the parent.

Ion exchange using either organic or inorganic media is very useful for some longer-lived nuclides. In this process the active ions such as cobalt-60 are exchanged for non-active ions such a sodium in a process very similar to that used in water treatment. Unlike conventional ion exchange, the media is not regenerated when saturated but is usually encapsulated for long-term disposal. Decontamination factors of between 10 and 100 are possible with some nuclides, but not all are suited to this form of treatment. It is relatively cheap and simple. The only problem being the long term disposal of the spent media.

The most intractable, usually long half-life, wastes may be treated by evaporation which produces a pure distillate that can be discharged to the environment and a highly active concentrate that can be immobilised by encapsulation or vitrification. It is very expensive in capital costs and also very energy intensive and can give rise to high operator dose rates. It is normally only used for very difficult materials such as reprocessing wastes or in locations where no radioactive discharge to the environment is permitted.   [30%]

(b)   Flow rate Q $0.063 \, \mathrm{m^3 hr^{-1}}$
      Collection time T 240 hrs
      Hold-up time   0 hrs
      Density ρ $1000 \, \mathrm{kg \, m^{-3}}$

Total collected volume $V = 0.063 \times 240 = 15.12 \, \mathrm{m^3}$

$$\lambda_{\mathrm{Sr\text{-}91}} = \frac{\ln 2}{9.5} = 0.07296 \, \mathrm{hr^{-1}}$$

$$\lambda_{\mathrm{Y\text{-}91}} = \frac{\ln 2}{58.5 \times 24} = 4.937 \times 10^{-4} \, \mathrm{hr^{-1}}$$

For Sr-91 $$P_{\mathrm{Sr\text{-}91}} = \frac{QA_{\mathrm{Sr\text{-}91}}\rho \times 1000[\mathrm{g/kg}] \times 3600[\mathrm{s/hr}]}{\lambda_{\mathrm{Sr\text{-}91}}}$$

$$\therefore \quad P_{\mathrm{Sr\text{-}91}} = \frac{0.063 \times 29.8 \times 1000 \times 1000 \times 3600}{0.07296} = 9.263 \times 10^{10} \, \mathrm{atoms \, hr^{-1}}$$

For Y-91 $$P_{\mathrm{Y\text{-}91}} = \frac{QA_{\mathrm{Y\text{-}91}}\rho \times 1000[\mathrm{g/kg}] \times 3600[\mathrm{s/hr}]}{\lambda_{\mathrm{Y\text{-}91}}}$$

$$\therefore \quad P_{\mathrm{Y\text{-}91}} = \frac{0.063 \times 0.973 \times 1000 \times 1000 \times 3600}{4.937 \times 10^{-4}} = 4.470 \times 10^{11} \, \mathrm{atoms \, hr^{-1}}$$

Decay of Sr-91 $$N_{\mathrm{Sr\text{-}91}} = \frac{P_{\mathrm{Sr\text{-}91}}}{\lambda_{\mathrm{Sr\text{-}91}}}\left[1 - \exp(-\lambda_{\mathrm{Sr\text{-}91}}T)\right]$$

$$\therefore \quad N_{\mathrm{Sr\text{-}91}} = \frac{9.263 \times 10^{10}}{0.07296}\left[1 - \exp(-0.07296 \times 240)\right] = 1.270 \times 10^{12} \, \mathrm{atoms}$$

The specific activity of an effluent component is given by

$$A = \frac{N\lambda}{V\rho \times 1000[g/kg] \times 3600[s/hr]}$$

$$\therefore \quad A_{Sr-91} = \frac{1.270 \times 10^{12} \times 0.07296}{15.12 \times 1000 \times 1000 \times 3600} = 1.702 \text{ Bq g}^{-1}$$

So, after ion exchange (DF10) $A_{Sr-91} = 0.1702 \text{ Bq g}^{-1}$

For Y-91

From the original Y-91 $N_{Y-91} = \frac{P_{Y-91}}{\lambda_{Y-91}}\left[1 - \exp(-\lambda_{Y-91}T)\right]$

$$\therefore \quad N_{Y-91} = \frac{4.470 \times 10^{11}}{4.937 \times 10^{-4}}\left[1 - \exp(-4.937 \times 10^{-4} \times 240)\right] = 1.012 \times 10^{14} \text{ atoms}$$

From the decay of Sr-91, using Bateman's equation (4M16 data sheet page 8)

$$N_{Y-91} = \lambda_{Sr-91}P_{Sr-91}\left[\frac{1 - \exp(-\lambda_{Sr-91}T)}{\lambda_{Sr-91}(\lambda_{Y-91} - \lambda_{Sr-91})} + \frac{1 - \exp(-\lambda_{Y-91}T)}{\lambda_{Y-91}(\lambda_{Sr-91} - \lambda_{Y-91})}\right]$$

$$\therefore \quad N_{Y-91} = 0.07296 \times 9.263 \times 10^{10}\left[\frac{1 - \exp(-0.07296 \times 240)}{0.07296(4.937 \times 10^{-4} - 0.07296)}\right.$$

$$\left. + \frac{1 - \exp(-4.937 \times 10^{-4} \times 240)}{4.937 \times 10^{-4}(0.07296 - 4.937 \times 10^{-4})}\right]$$

$$\therefore \quad N_{Y-91} = 1.983 \times 10^{13} \text{ atoms}$$

So, the total Y-91 $N_{Y-91} = 1.012 \times 10^{14} + 1.983 \times 10^{13} = 1.210 \times 10^{14}$ atoms

$$A_{Y-91} = \frac{N_{Y-91}\lambda_{Y-91}}{V\rho \times 1000[g/kg] \times 3600[s/hr]}$$

$$\therefore \quad A_{Y-91} = \frac{1.210 \times 10^{14} \times 4.937 \times 10^{-4}}{15.12 \times 1000 \times 1000 \times 3600} = 1.097 \text{ Bq g}^{-1}$$

So, after ion exchange (DF10) $A_{Y-91} = 0.1097 \text{ Bq g}^{-1}$

And the total activity is $A_{tot} = A_{Sr-91} + A_{Y-91} = 0.1702 + 0.1097 = 0.2799 \text{ Bq g}^{-1}$

The collection process significantly reduces the activity of the Sr-91, thanks to that isotope's short half-life in comparison to the filling time, but has negligible effect on the activity of the Y-91, due to that isotope's long half-life compared to the filling time – indeed the Y-91 activity actually goes up due to the decay of Sr-91.

The ion exchange part of the process has the same (large) effect on both isotopes, of course.   [70%]

Assessor's Comments:

All candidates: 31 attempts, Average raw mark 9.2/20,   Maximum 17, Minimum 1.

Comfortably the least popular question, attempted by only 32% of candidates, and the least well done.

Many answers to part (a) discussed waste storage rather than waste treatment methods. Some of the treatment methods described were not appropriate for liquid wastes, and a surprising number of answers did not mention hold-up and decay and/or ion exchange, despite the clues of their mention in part (b).

Part (b) was, in general, not well done, despite the similarity of the calculations required to one of the Examples Paper questions.

Calculations were sometimes on the right lines but missing important aspects, like consideration of the production of Y from Sr.

Many candidates were confused about units and/or failed to specify the units of quantities found, leading to confusion and error.

The omission of numerical working in several answers made it difficult to identify the source of errors and assign partial credit.

Many candidates calculated the necessary decay constants in units of s^{-1} rather than hr^{-1}, complicating matters considerably.

A couple of candidates made good attempts at working from first principles rather than following the methodology taught in lectures.

Several answers revealed a lack of understanding of the hold-up and decay terms of Bateman's equation.

Comparatively few comments on the effectiveness of the process took into account what one would expect given the half-lives of the isotopes being treated.